

## Performance Evaluation of Single-Carrier Broadband Transmission with Frequency Domain Equalization

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**Abstract** – One of the most challenging problems in high-rate data transmissions is to combat the inter-symbol interference introduced by the multipath propagation. Since adaptive equalization in time domain proved oftentimes to be a too expansive solution in terms of computational complexity and implementation costs, some different equalization approaches must be considered. In this paper we shall investigate the performance of two recent-years findings in this direction, i.e. OFDM (Orthogonal Frequency Division Multiplexing) and SC-FDE (Single Carrier with Frequency Domain Equalization).

**Keywords:** OFDM, SC-FDE, equalization, wireless

### I. INTRODUCTION

Wireless transmission systems are increasingly required to provide high-rate data transmission with high bandwidth efficiency. In radio-channel communications, the transmitted signal is subjected to multipath fading caused by phenomenon that affect the radio-wave propagation: reflection, diffraction and scattering. As a result of these phenomenon, the transmitted signal arrives at the receiver following different paths, with different strengths and delays. Moreover, oftentimes in mobile radio communications, the receiver's mobility and the motion of the objects situated between transmitter and receiver determine the temporal variation of the multiple radio propagation paths. Consequently the radio channel is far from an ideal data transmission medium, exhibiting both frequency selectivity and time-variant characteristics.

Multipath propagation causes time dispersion of the signal. Thus, the energy corresponding to a symbol overlaps with the energy corresponding to the subsequent symbols, the inter-symbol interference (ISI) arising. One of the most challenging issues in radio communication is to overcome this phenomenon, which, moreover, especially impairs the transmission when high-data rates (short duration of the information symbols, respectively) are demanded.

The classical approach in order to mitigate this problem is the use of adaptive equalization techniques at the receiver, but there are practical difficulties in operating this equalization in real-time conditions at several Mb/s and with compact, low-cost hardware [1]. In fact, the different types of time-domain equalizer structures, e.g. the MLSE (Maximum Likelihood Sequence Estimator), the linear filter equalizer and the DFE (Decision Feedback Equalizer) all have the common problem that the computational complexity grows at least quadratically with the desired bit rate, becoming totally unattractive [2]. Habitually, these equalizers are used in classical single-carrier schemes, when the data symbols (amplitude and/or phase modulated pulses which will in turn modulate a sinusoidal carrier) are sequentially transmitted one-by-one, the frequency spectrum of each symbol allowed to occupy the entire available bandwidth.

In this paper we will study two of the recent years approaches that counteract the ISI phenomenon in a distinct manner: OFDM (Orthogonal Frequency Division Multiplexing) and SC-FDE (Single-Carrier with Frequency Domain Equalization). OFDM splits the high-rate data stream into a number of lower speed substreams, each of them modulating a different carrier. The multiple subchannels that carry the data do not interfere each other since the correspondent subcarriers are all orthogonal one-another. Modern Fast Fourier Transform (FFT) processing techniques are used to generate and demodulate the signal.

OFDM appears to offer a better performance/complexity trade-off than conventional SC modulation with time domain equalization, since the equalization complexity merely grows slightly than linearly with the bit rate [3].

Because the transmitted OFDM symbol is a sum of a large number of slowly modulated subcarriers, it has a high peak-to-average power ratio, even if a constant envelope modulation (such as BPSK or QPSK) is performed on each subchannel. This

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generates difficulties regarding the RF power amplifiers that must be used in a practical implementation. The transmitter power amplifier requires an increased power back-off for obtaining a wide range of linearity in order to faithfully reproduce all the peaks of the signal envelope, which will significantly increase the cost of this component.

SC-FDE is an approach that counteracts this inconvenience, keeping a low complexity of the whole equalization process, similar to that of OFDM. The data are transmitted using a single carrier with a classical modulation scheme, which will eliminate the problem of an expensive peak-to-average power ratio. The whole complexity of the equalization is distributed at the receiver and is still based on FFT processing. The basic mathematics of this approach are practically the same as in OFDM, as we will see in the next section. A frequency domain - linear or adaptive - equalization must be performed in order to counteract the inherent time-dispersive nature of the radio channel.

In the next section of the paper we will study the equalization concepts proposed by the two approaches, which moreover could efficiently be used simultaneously in a dual-mode. The BER performance of both methods is investigated in the section III by means of computer simulation. The relevant conclusions regarding the two methods are outlined in the final section.

