Course 2: Channels
"You see, wire telegraph is a kind of a very, very long cat. You pull his tail in New York and his head is meowing in Los Angeles. Do you understand this? And radio operates exactly the same way: you send signals here, they receive them there. The only difference is that there is no cat."

Albert Einstein
Agenda

- Transmission media
- Transmission impairments:
  - Attenuation
  - Delay Distortion
  - Noise
- Channel capacity
  - Nyquist theorem
  - Shannon theorem
Transmission media

- Guided - wire
- Unguided - wireless
- Characteristics and quality determined by medium and signal
  - For guided, the medium is more important
  - For unguided, the bandwidth produced by the antenna is more important
- Key concerns are data rate and distance
Transmission impairments

- The received signal may differ from the transmitted one
- Analog - degradation of signal quality
- Digital - bit errors may occur
- Caused by
  - Attenuation and attenuation distortion
  - Delay distortion
  - Noise
Attenuation

- Signal strength decreases with distance
- Depends on the transmission medium
- Received signal strength:
  - must be high enough, allowing the signal to be detected
  - must be sufficiently higher than noise, such that the signal to be received accurately (Solution: repeaters, amplifiers)
  - higher the transmission frequency, higher the attenuation is (mainly concerns the analog signals)
The frequency dependency of the attenuation versus distance curve is particularly upsetting for analog data. Fig. 1 shows the specific attenuation curve for an analog telephone channel, whose frequency band spans from 300 to 3400 Hz. As for digital data, most of their energy is concentrated around a “fundamental frequency”, which is, basically, equal to the data rate. Consequently, the higher frequency components have lower energy and the frequency dependency of the attenuation has no meaningful impact.

Notice that in the figure the attenuation uses as a reference the signal power at 1000 Hz, and, consequently, both curves cross zero dB at that frequency.

For the un-equalized curve, the attenuation is much higher towards the superior edge of the dedicated band, which causes degradation of the voice quality. The equalized curve is much smoother, meaning that all the frequency components are similarly attenuated during the transmission. Such an equalization can be performed using amplifiers, that amplify more the higher frequency components and less the lower frequency part of the signal.
Delay distortion

- Specific to guided media (wires)
- Signal propagation speed depends on the frequency
- Frequency selectivity arises: various frequency components of the signal will arrive at receiver with different delays
- A kind of Inter Symbol Interference (ISI) occurs
- Particularly annoying for digital data
The so called “delay distortion” actually means that the velocity of propagation of different frequency components of the transmitted signal is a function of frequency. So, we deal again with a kind of “frequency selectivity” of the channel. For a band limited signal, the propagation speed tends to be higher in the center of the dedicated bandwidth and lower towards its edges. Thus, various frequency components arrive at reception with different delays and thus distort the signal. Equalization techniques must be used to alleviate the negative effect of the delay distortion.
Noises

- Definition: “unwanted signals that are inserted somewhere between transmission and reception”
- Four categories:
  - Thermal noise
  - Intermodulation noise
  - Crosstalk
  - Impulse noise
Thermal noise

- Thermal noise
  - Generated by the thermal agitation of electrons
  - Uniformly distributed in frequency
  - Generally modeled as white noise
- The amount of thermal noise in 1Hz
  \[ N_0 = kT \]
  \[ k = 1.3803 \cdot 10^{-23} \text{ J/}^\circ\text{K} \]
  \( N_0 \) is the power spectral density [Watts/Hz]
- The amount of thermal noise in W Hz
  \[ N = kTW \]

The thermal noise is present in all electronic circuits and transmission media. Generally, it is mathematically described as a "white noise", i.e. a noise whose power is uniformly distributed over the whole frequency bandwidth. This is the most common type of noise considered when modeling systems, and there is a huge amount of literature (information theory, coding, data transmission) built on the hypothesis that the noise that affects the signal is a white noise.

The power spectral density (PSD) of a white noise is constant and equals \( N_0/2 \) (if the representation considers the negative part of the frequency axis too), where \( N_0 \) depends on the temperature.

