Optimal Design of Second Order McClellan Transformation for Eightfold Symmetric Contours 2-D FIR Filters - Part I: The Algorithm

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Rezumat

Transformarea McClellan este un procedeu util de proiectare a filtrelor FIR bidimensionale prin intermediul unei transpuneri 1-D la 2-D. Algoritmul de proiectare calculează coeficienții transformării McClellan prin minimizarea eroației pătratice dintre conturul ideal al filtrului 2-D și cel furnizat de transformare. Tipul caracteristicii filtrului 2-D și simetria care impun restricții în proiectare, care devine astfel un problemă de minimizare a eroației pătratice. Scopul principal al lucrării este să extindă metoda în cazul transformării McClellan de ordinul 2. Principala dificultate constă în capacitatea de a asigura monotonia transformării în planul de frecvență 2-D. Rezolvarea problemei se face printr-o procedură iterativă descrisă în lucrare.

Partea I a acestei lucrări se ocupă cu deducerea algoritmului de proiectare a filtrului 2-D prin metoda McClellan. Partea II prezintă exemple de proiectare a filtrului 2-D prin această metodă. Pentru filtrile 2-D cu contur circular și rombid, rezultatele obținute sunt comparate cu cele furnizate de transformarea McClellan de ordinul 1.

I Introduction

The McClellan transformation converts a 1-D zero-phase FIR filter into a 2-D one through a substitution of variables [1], [2]. For filters of moderate order, this implementation can be considerably more efficient than all other direct design procedures. The use of McClellan transformation breaks the 2-D filter design problem up into two smaller problems, namely, the design of the transformation mapping contours and the design of the 1-D prototype.

The first order McClellan transformation uses the substitution [2], [5]:

$$\cos \omega = F(\omega_1, \omega_2) = t_{00} + t_{01} \cos \omega_1 + t_{11} \cos \omega_1 \cos \omega_2$$

The extended transformation is expressed as

$$\cos \omega = F(\omega_1, \omega_2) = \sum_{i=0}^{K} \sum_{j=0}^{I} t_{ij} \cos \omega_1 \cos j \omega_2 + \sum_{k=1}^{K} \sum_{l=1}^{I} s_{kl} \sin k \omega_1 \sin l \omega_2$$

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Mersereau et al. [3] and Nguyen et al. [4] points up that these transformations lead to quadrantly symmetric and centro-symmetric 2-D FIR filters. If \( F(\omega_1, \omega_2) \) satisfies the relation

\[
|F(\omega_1, \omega_2)| \leq 1
\]  

(3)

the 2-D filter transfer function assume values that are assumed by the 1-D filter prototype response.

As this paper refers only to the design of 2-D filters having an eightfold symmetry in their frequency response, i.e. 2-D circularly symmetric filters and diamond-shaped filters, those properties reflect simply in restrictions imposed on the McClellan transformation:

\[
F(\omega_1, \omega_2) = F(\pm \omega_1, \pm \omega_2) = F(\omega_1, \omega_2)
\]  

(4)

There are some consequences of the last equation. Firstly in (2), all \( s_{kj} \) are zero. [6]. Next, the cosine coefficients \( t_{ij} \) in (1) and (2) fulfill the equalities:

\[
t_{ij} = t_{ji}, \quad i \neq j
\]  

(5)

The main goal of an efficient 1-D to 2-D transformation is to realize an imposed shape of the 2-D filter frequency response. Usually, that means that the contours of the passband edge and eventually, of the stopband edge curves approximate ideal given curves. Recent works [5], [7] introduced the least squares minimization technique as an efficient and optimal way to calculate the coefficients of the transformation. A total squared error of the transformation is calculated along a desired contour in the 2-D filter frequency plane. The minimization of this error by a constrained least squares procedure gives the optimal value of the transform coefficients. It is shown on several different examples that this method gives better results than those reported in other McClellan transform methods.

All the work that has been done in the calculus of McClellan transform coefficients by least squares minimization techniques concerned only the case of the first order transform (1). This paper aim is to extend those results to the case of the second order transform, which is in the case of an octal symmetric frequency response, in concordance with (2) and (5), takes the form

\[
F(\omega_1, \omega_2) = t_{00} + t_{10}(\cos \omega_1 + \cos \omega_2) + t_{11}\cos \omega_1 \cos \omega_2 + t_{20}(\cos 2\omega_1 + \cos 2\omega_2)
\]

\[
+ t_{21}(\cos 2\omega_1 \cos \omega_2 + \cos \omega_1 \cos 2\omega_2) + t_{22}\cos 2\omega_1 \cos 2\omega_2
\]  

