Word Error Rate Statistics of a DFT-based MCM System in FSF Channels

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ABSTRACT

The BER (Bit Error Rate) and WER (Word Error Rate) of the frequency selective channels are analysed using a MCM (multicarrier modulation) technique, commonly referred to as OFDM (Orthogonal Frequency Division Multiplexing), and DBPSK modulation. The equalization technique is based on cyclic prefix added to the DFT based system, in order to combat ISI (the intersymbol interference). We considered channels with a gaussian noise and both Rayleigh and Rice fading conditions.

I. INTRODUCTION

Mobile radio communication systems are increasingly required to offer a variety of services and qualities for mobile users. Then, the modern mobile radio transceivers must be able to provide high capacity and variable bit rate (VBR) information transmission with high bandwidth efficiency. However, the radio channel signals are usually affected by fading and multipath delay spread phenomenon. Severe fading of the signal amplitude and intersymbol interference due to the frequency selectivity of the channel cause an unacceptable degradation of the system performance.

In such channels, the fading mechanism can be equally well characterized in both time and frequency domain [SKL ’97]. In fig. 1a, a multipath intensity profile, $s(\tau)$, versus time delay (\(\tau\)) is plotted. The term “time-delay” is used to refer to the excess-delay, representing the propagation delay of the signal that exceeds the moment of the first signal component arrival to the receiver. For a single transmitted impulse, the time $T_m$, between the first and the last received component, represents the maximum excess delay, during which the multipath signal power falls to some threshold level below that of the strongest component. For an ideal system (zero excess delay), the function $s(\tau)$ would consist of an ideal impulse, with weight equal to the total average received signal power.

A channel is said to exhibit frequency-selective fading if $T_m>T_s$. This condition is achieved if the received multipath components of a symbol arrive beyond the symbol duration. This category of fading degradation is called channel-induced ISI. A channel exhibits non selective, or flat fading if $T_m<T_s$. In this case, there is no channel-induced ISI distortion, but there still is a performance degradation, since the multipath components can add up destructively, to yield a reduction in SNR.

A completely analogous characterization of signal dispersion can be made in frequency domain (fig. 1b). $R(\Delta f)$ represents the Fourier transform of $s(\tau)$. A channel is referred to as frequency-selective, if $f_0<1/T_s$, where $1/T_s$ is considered the signal

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bandwidth. Thus, the FSF (frequency-selective fading) occurs when the spectral components of a signal are not equally affected by the channel. If \( f_0 > 1/T_s \), the channel exhibits flat fading. The techniques used in the single carrier mobile communication systems - channel coding and adaptive equalization- to combat fading and multipath propagation are practical difficult to use at high bit rate, due to the delay in coding and equalization process and to the high cost of hardware. The multicarrier modulation technique, commonly referred as OFDM is a main modulation scheme for digital audio and video broadcasting to avoid ISI introduced by frequency selective multipath fading. The basic idea behind this scheme is to spread-out the effect of fading over many bits. Rather then a few adjacent bits completely destroyed, we now have all the bits only slightly affected by the fading. In order to attend this goal, high-rate data are sent in parallel on a number of narrowband ISI free subchannels, each subchannel operating at a low data rate, to avoid the channel frequency selectivity. Adding a cyclic prefix for each block of data, the ISI can almost completely be avoided. The circular prefix must be longer that the maximum excess delay. Each carrier of OFDM signals is modulated with M-ary differential phase shift keying (MDPSK).

In this paper we study the BER and WER performances of OFDM-DBPSK in a frequency selective Rician and Rayleigh fading channel, rarely reported in the literature. Techniques for channel equalization based on redundant filter bank precoder have been introduced and well-situated in recent years [SGB `97, XIA `97]. The cyclic prefix system with DFT (Discrete Fourier Transform) matrices, which is commonly employed in discrete multitone (DMT) systems for twisted pair channels in telephone cables [BIN `90] can actually be used for the equalization of a much broader class of channels. One advantage of the method is that besides FFT, there is very little computation involved, and therefore the method is very efficient.

If a channel has zeros in \( |z| \geq 1 \) domain then there are some problems with traditional equalization: the channel noise can get severely be amplified. The cyclic prefix method does not require the channel to be minimum-phase. Equalization does not severely amplify noise as long as the zeros of the channel are not too close to the unit circle. The advantages are obtained at the expense of a slightly higher bandwidth expansion ratio compared to [SGB`97], but the expansion becomes negligible as the block length increases.

II. CYCLIC PREFIX

We assume that the channel is an L-th order FIR system:

\[
C(z) = \sum_{n=0}^{L} c(n)z^{-n} \tag{1}
\]

with additive noise (fig. 2).

The symbol stream is divided into block of length M, and after that, L zeros are introduced at the beginning of each block like a guard interval. The all M+L sequence occupies the same time interval like original block, resulting that for a given symbol rate, zero-prefix reduces the spacing between samples (fig. 3).

