Course 11: OFDM
Orthogonal Frequency Division Multiplexing

Agenda
- History of multi-carrier modulations
- The need for OFDM
- OFDM principles (parallel transmission, orthogonal carriers, IFFT modulator, cyclic prefix)
- OFDM: drawbacks and challenges
- OFDM in “real life”
**History**

- FDM = Frequency Division Multiplexing: a way to share a single transmission channel between several users

  - Every user has a dedicated bandwidth, guard intervals required
  - Rather a multiple access/multiplexing technique than a modulation
  - Used in PSTN, for the multiplexing of telephone calls

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**History of OFDM**

**The step towards OFDM**

- Kineplex receiver (1958): 20 orthogonal carriers are used to simultaneously convey data over a frequency selective radio channel
- A single transmitter !!!
- Bandwidth saving achieved, but prohibitive complexity
- The orthogonal carriers are very difficult to generate by analog circuits
The need for OFDM

- Most of the transmission channels are frequency-selective
  - frequency components from the input signal affected differently by the channel
  - ...meaning that the channel’s transfer function $H(f)$ is not flat over the bandwidth
  - this introduces Inter-Symbol interference (ISI)
  - ISI can be seen as a time-domain manifestation of the frequency selectivity
- OFDM is exceptionally robust to ISI
- Well suited to all the dispersive channels, and especially to the wireless channel

ISI: why so critical?

- In a large number of cases, the ISI is the most annoying phenomenon affecting the digital transmission
- E.g.: in a radio channel, the receiver gets multiple copies of the transmitted signal, with different delays and attenuations (=> time “spreading” of the signal)
- Example: single-carrier transmission at $R_S=10$Mbps
  - $T_S=1/R_S=0.1\mu s$
  - Multipath delay (time spreading): $10\mu s$

ISI for 100 symbols !!!
OFDM’s “trick”

- The symbol duration is increased, to be much longer than the multi-path delay
- N low-rate parallel flows transmitted
  \( R_p = 1/(NT_S) \) the data rate of each flow
  - Symbol duration: \( T = 1/NT_S \)
  - The overall rate is preserved (like in the single carrier case):
    \( R_S = NR_p \)
- Reminder: \( B \approx 1/T_S \)
- E.g: if \( N=1000 \) and \( T_S = 0.1 \mu s \), \( T = 100 \mu s \)

The transmitted symbol is ten times longer than the multipath delay!!!

Parallel transmission on multiple carriers

- Longer symbol duration on each parallel carrier
- The multiple carriers must be orthogonal

Principle of the parallel transmission on multiple carriers
Comments on the previous slide

- The direct effect of increasing the symbol duration is that its bandwidth decreases correspondingly.
- In OFDM, data is transmitted through a large number of narrow-band streams.

Orthogonal carriers

- The OFDM carriers are orthogonal, their frequencies being \( f_0, 2f_0, 3f_0 \) etc.

\[
\frac{2}{T} \int_{kT}^{(k+1)T} \sin(mf_0 t) \cdot \sin(nf_0 t) \cdot dt = \begin{cases} 
1, & \text{if } m = n \\
0, & \text{if } m \neq n
\end{cases} \tag{1}
\]

- Complex exponentials of limited duration used in practice
  - Their duration equals OFDM’s symbol time (T)
- The orthogonality is met if: \( f_0 = \frac{1}{T} \)
OFDM: the carriers and their spectra

In the time domain, every carrier covers an integer number of cycles (periods) during the symbol time \( T \)
- This is a condition for orthogonality

The carriers are time-bounded by a rectangular window, giving the symbol duration

The sinc shape of the spectrum corresponds to a sine carrier multiplied by a rectangular time window

At the central frequency of each carrier, all the other carriers cross zero
- This is the orthogonality frequency view
- Carriers must be separated by \( 1/T \) on the frequency axis
OFDM signal generation

- The signal corresponding to an OFDM symbol:

\[ s_s(t) = \frac{1}{N} \sum_{n=0}^{N-1} A_n e^{j(\omega_n t + \varphi_n)}, \text{ pentru } t \in [kT, (k+1)T] \] (2)

- If \( s_s(t) \) is sampled every \( T_S \) seconds:

\[ s_s(kT_S) = \frac{1}{N} \sum_{n=0}^{N-1} A_n e^{j\varphi_n} e^{jn\Delta \omega k T_S}, k = 0,\ldots,N-1 \] (3)