(6)

Let's denote by \( G(\eta_1, \eta_2) \) the inverse Fourier transform of \( F(\omega_1, \omega_2) \), a \((2P+1) \times (2P+1)\) samples function centered at the origin. The derivation of the frequency response of 2-D FIR filter, starting from a 1-D odd-length symmetric \( 2L+1 \) cells filter, suppose a \( L \)-fold convolution in the spatial domain of \( G(\eta_1, \eta_2) \) with itself. The resulting 2-D FIR filter will have therefore \((2LP+1) \times (2LP+1)\) samples centered at the origin. In conclusion, the use of a first order McClellan transformation gives a 2-D symmetric \((2L+1) \times (2L+1)\) FIR filter, while a second order transform \((P = 2)\) will give a \((4L+1) \times (4L+1)\) samples filter.

Despite this major drawback, the second order McClellan transformation may offer serious advantages as compared with the first order transformation. Firstly, in the case of octal symmetric frequency response 2-D filters, it uses six independent coefficients instead of three as in the case of the first order transformation. It permits thus, the approximation of an ideal contour in 2-D plane with an increased precision. Then, a more important issue becomes possible with the use of the second order transform. In the case of a first order transformation, the optimal approximation is made only along the contour of the passband edge of the 2-D filter. The second order transformation permits to realize that optimization all along two curves in the 2-D plane: the passband edge and also, the stopband edge of a low-pass filter.

Far greater, the accuracy of the approximation along the two reference contours can be easily controlled via a weighting coefficient.

For all that, the implementation of the least squares minimization design in the case of the second order McClellan transformation is not a trivial problem. Unlike the first order transform, in the case of the second order transformation, difficulties arise due to the necessity to maintain a bijective relation between the 1-D and 2-D frequency domains. For instance, the appliance of the least squares minimization procedure in the case of a second order McClellan transformation leads to a nonmonotonic behavior of \( F(\omega_1, \omega_2) \) in the 2-D plane. As a consequence, a single 1-D frequency point maps in two or more nonadjacent isopotential contours in 2-D frequency plane. To overcome this major difficulty, the partial derivatives of \( F(\omega_1, \omega_2) \) are constrained to maintain the same sign over the entire quarter plane used to implement the transformation. The result is a monotonic behavior for the transfer function of a 2-D filter, an imperative condition for having a low-pass or a high-pass frequency response. The additional constraints on derivatives are imposed by an iterative application of the Lagrangian method [8] to solve an inequalities constrained least squares minimization problem. As the paper points out, these additional constraints imposed on derivatives of \( F(\omega_1, \omega_2) \) strongly reduce the advantages of using a higher order transform instead of a first order one.

This paper is organized in two parts: Part I devotes to the derivation of the constrained least squares algorithm. Part II contains different examples of 2-D filters designs that use the method. The geometrical accuracy of these designs is compared for circular and diamond-shaped 2-D filters with the results given by the first order optimal transformation.

In Part I Section 2 extends the constrained least squares design method of the optimal first order McClellan transform [7]. The minimization of the squared error of a second order 1-D to 2-D transformation is made along two eightfold symmetric reference contours in 2-D frequency plane. A weighting coefficient \( w \) can control the amount of the error along the two contours. As the resulting transformation function \( F(\omega_1, \omega_2) \) often do not lead to realizable 2-D filters, Section 3 introduces new inequalities constraints that give a monotonic behavior of the function in the case of a eightfold symmetric second order transformation. Namely, for a low-pass filter, the partial derivatives of \( F(\omega_1, \omega_2) \) must have a negative value over the entire frequency plane. The robust and versatile iterative algorithm presented in Section 4, uses Kuhn-Tucker conditions to implement these new constraints. The result is an optimized octal symmetric two-contour design procedure of a second order McClellan transformation, which showed convergence in all cases. Finally, the conclusions are made in Section 5.

II Optimal McClellan Transformation Design

The least squares design of an optimal second order 1-D to 2-D FIR filter transformation has as a starting point two ideal contours in the frequency plane of the 2-D filter, denoted by

\[
C_B(\omega_1, \omega_2) = 0, \quad C_S(\omega_1, \omega_2) = 0
\]  

(7)

The first of them represents in the case of an octal symmetric low-pass frequency response, the passband edge of the filter, the second one, the stopband edge. Each of them corresponds by the transformation to two distinct points \( \omega_B \) and \( \omega_S \), the limits of bandpass and stopband of 1-D FIR filter. An optimal McClellan transformation design means to establish these two 1-D frequencies and the corresponding transformation coefficients that lead to a minimal squared error between these two ideal contours and the isopotential curves described in the 2-D plane by the transformation.
To measure this quadratic error, \( N \) equally spaced points are taken on each of two imposed contours. Their co-ordinates, denoted by \( \omega_B^{n}, \omega_S^{n} \) for the first curve, and \( \omega_B^{n+1}, \omega_S^{n+1} \) for the second one, \( 0 \leq n \leq N-1 \), satisfy the equations (7).