The bandwidth expansion factor \( \gamma = (M+L)/M \) represents the excess bandwidth. By making M large enough, it can reduce \( \gamma \). From the measurement of output block - that depends on the input and the noise - it can recover the corresponding input block. Ignoring noise for the moment, we have for k-th block:

\[
\begin{bmatrix}
y(J_{k}) \\
y(J_{k+1}) \\
y(J_{k+M-1})
\end{bmatrix} = C_{\Delta} \begin{bmatrix}
s(J_{k}) \\
s(J_{k+1}) \\
s(J_{k+M-1})
\end{bmatrix} \tag{2}
\]

where \( J = k(L+M)+L, \ M>0, \) and

\[
C_{\Delta} = \begin{pmatrix}
c(0) & 0 & \cdots & 0 \\
c(1) & c(2) & \cdots & 0 \\
c(M-1) & c(M-2) & \cdots & c(0)
\end{pmatrix}
\]

This is a lower triangular Toeplitz matrix representing causal convolution. Assume that \( c(0) \neq 0 \) (it can extract delays from \( C(z) \) if it is necessary). Then \( C_{\Delta} \) is nonsingular and its inverse \( C_{\Delta}^{-1} \) is also a
lower triangular Toeplitz matrix, like $C\Delta$, having the terms $h(n)$ the first $M$ coefficients of the inverse:

$$\frac{1}{C(z)} = \sum_{n=0}^{\infty} h(n)z^{-n}$$

(3)

In practice the channel adds the noise so $y(n) \rightarrow y(n) + e(n)$ in the former relations and so $s(n)$ is not recovered exactly. This noise could be severely amplified by the inversion process if $C(z)$ has some zeros outside the unit circle.

Instead of using a zero-prefix, it can also use a cyclic prefix; this method performs even if the channel is not a minimum phase. It is sufficient, but not necessary that the channel be free from unit circle zeros [VV-2]. The L symbols at the end of each block are copied into the beginning of that block, to form the cyclic prefix (fig. 4).

This assumes $L\leq M$; if $L>M$, the matrix becomes circulant and eq. (2) becomes (4). The last input symbols $s(n)$ in the $m$-th block are related to the last $M$ output symbol $y(n)$ in the $m$-th block.

$$y(n) = Cs(n)$$

(4)

where:

$$s(n) = [s(mM), s(mM+1)…s(mM+M-1)]$$

$$y(n) = [y(m(L+M)), y(m(L+M)+1)…$$

$$……y(m(L+M)+L-1)],$$

$C$ is a circulant matrix with the element of top row coming from the channel impulse response $c(n)$; for example when the channel order is $L=3$ and $M=6$.

$$C = \begin{pmatrix}
  c(0) & 0 & 0 & c(3) & c(2) & c(1) \\
  c(1) & c(0) & 0 & 0 & c(3) & c(2) \\
  c(2) & c(1) & c(0) & 0 & 0 & c(3) \\
  c(3) & c(2) & c(2) & c(0) & 0 & 0 \\
  0 & c(3) & c(2) & c(1) & c(0) & 0 \\
  0 & 0 & c(3) & c(2) & c(1) & c(0)
\end{pmatrix}$$

If the channel is known, it can perform the equalization by inverting (4), assuming $C$ is nonsingular [VV-1]. The eigenvalues of the $M \times M$ circulant are equal to the DFT coefficients of the top row [PAP '97] in reversed order. These eigenvalues are:

$$\eta(k) = \sum_{n=0}^{M-1} c(n)W^{-nk}$$

where $W=e^{j2\pi k/M}$ and $C(e^{j\omega})$ represents the channel frequency response. Thus $\eta(k)$ are obtained by uniformly sampling $C(e^{j\omega})$ at $M$ frequencies. For $L<n<M$, $c(n)=0$. The circulant matrix can be diagonalized with DFT matrix:

$$C = W^{-1}\Lambda C W$$

and:

$$C_M[k]= \sum_{n=0}^{L} C(n)W^{nk} = M - \text{point DFT of } c(n)$$

is a permuted version of the eigenvalues $\eta(k)$. The implementation of the communication system can be represented as shown in fig. 5.

Fig. 4. Explanation of how cyclic prefix is inserted

Fig. 5. Block diagram description of the system based on cyclic prefix. (a) Transmitter, and (b) receiver

Fig. 6. (a) A simplified schematic of the cyclic prefix system, and (b) practically useful rearrangement similar to conventional DMT system
The diagonal elements of $\Lambda_c^{-1}$ are $1/C_M(k)$ and can be regarded as a set of DFT-domain equalizers. All the complexity is done at the receiver, but in fig. 6 the DFT is done at the transmitter as in DMT system. If the channel is known it can move also $\Lambda_c^{-1}$ and $W$ to the transmitter part, yielding a useful configuration for cases where the receiver has to be the simplest. If the channel is non-minimum phase, $C$ is non-singular and it can invert (4) to obtain the input symbol stream. In the cyclic prefix method, the DFT coefficients are bounded $|C_M[k]| \geq 1$, so that the diagonal elements $1/C_M[k]$ of the equalizer $\Lambda_c^{-1}$ do not amplify the noise.

