Optimal 1-D to 2-D FIR Filter McClellan Transformations by Standard Least Squares Minimization Techniques

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Abstract

The paper proposes a new and powerful technique to determine the optimal values of coefficients of McClellan transformation used to map very effectively 1-D filter prototypes in 2-D FIR filters. The new algorithm permits the mapping of 1-D prototype in an arbitrary chosen 2-D contour. The algorithm minimizes all along the given contour, in a least squares sense, the total error between the ideal contour of the imposed boundary of the 2-D filter bandpass and an isopotential curve described by the McClellan transform. Additional initial designing conditions can be expressed as constraints for this minimization problem, which is solved as a constrained least squares minimization problem. Numerical examples, which include circularly symmetric and diamond-shaped filters, elliptically symmetric and 2-D fan filters demonstrate by lower error values attained by comparison with previous works the effectiveness and the usefulness of this method.

1. Introduction

The McClellan transformation [1] is well known as an efficient way to design 2-D FIR digital filters by a change of variables from 1-D frequency domain to 2-D frequency space. The transformation employs the substitution [1], [2]:

\[
\cos \omega = F(\omega_1, \omega_2) = t_{00} + t_{10} \cos \omega_1 + t_{01} \cos \omega_2 + t_{11} \cos \omega_1 \cos \omega_2
\]  

(1a)

where:

\[
|F(\omega_1, \omega_2)| \leq 1
\]  

(1b)

Using this substitution, the design of a 2-D FIR filter consists of two distinct steps: in the first one, the design of 1-D filter prototype; the last step maps the prototype 1-D frequency response by a McClellan transformation into the desired shape of a 2-D FIR filter.

Originally, the McClellan transformation coefficients were determined by scaling and optimization techniques [2], requiring large computationally efforts. More recently, Pei and Shyu [3] employed eigenfilter design techniques to calculate a quadratic measure of the total error of the transformation along the desired contour in the 2-D filter frequency plane. They search the optimal transformation coefficients for a given contour by a variational technique that establishes the minimum of this error, an operation
demanding also an important computing effort. Even if the errors in [3] are much smaller than the results reported in previous papers [4] [5], this technique remains highly inefficient as on the one hand the solutions are not the real optimal ones, viewing the above revealed deficiencies and on the other hand they didn’t find an unified treatment for all 2-D contours considered.

This paper uses the very powerful technique of constrained least squares minimization to calculate the optimal values of McClellan coefficients. This technique has successfully been used recently to design 1-D FIR filters with imposed constraints [6], [7]. In the case of McClellan transform, the constraints express some basic properties of the 2-D contour to be designed and the minimization is made on the squared error of the transformation along the imposed 2-D curve. Unlike the work reported in [3], all considered cases are solved by a unified treatment and the results reported are better.

The paper is organised into six sections. Section 2 introduces the calculus of the McClellan transform coefficients by the constrained least squares minimization method for an arbitrary shape of a 2-D quadrantly symmetric FIR filter. The following three sections discuss the design based on the method of four most frequently encountered contours of 2-D FIR filters: in Section 3 circular symmetric and diamond-shaped filters, in Section 4 elliptical symmetric filters and in Section 5 fan filters with arbitrary inclination. The results are permanently compared with those of [3]. Section 6 is conclusions.

2. Optimal McClellan Transformation Design

The McClellan transformation maps each point $\omega_c$ from the frequency axis of the 1-D FIR filter into an isopotential contour in the frequency plane of the 2-D filter. The design of an optimal transformation starts from a given contour $C(\omega_1, \omega_2) = 0$ in the 2-D plane, which represents usually the pass-band boundary of the 2-D filter to be implemented. The optimization of McClellan transform on this contour means to find the 1-D frequency point $\omega_c$ and the corresponding transformation coefficients that lead to a minimal deviation between this contour and the isopotential curve described in the 2-D frequency plane by these variables.