## II. OFDM AND SC-FDE: BASIC CONCEPTS

### A. OFDM

OFDM represents an optimized version of the multicarrier modulation techniques. The finding of this approach was to replace a single-carrier serial transmission at a high data rate with a number of slower parallel data streams that will simultaneously modulate orthogonal carriers. By creating  $N$  parallel substreams, the bandwidth of the modulation symbol is reduced by the same factor, or, equivalently, the duration of the modulation symbol will be  $N$  times higher. The lengthening of the transmitted symbol will significantly reduce its sensitivity to ISI. The  $N$  parallel transmission channels do not interfere each other, since their correspondent subcarriers are orthogonal. This is in fact the basic idea that lies behind OFDM. The generation of the multiple carriers is done by performing Inverse Fast Fourier Transform (IFFT) processing at the transmitter. To the receiver, data are recovered using FFT processing, which extracts the subcarriers. In addition, a cyclic prefix is inserted in front of each symbol, in order to prevent two consecutive blocks to interfere because of the time-dispersive channel character and, furthermore, to facilitate the equalization process to the receiver. In the figure 1, the block diagram of an OFDM system is shown.

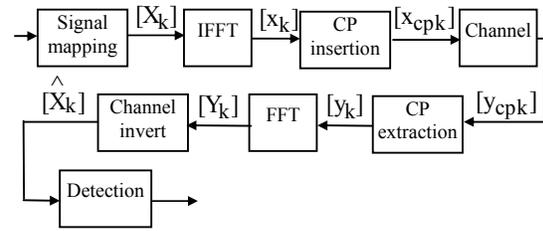


Fig.1: Block diagram of an OFDM system

The data are first grouped in blocks of  $M$  bits. Each  $m$  bits are then mapped to one of  $2^m$  complex valued symbols  $X_k$ ,  $k=0,1,\dots,N-1$  using a digital modulation scheme.

The  $N$  time domain signal samples forming an OFDM symbol ( $x_0, x_1, \dots, x_{N-1}$ ) are obtained through IFFT processing according to (1):

$$x[n] = \sum_{k=0}^{N-1} X[k] \cdot e^{jk \frac{2\pi}{N} n}, \quad n=0,1,\dots,N-1 \quad (1)$$

Afterwards,  $L-1$  cyclic prefix samples are added in front of the signal sequence, that becomes:

$[x_{cp}] = (x_{N-L+1}, x_{N-L+2}, \dots, x_0, \dots, x_{N-1})$ . This signal is analog converted and modulates a RF carrier, before being transmitted in the channel. If we consider the equivalent baseband discrete model of the channel as a FIR filter of order  $L$ , then the Z-domain channel response is given by:

$$H(z) = \sum_{n=0}^{L-1} h[n] \cdot z^{-n} \quad (2)$$

Since we must assume our channel to be time variant, its impulse response will depend on the time at which the impulse is applied. We shall assume, however, that the channel impulse response is static (will not change) for the duration of an IFFT frame.

The equivalent baseband signal at the channel output can be obtained by the well known operation of convolution:

$$y_{cp}[n] = x_{cp}[n] * h[n] \quad (3)$$

Discarding the  $L-1$  CP samples from the received sequence, the remaining (useful) signal can be expressed as:

$$y[n] = x[n] \circledast h[n] \quad (4)$$

where “ $\circledast$ ” denotes the circular convolution operator. The relation above is in fact the main reason that lies behind the use a cyclic prefix. Its insertion will transform the convolution between the data sequence and the channel impulse response into a circular convolution [4], which preserves the temporal support of the signal, thus avoiding the interference of two successive OFDM symbols due to the time dispersive

nature of the channel. If the cyclic prefix duration spans more than the multipath delay spread of the channel, the interference from the previous transmitted blocks is totally eliminated through this operation of CP insertion/extraction. Furthermore, the equalization process is facilitated at the receiver, a simple “channel inversion” allowing theoretically a perfect data recovering, as we will see next.