Notice that, larger the bandwidth, larger will be the amount of thermal noise “seen” by the receiver. We may therefore say that larger bandwidth transmissions are more affected by the thermal noise, compared to the narrower bandwidth transmissions.
Other types of noises

- **Intermodulation**
  - Produces components having frequencies $f_1 + f_2$ and $f_1 - f_2$
  - Caused by non-linearity of the channel's transfer function
- **Crosstalk**
  - A signal from one line is picked up by another
  - Electrical coupling between nearby twisted pairs
- **Impulse**
  - Irregular pulses or spikes
  - *e.g.* External electromagnetic interference
  - Short duration and high amplitude
  - Important source of errors for the digital signals

**Inter-modulation noises** occur because of the non-linearity of the transfer function in transmitter, channel and/or receiver. Normally, they can be modeled as linear circuits, i.e. the system's output will be a linear combination of the components of the input signal. But, because of malfunctioning, or, for example, because of excessive signal strength, the output will not satisfy the linearity condition. This is the scenario when inter-modulation occurs, mainly producing frequency components which are the sum and the difference of the input frequency components.
Crosstalk noise

- Far end and near end crosstalk
- Near end crosstalk predominates
- Near end crosstalk falls off with frequency

\[ a_p = 20 \lg \left( \frac{U_{1,4}}{U_{2,4}} \right) + 10 \lg \left( \frac{Z_1}{Z_2} \right) \]

Crosstalk is an effect that occurs between neighbor pairs of wires. The electrical current flowing through one wire will create an electromagnetic field which is captured by the neighbor wire. Physically, this is very similar to the radio transmissions: one wire acts as a transmitting antenna, whereas the other one will be the receiving antenna.

Near End Crosstalk (NEXT) is measured at one end only of a cable, by transmitting a signal into one pair and measuring the resulting signal power on an adjacent pair at the same end.

Far End Crosstalk (FEXT) is measured at both ends of a cable, by transmitting a signal into one pair at one end and measuring the resulting signal power on an adjacent pair at the other end.
Phase jitter

- Definition: a random distortion of signal durations caused by the rapid fluctuation of the frequency of the transmitted signal
- May have different meanings, depending on the application
- Example: the difference (in periods) between two successive clock cycles, the difference (in phase) between the initial phase of the carrier for two transmitted symbols
- Causes: imperfections of the transmission media, the noise of the electronic devices used
Other noises

- **Fluctuation noise**: caused by the power supply networks, radio stations etc
- Uniformly distributed in the useful bandwidth
- **Oscillation noise**: parasite harmonics of 50Hz
- **Pulse noise**: issued from crosstalk (pulses transmitted in the neighbor lines) or because of the switches from the telephone exchange
Other distortions

- **Frequency deviation** of the oscillator from the receiver, compared to the transmitter
- **Echoes**: at the transitions between 2 wires and 4 wires
- Counteracted by echo suppressors (echo attenuations >19dB)
- **Short duration cuts** of the signal, caused by power supply back off activation, redundancy mechanisms in case of failure
- They are defined as a decrease of at least 6dB of the signal level, for a duration ranging from 3 to 300 ms
Channel capacity

- Definition: the rate at which data can be transmitted over a given communication path, under given conditions
- Four important concepts in defining capacity
  - Data rate
    - In bits per second
    - Rate at which data can be transmitted
  - Bandwidth
    - In Hertz
    - Constrained by transmitter (regulations) and medium Noise
  - Noise
  - Bit Error Rate (BER)
Nyquist formulation

- For noise-free channels:
  \[ C = 2W \log_2 M \]
- \( W \) is the bandwidth, \( M \) is the number of signaling levels
- Question: What is the capacity of a telephone line modem that uses 8 signaling levels?
- Answer:
  \[ C = 2 \cdot 3100 \cdot 3 = 18600 \text{ bps} \]

For a noiseless channel, Nyquist formula defines the theoretical maximum bit rate. According to this formula, there are two means to enhance the data rate. First, if we use a large number of signaling levels \( M \), the capacity increases logarithmically. But, there are practical limitations when choosing \( M \). Thus, the receiver will decide much easier if it has to distinguish between only two signaling levels (\( M=2 \)) than in the case of a much larger number of levels (e.g. \( M=64 \)). By the other hand, another way to increase the data rate is by increasing the bandwidth. As already seen, bandwidth is always a scarce resource, limited by physical constraints and regulations.

Another example: We need to send information at a data rate of 256 kbps, over a noiseless channel with 16KHz of bandwidth. How many signaling levels do we need?