- Inverse Fast Fourier Transform

\[ g(kT_S) = \frac{1}{N} \sum_{n=0}^{N-1} G \left( \frac{n}{N T_S} \right) e^{jn2\pi k/N} \] (4)

Comments on the previous slide

- The data symbol to be transmitted (from eq. 2) is: \( X[n] = A_n e^{j\varphi_n} \)
  - This symbol corresponds to a certain modulation scheme (e.g. BPSK, QPSK, QAM etc)
- The signal from (2) corresponds to a single OFDM symbol (the time spanning is \( T \))
- In equations (3) and (4), \( T_S \) is the sampling time, and it matches the duration of the serial symbol to be transmitted
- Eq. (3) and (4) are equivalent if: \( A_n e^{j\varphi_n} = G \left( \frac{n}{N T_S} \right) \text{ and } \Delta \omega = \frac{2\pi}{T} \)
- The data symbols to be transmitted can be regarded as complex valued “frequency-domain samples”
- Eq. 4 is very close to IFFT’s formula
IFFT modulator

- A discrete version of the OFDM symbol is obtained by applying IFFT to the data sequence to be transmitted

\[
x[n] = \sum_{k=0}^{N-1} X[k] \cdot e^{\frac{jk2\pi}{N}n}, \quad (1.5)
\]

\[
x[n] = x[0] \cdot e^{\frac{j02\pi n}{N}} + x[1] \cdot e^{\frac{j2\pi}{N}n} + \cdots + x[N-1] \cdot e^{\frac{j(N-1)2\pi n}{N}}
\]

Comments on the previous slide

- FFT is a fast algorithm for the DFT implementation
- The orthogonal carriers are the complex exponentials
- Usage of IFFT cancels the need for analog oscillators generating the orthogonal carriers
- \(N\) is the total number of (multiple) carriers used in transmission
- Demodulator uses the direct transform (FFT)

OFDM signal is generated in the baseband by signal processing only!!!
Cyclic prefix: why?

- If the channel is not ideal (is time-dispersive), the successive OFDM blocks will interfere.
- This happens because each symbol is dispersed in time by the channel (see slide 6).

Inter-Block Interference (IBI) arises:
- Sometimes referred to as Inter-(OFDM)Symbol Interference.

OFDM’s cyclic prefix:

- Sometimes referred to as guard time.
- Cancels IBI.
- Simplifies equalization.
- Eases synchronization (a signal is always transmitted).
- Cyclic Prefix is removed by the demodulator.
Comments on the cyclic prefix

- IBI cancellation
  - Occurs only if the CP is longer than the multipath delay
  - Longer CP duration means higher protection against ISI, but smaller efficiency
- Synchronization is simpler if a signal is always transmitted
  - E.g. a peak in the autocorrelation of the OFDM symbol could indicate the OFDM symbol start
- Equalization is simpler, because the linear convolution \((x[n] \ast h[n])\) is transformed into a periodic one
  - This allows to use a very simple, “one-tap”, frequency domain equalizer
  - Every received sample \(R[k]\), must be divided by \(H[k]\), the channel’s response at the \(k\)-th frequency line

The transmitter

- \(d_n\)
- \(N\) data symbols: (in frequency-domain)
- Serial-to-parallel
- IDFT (IFFT)
- Parallel-to-serial
- Base-band signal (time-domain)

- \(k\) is the index of the OFDM symbol
- The base-band signal is converted to an analog signal, and modulates a high-frequency carrier before being transmitted
- \(N\) is a power of 2 (256, 512 etc)
OFDM: drawbacks and challenges

- High sensitivity to time and frequency synchronization errors
  - The channel “attacks” the carriers orthogonality
- High value of Peak-to-Average-Power Ratio (PAPR) (highly variable envelope)
- Overhead due to CP
- Insufficient out-of-band attenuation
OFDM in real world

- Most of the wireless transmission standards
  - WiMAX (IEEE 802.16), WiFi (IEEE 802.11), LTE (3GPP release 8), DAVB
  - Proprietary solutions: Flash OFDM (Flarion)
- Wired broadband access
  - xDSL
- Data transmission through power line (PLC = Power Line Communications)

Acknowledgement

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*Orthogonal Frequency Division Multiplexing (OFDM): Concept and System-Modeling*