Since  $x[n]=\text{IFFT}\{X[k]\}$  and taking into account the FFT demodulator, the received symbols  $Y[k]$  can be expressed as:

$$Y[k]=\text{FFT}\{\text{IFFT}\{X[k]\} \otimes h[n]\} \quad (5)$$

But, the FFT of a circular convolution of two discrete time signals yields a spectral multiplication:

$$Y[k]=\text{FFT}\{\text{IFFT}\{X[k]\}\} \cdot \text{FFT}\{h[n]\} = X[k] \cdot H[k], \\ k = 0, 1, \dots, N-1 \quad (6)$$

where  $H[k]$  represents the sampled frequency response of the equivalent discrete channel, corresponding to the frequencies  $\Omega_k=k(2\pi/N)$ . The crucial consequence of the relation above is that the modulation symbol  $X[k]$  could be recovered at the receiver by a simple pointwise division operation, commonly referred to as a “one-tap frequency domain equalizer”. Thus, the CP theoretically eliminates both IBI (each block preserves its temporal support) and inter-carrier interference (ICI) (each serial symbol received on the  $k$ -th carrier will depend only on the corresponding  $k$ -th carrier transmitted symbol, not being affected by the adjacent carriers).

### B. SC-FDE

SC-FDE is an alternative equalization approach, which eliminates some of the OFDM disadvantages (especially the high peak-to-average power ratio), while keeping approximately the same low complexity. The block-diagram of a SC-FDE system is illustrated in the figure 2.

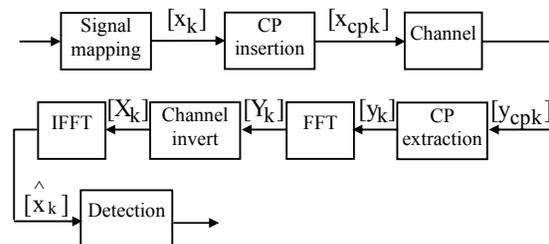


Fig.2: Block diagram of a SC-FDE system

The data are still transmitted in blocks of  $N$  samples, but this time the transmission is a classical serial single-carrier transmission. The data are encoded using a digital modulation scheme, obtaining the sequence  $[x_k]$ , where  $k=0, \dots, N-1$ . A cyclic prefix is added in front of each data block. If the cyclic prefix duration is longer than the multipath delay

spread of the channel, the interference introduced by temporal dispersion of the previous transmitted block is totally absorbed by the circular extension, which is discarded to the receiver. Moreover, since the equalization process is still based on FFT processing, the appearance of periodicity that cyclic prefix confers to the signal will facilitate this process. The mathematics of this method are essentially the same as for OFDM with the difference that in SC-FDE, all the equalization complexity is allocated to the receiver. The data block  $[y_k]$  arrived to the receiver is first FFT-processed, then the influence of the frequency-selective channel impulse response is eliminated by an simple channel inversion operation. In adaptive SC-FDE, the adaptation of FDE transfer function can be done using least mean square (LMS), root least square (RLS), or least-square minimization methods. An inverse FFT returns the equalized signal in the time domain prior to the detection of data symbols. In terms of complexity, for channels with severe delay spread, SC-FDE is simpler than time domain equalization, because equalization is performed on a data block at a time, using a computationally efficient FFT algorithm.

Furthermore, as proposed in [2], OFDM and SC-FDE could simultaneously be used with high efficiency in a transmission modem. It is easy to notice from the presented block-diagrams that the two types of systems mainly differ in the placement of IFFT operation. A system in which a radio modem can be configured to work in both OFDM and SC-FDE mode could be simply implemented by switching the IFFT block between transmitter and receiver. As observed in [2] since SC-FDE concentrates all the complexity to the receiver, such a system could be appropriated for an uplink, thus the most complex part of the communication issues being solved by the base station. Using OFDM in downlink will reduce the complexity of the processing that must be done by the mobile station. Such an arrangement has two obvious advantages: concentrates the main amount of processing in the base station and reduces the power consumption of the mobile station that uses a single-carrier mode for transmission and an OFDM mode for reception.

### III. EXPERIMENTAL RESULTS

BER performances of both OFDM and SC-FDE systems were studied by means of computer simulation. Both Rice (Line-of-Sight) and two-ray Rayleigh (Non-Line-of-Sight) conditions with perfect channel knowledge were taken into account in order to simulate the signal propagation in the radio channel. As a parameter of the simulation, the relative power of the two multipath components denoted by  $P_1$  and  $P_2$  in a Rayleigh fading channel was considered. The Rice factor  $K$ , defined as the ratio between the power of line-of-sight deterministic signal and the power of the multipath components is also modified during the simulations. The fading was modeled as

quasi-static that is it remains unchanged during the transmission of a data block. For the simulated coded transmissions, a BCH encoder with rate 1/2 was implemented. This block-code can correct 6 bits in a block of length  $N=64$ , as was taken in all the simulations. The binary information is mapped using two classical constant-envelope modulation schemes: QPSK and DBPSK respectively.