Answer: \( \log_2 M = C/2W = 256 \times 10^3/2 \times 16 \times 10^3 = 8 \). It follows that \( M = 2^8 = 256 \) levels.

Comment: This shows quite a large number of levels which are required, and the receiver task is difficult. The reason behind is that, through quite a low bandwidth we try to send a fairly high data rate.
Shannon’s theorem represents one of the most famous elaborations from the scientific world, perhaps comparable with Einstein’s theory of relativity. This theorem lies at the basis of hundreds of books that founded a new scientific direction, called information and coding theory.
Formulation

**Theorem 2:** Let \( P \) be the average transmitter power, and suppose the noise is white thermal noise of power \( N \) in the band \( W \). By sufficiently complicated encoding systems it is possible to transmit binary digits at a rate

\[
C = W \log_2 \left( \frac{P + N}{N} \right)
\]

(19)

with as small a frequency of errors as desired. It is not possible by any encoding method to send at a higher rate and have an arbitrarily low frequency of errors.

- This is the original Shannon formulation.
- The widely used form is:
  \[
  C = W \log_2 \left( 1 + \frac{S}{N} \right)
  \]
- For high values of SNR
  \[
  C = \frac{W}{3} \frac{S}{N} \text{ [dB]}
  \]

Signal to noise ratio
The strength of Shannon’s theorem lies in its degree of generality, while its complexity is low. Basically, Shannon states that the data rate at which information can be sent through a channel is limited by the bandwidth and by the signal to noise ratio (SNR). By its formulation, Shannon defined a theoretical capacity bound that describes the maximum theoretical bit rate which can be achieved through a channel with certain characteristics. This bound is taken as a reference point, and, for example, the performance of an error correcting code is largely evaluated by comparing the experimental BER with the theoretical error probability issued from Shannon’s law.

Notice that, according to this theorem, the main resource of a communication system (that can increase the capacity) is the bandwidth \( W \): if the bandwidth increase \( N \) times, the same happens with the capacity. Another valuable resource is the signal power, \( S \), but the capacity increase with respect to \( S \) is only logarithmic.
Notice that Shannon’s capacity formula takes into account only the presence of the thermal noise, generally modeled as as white Gaussian noise. Although most of the information theory relies on this assumption, in practice we oftentimes meet other types of noises that perturb the useful signal. If Shannon’s formula still draw a maximal capacity curve, in the case where the signal is perturbed by other noises than the thermal noise, the performance of the system is further reduced.
Transmission efficiency

- C/W represents the transmission efficiency [bits/s/Hz]
- C/W is a critical parameter in any transmission system
Further parameters: $E_b/N_0$

- $E_b/N_0$ is the energy per bit to noise power density per Herz [Joule/Watt/Hz]

\[
\frac{E_b}{N_0} = \frac{ST_b}{N_0} = \frac{S}{N_0 R} = \frac{S}{kTR}
\]

- BER is a decreasing function of this ratio
- $E_b/N_0$ does not depend on the bandwidth, while SNR does (because $N=WN_0$)
- If $R$ increases, $E_b/N_0$ will degrade, impacting the error probability

In the formula above, the energy utilized for the transmission of one bit is calculated as the product between the bit duration ($T_b$) and the power dedicated for the transmission of the bit. Notice that, unlike SNR, $E_b/N_0$ is independent of the allocated bandwidth (does not depend on $W$). Nevertheless, this ratio depends on the data rate $R$: higher the data rate is, lower will be the quality of the transmission (i.e.: lower will be $E_b/N_0$). This illustrates the trade-off that must be made between the data rate we wish to achieve and the quality of the transmission.
Exercises

- What is the channel capacity for a teleprinter channel with a 300Hz bandwidth and a signal-to-noise ratio of 3dB?
- A digital signaling system is required to operate at 9600bps.
  - If a signal element encodes a four-bit word, what is the minimum required bandwidth of the channel?
  - Repeat previous computation for an eight-bit word.

SNR=3dB => its linear value (which is used in Shannon’s formula) is SNR=2.
The result is $C=300\log_2(1+2)=475$ bps

$C=2W\log_2 M$. In the first example $M=16$ (we need to encode 4 bits symbols).
$W=C/(2\log_2 M)=9600/8=1200$Hz.
For the second example $M=2^8=256$ signaling levels. The final result is $W=600$Hz.