### III. TRANSCEIVER SYSTEM AND FADING CHANNEL MODEL

The block diagram of the OFDM-DBPSK system is shown in fig. 8. The serial information data are first grouped into blocks of length $M$. The symbols are parallelized by the S/P converter and then applied to a binary differential encoder, being encoded in relative phase of symbols in the adjacent subchannels (consecutive symbols in a DFT frame). The data are passed to the IFFT modulator, a cyclic prefix of length $L < M$ is inserted in order to mitigate ISI distortion. The symbols are then converted to a serial stream.

The transmitted signal $s(t)$ is subjected to multipath fading and AWGN, so the received signal can be written as:

$$ r(t) = \int_0^\infty s(t-\tau)h(t,\tau)d\tau + n(t) $$

where $n(t)$ is a complex gaussian noise and $h(t,\tau)$ is the impulse response of the multipath fading channel at time $t-\tau$. The frequency selective Rician fading channel here is modelled as a 3-ray tapped delay line with one line of sight (LOS) path and two multipath components.

$$ h(t,\tau) = \sqrt{2P_s}\delta(t) + \sqrt{P_1}h_1(t)\delta(t-\tau_1) + \sqrt{P_2}h_2(t)\delta(t-\tau_2) $$

where $P_s$ is the power of LOS signal, $P_1$ and $P_2$ are the powers of the multipath signals, $\tau_1$ and $\tau_2$ are the delays of the first and second multipath respectively, and $0 < \tau_1 < \Delta < \tau_2$. $h_1(t)$ and $h_2(t)$ are independent slowly varying complex Gaussian random processes with maximum Doppler shift $f_m$ and are normalized as:

$$ E[|h_1(t)|^2] = E[|h_2(t)|^2] = 2 $$

$$ E[|h_1(t)h_2^*(t+T_s)|] = E[|h_2(t)h_1^*(t+T_s)|] = 2J_0(2nf_mT_s) $$

Since the multitone duration, $T_s$, is much longer than the serial symbol duration, $T$, the Doppler shift should not be ignored in (8) [LTA '90].

A parameter characterizing the nature of the Rician fading channel is the Rice factor defined as the ratio of LOS component power $P_s$ and the multipath components power $P_d = P_1 + P_2$, i.e. $K = P_s/P_d$. As special case the channel is AWGN channel (no multipath components) when $K \rightarrow \infty$, and a Rayleigh fading channel (no LOS component) when $K = 0$.

At the receiver side, the cyclic prefix is removed, the serial symbols are parallelized and applied to the FFT modulator. A differential detection allows the reconstruction of the transmitted symbols.
IV. MEASUREMENTS AND CONCLUSIONS

Fig. 8. Comparison between the BER performances of a flat fading channel and a two-ray FSF channel.

Fig. 9. Comparison of the WER between a flat fading channel and a two-ray channel, $M=64$, $f_m^*T_s=0.01$, $\rho=\tau/T=1$.

Fig. 10. BER performances of a flat fading channel and of a FSF channel with and without a BCH encoder.

Fig. 11. Comparison of the average WER for a two-ray fading channel, $M=64$, $f_m^*T_s=0.01$, $\rho=\tau/T=1$, for different SNR values.

Fig. 12. Comparison of the average WER between a Ricean fading channel and a Rayleigh fading channel, $M=64$, $f_m^*T_s=0.01$, $\rho=\tau/T=1$, $K=10$, SNR=20dB.

Fig. 13. Comparison of the average WER for a 3-ray Ricean fading channel, $M=64$, $f_m^*T_s=0.01$, $\rho=\tau/T=1$, $K=10$, for different SNR values.
From these simulations, it can be noticed that the BER of a 2-ray Rayleigh fading channel (we considered an equally amount of power for the two rays) is almost identical with that of a flat fading channel, until the values of SNR smaller than 22dB, when the BER becomes significantly better for the flat fading channel. Even if the BER of the two-ray channel is at best equal with the BER performances in a flat fading channel (fig.8) the average word error rate is better than that of a flat fading channel (fig.9). If a block code that can correct 6 errors in a block of 64 bits is employed, the frame error rate for the two-ray channel will be $10^{-3}$, almost then times better than the frame error rate for a flat fading channel. Using this conclusion, we emphasized a significant BER performance improvement (fig.10) when a BCH encoder that can correct 10 errors in a block of 64 bits is used, (a coding gain up to 15 dB for the FSF channel). The price of this spectacular improvement is, of course, a two time higher transmission rate needed in order to transmit the same amount of data, due to the redundancy introduced by the BCH encoder. A comparison of the average WER for different SNR values was made (fig. 11). The illustrated result is a normal WER improvement if the average SNR increases. The BER performance of a 3-ray (Rice fading) channel is better than that of a 2-ray Rayleigh fading channel (fig. 12), as we would expect, due to the presence of the LOS component. In figure 13, we plotted the average WER for a 3-ray Ricean fading channel, considering different SNR values. The performance improves when SNR increases. At a certain SNR, the WER performance of the 3-ray Ricean fading channel is better than that of the 2-ray Rayleigh fading channel.

References:


[VV-1] Vrcely B., Vaidyanathan P. P. “Pre - and post – processing for optimal noise reduction in cyclic prefix based channel equalizers”.