In the figure 3, a comparison of BER performances of OFDM and SC-FDE methods is illustrated. One can observe that for low values of SNR, OFDM performs generally better than SC-FDE, especially for coded transmission. When SNR exceeds the critical range of 12dB to 15dB, SC-FDE becomes more reliable, mainly when uncoded data blocks were transmitted. This confirms that, while desirable, the coding is not strictly imposed in single carrier method, unlike in OFDM where the coding is mandatory in order to combat the high amount of errors on the carriers attenuated by the frequency-selective channel response.

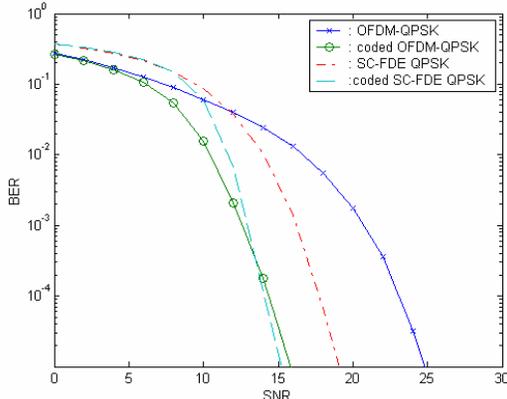


Fig. 3: A comparison of OFDM and SC-FDE with QPSK modulation, Rayleigh fading channel,  $P_1=P_2$

In the figures 4 and 5 the influence of the Rice factor ( $K$ ) and of the relative power of the second multipath ( $P_2$ ) on BER performance of both systems is aimed. It is obvious that the two systems behave

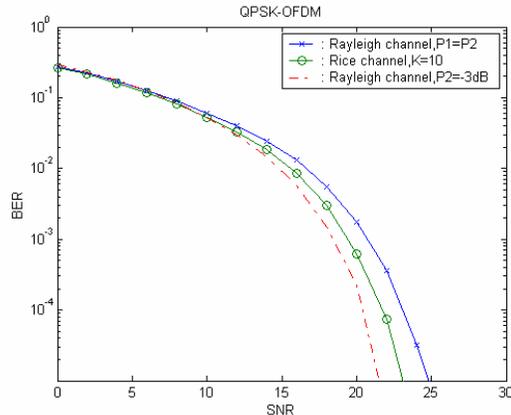


Fig. 4: The influence of  $P_2$  and  $K$  parameters on BER performance of an QPSK-OFDM system

almost identically in respect to these parameters. BER performance slightly improves when a LOS component is introduced, or when the relative power of the second multipath decreases. The effectiveness of the last parameter becomes clear especially for high SNR values, when the spread of the results is within about 1dB (for SC-FDE) and 4dB (for OFDM). Thus, the OFDM system proved to be slightly more sensitive to the considered parameters than the other system.

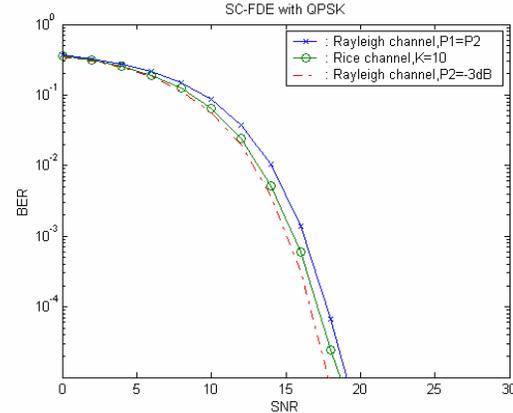


Fig. 5: The influence of  $P_2$  and  $K$  parameters on BER performance of a SC-FDE with QPSK system

The influence of the cyclic prefix length for the performance of both studied systems is shown in the figure 6. As mentioned previously, if the cyclic prefix