To express in the squared error sense the deviation between the two curves, the squared distance between them is calculated in $N$ equally spaced points taken on the imposed contour. Their co-ordinates $\omega_{1n}$ and $\omega_{2n}$, $0 \leq n \leq N - 1$ respect the contour describing equation: $C(\omega_{1n}, \omega_{2n}) = 0$. Using the transform definition (1a), the squared error on the imposed contour has the expression:

$$E = \sum_{n=0}^{N-1} (\cos \omega_c - t_{00} - t_{10} \cos \omega_{1n} - t_{01} \cos \omega_{2n} - t_{11} \cos \omega_{1n} \cos \omega_{2n})^2 \quad (2)$$

This error can be minimised using the standard techniques of linear algebra,
yielding the optimal values of all five parameters involved, \( \cos \omega_c, t_{00}, t_{10}, t_{01} \) and \( t_{11} \).

To reach this aim, the equation (2) must take a matrix form. To do this, firstly, the five unknown variables of the problem are included in the solution vector \( t \):

\[
(t_0 \ t_{10} \ t_{01} \ t_{11})^T
\]

(3)

where the superscript \( T \) denotes the transposing operation. The second vector to be introduced is:

\[
c(\omega_{1n}, \omega_{2n}) = [1 \ -1 \ -\cos \omega_{1n} \ -\cos \omega_{2n} \ -\cos \omega_{1n} \cos \omega_{2n}]
\]

(4)

These two vectors permit to replace the term in brackets from (2) by their product. One step more ahead toward the final aim, the \( N \times 5 \) different vectors \( c(\omega_{1n}, \omega_{2n}) \) are gathered in the \( N \times 5 \) matrix \( C \):

\[
C = [c^T(\omega_{10}, \omega_{20}) \ c^T(\omega_{11}, \omega_{21}) \ \ldots \ c^T(\omega_{1N-1}, \omega_{1N-1})]^T
\]

(5)

Using (3), (4) and (5) the matrix form of the squared error equation (2) is, therefore, expressed as:

\[
E(t) = (Ct)^T (Ct) = t^T C^T C t
\]

(6)

The constraints represent the second necessary element needed to define a 1-D to 2-D transformation. They express some peculiarities of the transform which depends on the contour to be designed, for instance in which manner specific points of the 1-D frequency axis maps in the 2-D frequencies plane, or supplementary symmetries on the same contour. Generally speaking, each constraint leads to one linear equation between the transform coefficients. Obviously, the number of these equations \( L \) is lower than \( 5 \), the number of unknowns. The result, as the following Sections will point out, is that these constraints can also be written in a matrix form:

\[
S t = K
\]

(7)

where \( S \) is a \( L \times 5 \) matrix and \( K \) a \( L \times 1 \) vector.

Now the design task of our problem is the minimization of squared error \( E(t) \) in (6) subject to the constraints (7). The solution of the problem uses an \( L \times 1 \) auxiliary vector \( \lambda \) to minimize the Lagrangian function:

\[
\Lambda(t, \lambda) = E(t) + \lambda^T (S t - K)
\]

(8)

This leads to a very convenient matrix equation [6], [7] that gives the optimal solution vector \( t \):

\[
\begin{bmatrix}
2 C^T C & S^T \\
S & 0
\end{bmatrix}
\begin{bmatrix}
t \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
0 \\
K
\end{bmatrix}
\]

(9)

In conclusion, the calculus of an optimal McClellan FIR filter transformation needs some common steps, indifferently of the chosen contour of the 2-D FIR filter to be designed. First of all, the matrix \( C \) is calculated, based on a convenient partition of points \( (\omega_{1n}, \omega_{2n}), 0 \leq n \leq N - 1 \) on the desired contour. Secondly, the constraints of the transformation must be formulated and, then implemented as \( S \) and \( K \). At last the
system (9) gives the optimal solution, denoted by \( \mathbf{t}^* = [\cos \omega_c^* \ t_{00}^* \ t_{01}^* \ t_{11}^*]^T \).