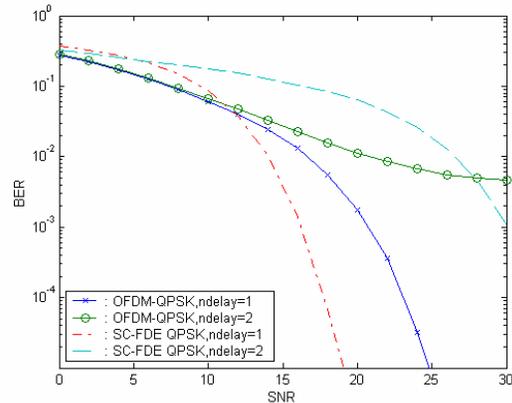


Fig. 6: The influence of  $ndelay$  parameter on BER performances of both OFDM-QPSK and SC-FDE QPSK systems

spans more than the channel impulse response duration, the interference introduced by the previous transmitted blocks is entirely eliminated [5]. In order to study the influence of cyclic prefix duration, the parameter “ $ndelay$ ” is defined as the ratio of the multipath delay spread of the channel and the cyclic prefix duration. If in the past simulations we avoided this issue by taking a unit value for  $ndelay$ , this time we also study the case when the multipath delay spread of the channel spans two times the duration of the cyclic prefix. It is obviously regarding the figure 6 that SC-FDE is generally more sensitive than OFDM

to a cyclic prefix that is insufficient in order to counteract the time-dispersive nature of the channel. Though, one can notice that for high values of SNR, SC-FDE overcomes OFDM.

All simulations were repeated using a differential modulation scheme, namely DBPSK. In the figure 7, BER performance of both systems with and without encoding is illustrated

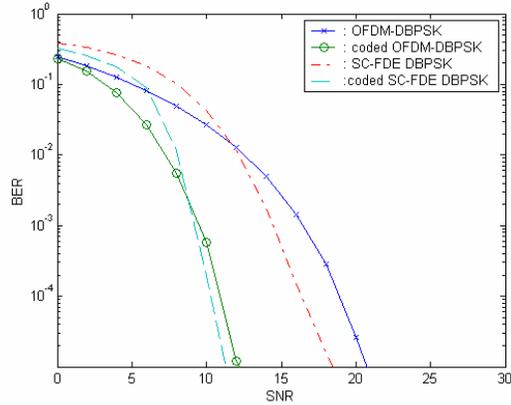


Fig. 7: A comparison of OFDM and SC-FDE with DBPSK modulation, Rayleigh fading channel,  $P_1=P_2$

The figure 7 confirms the conclusions previously inferred for QPSK modulation. The critical range where SC-FDE becomes more reliable moves 1-2dB to the left when compared to the first case.

The figures 8 and 9 emphasize the influence of K and  $P_2$  parameters on both presented methods. The same insignificant spread of the results like in case of QPSK modulation can be observed.

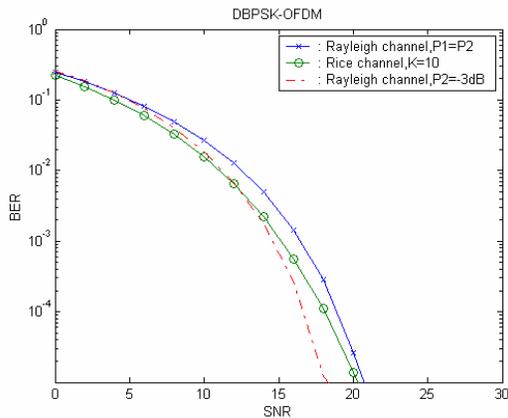


Fig. 8: The influence of  $P_2$  and K parameters on BER performance of a DBPSK-OFDM system

In the figure 10 the influence of the cyclic prefix length is studied. The conclusions that can be drawn slightly differ from those observed in the case of QPSK modulation. SC-FDE drastically improves its performance at high SNR values, despite the insufficient duration of the cyclic extension.

In the figures 11 and 12, a comparison of the performances of both systems is done, taking into

account the modulation method used. As expected, DBPSK generally performs better than QPSK.

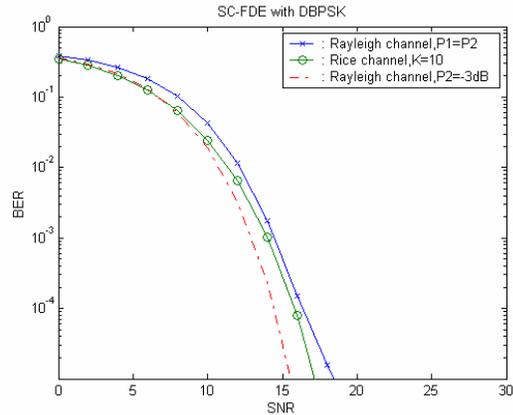


Fig. 9: The influence of  $P_2$  and K parameters on BER performance of a SC-FDE with DBPSK system

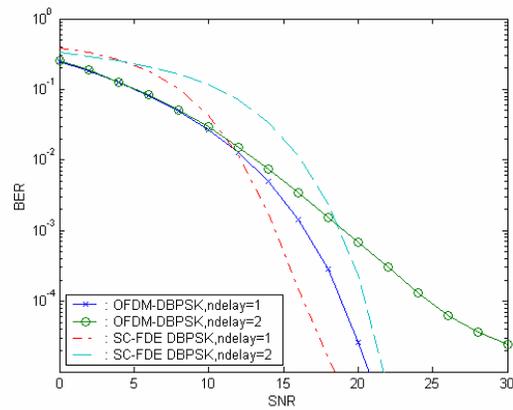


Fig. 10: The influence of  $ndelay$  parameter on BER performances of both OFDM-DBPSK and SC-FDE DBPSK systems

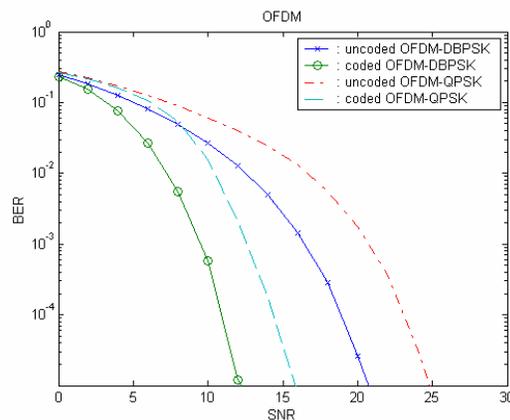


Fig. 11: A comparison of DBPSK and QPSK modulations when used in an OFDM system

There are though some refinements that make the difference between SC-FDE and OFDM. While in both QPSK and DBPSK used with OFDM the coding gain is approximately the same (within 9dB to 10dB),

SC-FDE seems to not improve in the same manner its performances. Thus, the coding gain of SC-FDE system with QPSK modulation is significantly inferior to the same system used with DBPSK modulation. Furthermore, the coded SC-FDE systems, as previously seen, performs generally worse than coded OFDM, observation confirmed by the performance curves in the figure 12. Thus, the coding gain in this case covers a range of values between approximately 4dB and 8dB.

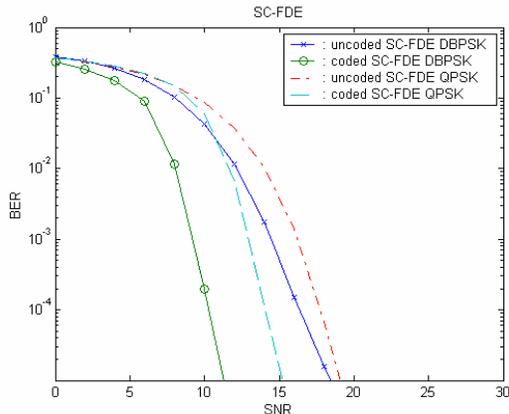


Fig. 12: A comparison of DBPSK and QPSK modulations when used in a SC-FDE system

#### IV. CONCLUSIONS

In this paper, a comparison of two of the recent years equalization approaches, OFDM and SC-FDE, was performed. The basic mathematics of these equalizations methods were studied, and it is shown that they are essentially the same. Furthermore, the two methods offer approximately the same conditions in terms of computational complexity and signal processing involved. Though, when compared to OFDM, SC-FDE seems to provide some advantages regarding its practical implementation. The BER performance of both systems in similar fading conditions is investigated in the section III by means of computer simulation. Both Rayleigh and Rice fading conditions were taken into account for modeling of the radio channel. The performances of the two systems are essentially the same in the considered conditions. There are yet some differences that could be observed. While uncoded SC-FDE generally performs better at important values of SNR, when coded, OFDM becomes more reliable. If a cyclic prefix that cannot entirely combat the time dispersion of the signal introduced by the channel is used, OFDM performs generally better than SC-FDE. Both systems slightly improve their performances when a LOS component appears or when the power of the delayed multipath signal decreases.

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