It is essential to appreciate the quality of the solution. As in [3] and [4], the deviation of the computed isopotential curve by respect to the ideal contour is computed along the contour:

\[
D = \left( \sum_{n=0}^{N-1} \left( \cos \omega_c^* - t_{00}^* \cos \omega_1n - t_{01}^* \cos \omega_2n - t_{11}^* \cos \omega_1n \cos \omega_2n \right)^2 \right)^{1/2}
= \left( (\mathbf{Ct}^*)^T (\mathbf{Ct}^*) \right)^{1/2}
\]

(10)

### 3. Optimal McClellan transformation for circular symmetric and diamond-shaped 2-D filters

The design of an optimal 1-D to 2-D filter transformation for a circularly-symmetric contour use as initial data \( \omega_r \), the radius of the desired circular passband boundary. Viewing the quadrantally symmetry of the McClellan transform, \( N \) equally spaced points \( (\omega_{1n}, \omega_{2n}) \), \( 0 \leq n \leq N-1 \) are taken along the first quadrant of the circle of radius \( \omega_r \):

\[
\omega_{1n} = \omega_r \cos \left( \frac{n\pi}{2N} \right), \quad \omega_{2n} = \omega_r \sin \left( \frac{n\pi}{2N} \right), \quad 0 \leq n \leq N-1
\]

(11)

The matrix \( \mathbf{C} \) of (5) is calculated using that partition. There are three constraints to be imposed in this case. Firstly, the 1-D filter point \( \omega = 0 \) maps to \( (0,0) \), then, secondly \( \omega = \pi \) maps to \( (\pi,\pi) \). The last condition is obvious: the transform is symmetric with respect to \( \omega_1 \) and \( \omega_2 \). The constraints give the matrices \( \mathbf{S} \) and \( \mathbf{K} \) of (7).

In the first step, the optimal McClellan transform developed here was used to calculate, starting from a given set of cut-off radius \( \omega_r \) of the 2-D filter, the corresponding values of the cut-off frequency of 1-D filter \( \omega_c \) and the transformation coefficients. As the constraints lead in this case to: \( t_{10} = t_{01} = 1/2 \) and \( t_{00} = -t_{11} \), only the parameter \( t_{11} \) is included in Table I. For reference, the table contains values of the minimal deviation \( D \) in (10) calculated for a number of \( N = 50 \) points taken on the \( \omega_r \) radius circle for the optimal transform method and for the algorithm in (3). The optimal approach gives much lower deviation error, a first confirmation on the quality of the method. Figure 1 presents also the isopotential contours of circular shape given by the algorithm for different pass-band radius \( \omega_r \).
TABLE 1 1-D to 2-D Optimal Circularly Symmetric Filter Design for Different Radius \(\omega_r\), the Corresponding 1-D Frequency \(\omega_c\), \(t_{11}\) Transform Coefficient and Deviation \(D\)

<table>
<thead>
<tr>
<th>(\omega_r / \pi)</th>
<th>(\omega_c / \pi)</th>
<th>(t_{11})</th>
<th>Deviation (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>New Method</td>
</tr>
<tr>
<td>0.1</td>
<td>0.086585</td>
<td>0.25124</td>
<td>4.7117\times10^{-10}</td>
</tr>
<tr>
<td>0.2</td>
<td>0.17306</td>
<td>0.25501</td>
<td>1.1721\times10^{-7}</td>
</tr>
<tr>
<td>0.3</td>
<td>0.25932</td>
<td>0.26149</td>
<td>2.9900\times10^{-6}</td>
</tr>
<tr>
<td>0.4</td>
<td>0.34524</td>
<td>0.27098</td>
<td>2.9682\times10^{-5}</td>
</tr>
<tr>
<td>0.5</td>
<td>0.43070</td>
<td>0.28397</td>
<td>1.7564\times10^{-4}</td>
</tr>
<tr>
<td>0.6</td>
<td>0.51557</td>
<td>0.30117</td>
<td>7.4935\times10^{-4}</td>
</tr>
<tr>
<td>0.7</td>
<td>0.59976</td>
<td>0.32365</td>
<td>2.5528\times10^{-3}</td>
</tr>
<tr>
<td>0.8</td>
<td>0.68328</td>
<td>0.35296</td>
<td>7.3849\times10^{-3}</td>
</tr>
<tr>
<td>0.9</td>
<td>0.76665</td>
<td>0.39146</td>
<td>1.8894\times10^{-2}</td>
</tr>
<tr>
<td>1.0</td>
<td>0.85259</td>
<td>0.44284</td>
<td>4.4014\times10^{-2}</td>
</tr>
</tbody>
</table>

As an example, the case of a 2-D circularly symmetric FIR filter design is considered. The pass-band boundary is \(\omega_r = 0.7\pi\). Table I gives in this case the data for an optimal 1-D to 2-D transform:

\[
\omega_c = 0.59976 \cdot \pi \\
t_{11} = -t_{00} = 0.32365, \quad t_{10} = t_{01} = 0.5
\]

![Fig. 1](image1.png) The isopotential contours corresponding to different \(\omega_r\) for a circular symmetric filter design as in Table I

![Fig. 2](image2.png) The isopotential contours for the circular symmetric filter design with radius \(\omega_r = 0.7\pi\)
The isopotential contours, which correspond at these parameters, are shown in Fig. 2. A 1-D 31 length prototype lowpass filter with passband \([0, 0.59976\pi]\) and stopband \([0.7\pi, \pi]\) is designed with the well-known Parks-McClellan algorithm [8]. Next, a McClellan transform with the computed coefficients maps the 1-D prototype in the 31×31 circular symmetric 2-D filter of Fig. 3.

The design of a diamond-shape filter by the optimal McClellan transformation uses the specifications in Fig. 4. The transform parameters are optimized on the first quadrant contour, where \(N\) equally spaced points are taken:

\[
\omega_{1n} = \frac{n\omega_d}{N}, \quad \omega_{2n} = \left(1 - \frac{n}{N}\right)\omega_d; \quad 0 \leq n \leq N-1
\]

The constraints remain the same as in first case. That means: \(t_{10} = t_{01} = 1/2\), \(t_{00} = -t_{11}\) and only the parameter \(t_{11}\) must be effectively calculated. Table II lists for different values of the cut-off frequency \(\omega_d\) of diamond-shaped filter, the corresponding values established by the optimal McClellan transformation for the parameter \(t_{11}\) and for the cut-off frequency \(\omega_c\) of the corresponding 1-D low pass filter. The deviations calculated for
$N = 50$ as in (3) gives for the optimal method better values.

TABLE II  1-D to 2-D Optimal Diamond-Shaped Filter Design for Different $\omega_d$, the Corresponding 1-D Frequency $\omega_c$, $t_{11}$ Transform Coefficient and Deviation $D$

<table>
<thead>
<tr>
<th>$\omega_d / \pi$</th>
<th>$\omega_c / \pi$</th>
<th>$t_{11}$</th>
<th>Deviation $D$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>New Method</td>
</tr>
<tr>
<td>0.1 0.005 62</td>
<td>-0.496 47</td>
<td>4.103 10-5</td>
<td>4.410 07×10-5</td>
</tr>
<tr>
<td>0.2 0.022 33</td>
<td>-0.485 74</td>
<td>6.348 72×10-4</td>
<td>6.822 45×10-4</td>
</tr>
<tr>
<td>0.3 0.049 76</td>
<td>-0.467 36</td>
<td>3.033 04×10-3</td>
<td>3.284 21×10-3</td>
</tr>
<tr>
<td>0.4 0.087 35</td>
<td>-0.440 53</td>
<td>8.794 57×10-3</td>
<td>9.461 21×10-3</td>
</tr>
<tr>
<td>0.5 0.134 48</td>
<td>-0.404 08</td>
<td>1.902 38×10-2</td>
<td>2.048 42×10-2</td>
</tr>
<tr>
<td>0.6 0.190 57</td>
<td>-0.356 38</td>
<td>3.335 75×10-2</td>
<td>3.595 83×10-2</td>
</tr>
<tr>
<td>0.7 0.255 17</td>
<td>-0.295 21</td>
<td>4.873 62×10-2</td>
<td>5.260 82×10-2</td>
</tr>
<tr>
<td>0.8 0.328 08</td>
<td>-0.217 76</td>
<td>5.800 57×10-2</td>
<td>6.271 66×10-2</td>
</tr>
<tr>
<td>0.9 0.409 46</td>
<td>-0.120 60</td>
<td>4.839 66×10-2</td>
<td>5.242 51×10-2</td>
</tr>
<tr>
<td>1.0 0.500 00</td>
<td>0. 0 0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The example considered for this case refers to the design of a diamond-shaped 2-D filter with $\omega_d = 0.6\pi$. The Table II gives the optimal values:

\[
\omega_c = 0.19057 \cdot \pi
\]

\[
t_{11} = -t_{00} = -0.35638, \quad t_{10} = t_{01} = 0.5
\]

and Fig. 6 presents the isopotential contours of the 2-D filter for this example. Next, a 61 length 1-D prototype filter is designed with passband $[0, 0.19057\pi]$ and stopband $[0.23\pi, \pi]$. The McClellan transformation with the values in (14) leads to the optimal...
2-D 61×61 diamond-shaped filter having the frequency response in Fig. 7.

4. Optimal McClellan transformation for 2-D quadrantly elliptically symmetric filters

The passband boundary of low-pass quadrantly elliptically symmetric 2-D filter is described by the equation:

\[ C(\omega_1, \omega_2) = \left( \frac{\omega_1}{a} \right)^2 + \left( \frac{\omega_2}{b} \right)^2 - 1 = 0 \]  \hspace{1cm} (15)

where \( a \) and \( b \) are the passband frequency edges on the \( \omega_1 \) axis and \( \omega_2 \) axis, respectively. To form the coefficient matrix \( C \), a convenient set of \( N \) equally spaced points \((\omega_{1n}, \omega_{2n})\) on the first quadrant contour is defined by:

\[ \omega_{1n} = \sqrt{\frac{a^2 b^2}{a^2 \tan^2 \theta_n + b^2}}, \quad \omega_{2n} = \omega_{1n} \cdot \tan \theta_n \]  \hspace{1cm} (16)

where \( \theta_n = \frac{n \pi}{2N} \),  \( 0 \leq n \leq N - 1 \)

In this case of an elliptically symmetric boundary, the constraints formulation is related to \( a \) and \( b \) size ratio. Considering only the case \( b > a \), the imposed constraints are: (i) maps to \((0, 0)\) and (ii) \( \omega = \pi \) maps to \((\pi, 0)\). The two linear equations written on these constraints give the matrices \( S \) and \( K \) of (7). It is also obvious that \( t_{01} = t_{00} \) and \( t_{11} = 1 - t_{10} \).

A design example with the same specification as [3], i.e., \( a = 0.25\pi \), \( b = 0.5\pi \), is illustrated here. The corresponding frequency of 1-D prototype and the coefficients given by the optimal method are:

\[ \omega_c = 0.249 \, 94 \cdot \pi \]
\[ t_{00} = -t_{01} = -0.052872, \quad t_{10} = 0.760474, \quad t_{11} = 0.239526 \]  \hspace{1cm} (17)

Fig. 8 presents for this case the isopotential contours of a 2-D filter designed by the optimal algorithm for these specifications. The 1-D prototype filter has 31 cells and a passband edge at \( \omega_c \) from (17), the stopband edge being \( 0.35\pi \) and is designed by the Parks-McClellan algorithm. Fig. 9 gives the frequency response of 31×31 quadrantly elliptically symmetric filter obtained by a McClellan transform which uses the coefficients in (17). Finally, if the deviation \( D \) of the cut-off isopotential contour with the ideal one computed for \( N = 50 \) points is in [3] \( 1.291 \, 860 \times 10^{-3} \), the corresponding value obtained by the optimal method is only \( 1.078 \, 539 \times 10^{-3} \), a confirmation of the quality of this new method.
5. Optimal McClellan transformation for 2-D FIR fan filters with arbitrary inclination

To design a 2-D fan filter of arbitrary inclination, the procedure starts with the passband edges specification in the first quadrant of the 2-D frequency plane. As points Fig. 10, the inclination angle $\theta$ specifies completely the 2-D fan filter. By simplifying reasons only the case $0^\circ < \theta \leq 45^\circ$ will be further illustrated. The set of $N$ equally-spaced points whose cosines make up the matrix $C$ in (5), are taken on the straight line of Fig. 10 as follows:

$$\omega_{1n} = \frac{n \pi}{N}, \quad \omega_{2n} = \omega_{1n} \cdot \tan \theta, \quad 0 \leq n \leq N - 1$$ (18)

![Fig. 8 Isopotential contours of a quadrantally elliptically symmetric 2-D filter design with $a = 0.25\pi$ and $b = 0.5\pi$](image1)

![Fig. 9 Frequency response of 2-D 31×31 quadrantally elliptically symmetric filter with $a = 0.25\pi$ and $b = 0.5\pi$](image2)

![Fig. 10 The first quadrant specification of an ideal 2-D fan filter](image3)

![Fig. 11 The isopotential contours corresponding to different $\theta$ for fan filter designs as in Table III](image4)
The settling of constraints represent the second step in the design of an optimal McClellan transformation. The constraints are the following: (i) the origin $\omega = 0$ maps in $(0, \pi)$ and $\omega = \pi$ maps in the $(\pi, 0)$ point of the 2-D plane. The matrices $S$ and $K$ of (7) represent the result of these constraints, even these relationships between the coefficients of the transformation become obvious: $t_{10} = 1 + t_{01}$ and $t_{00} = t_{11}$.

Table III lists optimal values of $t_{01}$, $t_{11}$ and $\omega_c$ established for a set of inclination angles $\theta$ between $5^\circ$ and $45^\circ$. The deviation $D$ between the computed isopotential curves and the ideal ones is calculated for $N = 50$ points and shows lower values of the error than those reported in [3].

TABLE III 1-D to 2-D Optimal Fan Filter with Different Orientation Design for Different Orientation $\theta$, the Corresponding 1-D Frequency $\omega_c$, $t_{01}$ and $t_{11}$ Transform Coefficients and Deviation $D$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\omega_c/\pi$</th>
<th>$t_{01}$</th>
<th>$t_{11}$</th>
<th>Deviation $D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^\circ$</td>
<td>0.944 77</td>
<td>-0.798 73</td>
<td>-0.193 76</td>
<td>3.272 $3 \times 10^{-5}$</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>0.889 53</td>
<td>-0.784 57</td>
<td>-0.185 63</td>
<td>1.295 $2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$15^\circ$</td>
<td>0.834 26</td>
<td>-0.761 50</td>
<td>-0.172 27</td>
<td>2.856 $4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>0.778 96</td>
<td>-0.730 28</td>
<td>-0.153 96</td>
<td>4.907 $2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$25^\circ$</td>
<td>0.723 58</td>
<td>-0.692 01</td>
<td>-0.131 07</td>
<td>7.237 $2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>0.668 10</td>
<td>-0.648 03</td>
<td>-0.103 99</td>
<td>9.421 $1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$35^\circ$</td>
<td>0.612 44</td>
<td>-0.599 99</td>
<td>-0.073 06</td>
<td>1.058 $1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>0.556 48</td>
<td>-0.549 86</td>
<td>-0.038 46</td>
<td>8.843 $4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>0.5</td>
<td>-0.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Fig. 12 Isopotential contours for a fan filter design with $\theta = 40^\circ$

Fig. 13 Frequency response of 2-D $31 \times 31$ fan filter with $40^\circ$ orientation designed by the optimal McClellan transform
For example, a design case of a 2-D fan filter with the same specifications as [3], i.e., $\theta = 40^\circ$, is illustrated here. The optimal algorithm gives for the corresponding 1-D cut-off frequency $\omega_c$ and for the transformation coefficients the following values:

$$\omega_c = 0.55648 \cdot \pi$$

$$t_{00} = t_{11} = -0.03846, t_{10} = 0.45014, t_{01} = -0.54986$$

(19)

A 1-D 31cells prototype lowpass filter with passband $[0, \omega_c]$ and stopband $[0, 0.68 \cdot \pi]$ is designed using the same Remez algorithm as in the preceding examples. Then, a McClellan transformation with the parameters given in (19) calculates the coefficients of the 31×31 2-D fan filter, whose isopotential contours in the first quadrant are represented in Fig. 12 and frequency response in Fig. 13.

6. Conclusions

This paper introduces the very powerful technique of constrained least squares minimization to calculate the optimal values of the coefficients of the well known 1-D to 2-D McClellan filter transform. The algorithm minimizes in a least squares sense the total error between the ideal contour of the imposed boundary of the 2-D filter bandpass and an isopotential curve described by this transform all along the ideal contour. Additional initial designing conditions can be expressed as constraints for this minimization problem. All the procedure needs in the first step to calculate very small size matrices and next to solve an ordinary linear equations system. The procedure can design arbitrary shape transformation contours of 2-D filter, giving a unified and efficient approach for this problem. Numerical examples, which include circularly symmetric and diamond-shaped filters, elliptically symmetric and 2-D fan filters demonstrate by lower error values attained by comparison with previous works the effectiveness of this method.

7. REFERENCES