The quadratic errors, \( E_B \) and \( E_S \), between the ideal contours and the corresponding isopotential curves represented by the transformation are well approximated by the following sums

\[
E_B = \sum_{n=0}^{N} \left[ \cos \omega_B^{n} - F(\omega_B^{n}, \omega_{12}^{n}) \right] \quad E_S = \sum_{n=0}^{N} \left[ \cos \omega_S^{n} - F(\omega_B^{n+1}, \omega_{12}^{n+1}) \right] \tag{8}
\]

where \( \omega_B \) and \( \omega_S \) are the passband and the stopband frequencies of the 1-D prototype filter.

A total weighted squared error of the transformation relative to the two imposed contours is expressed by

\[
E = E_B + wE_S \tag{9}
\]

The introduction of the weighting coefficient \( w \) in (9) permits a very simple control of the precision of transformation along the two reference contours.

The error (9) is minimized by the least squares minimization technique, yielding the optimal values of the transformation parameters \( t_{ij} \) and of the points \( \omega_B \) and \( \omega_S \). This operation is accomplished by rewriting in a matrix manner the equation (9). In this aim, a first vector denoted by \( T \) includes the unknowns of the problem: \( \cos \omega_B \), \( \cos \omega_S \) and the transformation coefficients \( t_{ij} \).

\[
T = \begin{bmatrix} \cos \omega_B & \cos \omega_S & t_{00} & t_{10} & t_{01} & \ldots & t_{22} \end{bmatrix} \tag{10}
\]

where the superscript \( t \) denotes the transposing operation. If the independent coefficients of the transformation \( t_{ij} \) are in number of \( M \), then \( T \) is a \((M+2 \times 1)\) size vector. A second \( 2N \times (M+2) \) size matrix consisting of cosine terms in equations (8) is denoted by \( C \). Finally, a \( 2N \) by \( 2N \) squared diagonal matrix named \( W \) includes the weights assumed for the errors on the imposed contours: \( w \) on the first curve, \( w \) on the second one. With these new introduced symbols, the error equation (9) is written as

\[
E(T) = (CT)^T W(CT) \tag{11}
\]

The constraints represent the second element needed to build up the algorithm. Generally speaking, there are two categories of constraints. The first of them expresses the octal symmetric character of the frequency response of 2-D filter by equations (5). The other constraints describe biunivoc relationships between specific points of the 1-D filter frequency axis and corresponding points in the 2-D filter frequency plane. They impose a low-pass or a high-pass character for the 2-D filter. For instance, a low-pass eightfold symmetric 2-D filter design uses the following constraints

\[
\cos 0 = F(0,0) \quad \cos \pi = F(\pi, \pi) \tag{12}
\]

Each constraint gives one linear equation. These equations are written matricially as

\[
ST = K \tag{13}
\]

where \( S \) is an \( L \times (M+2) \) matrix and \( K \) an \( L \times 1 \) vector, \( L \) being the total number of constraints.

The formulation of the design task imposes the minimization of \( E(T) \) in (11) subject to constraints given by (13). The solution uses an \( L \times 1 \) auxiliary vector \( \lambda \) to minimize the Lagrangian function

\[
\Lambda(T, \lambda) = E(T) + \lambda^T (ST - K) \tag{14}
\]

The solution is obtained by equating with zero the gradients of (14) with respect to \( T \) and \( \lambda \).

\[
\begin{bmatrix} 2C^T WC \quad S^T \end{bmatrix} \begin{bmatrix} T \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ K \end{bmatrix} \tag{15}
\]

Denoting the optimal solution of the constrained least squares minimization problem by \( T^* = [\cos \omega_B^*, \cos \omega_S^*, t_{00}^*, t_{10}^*, t_{01}^*, \ldots, t_{22}^*] \), the determination of the quality of this solution is made also in a matrix form. Integral deviations of the isopotential curves described by the transformation with the ideal contours are defined as in [5] and [7]. Those deviations are calculated on each of two imposed contours, \( D_B(T^*) \) respectively \( D_S(T^*) \). Using appropriate weighting matrices, namely \( W_B \) respectively \( W_S \), the deviations are calculated matricially

\[
D_B(T^*) = \left[ \sum_{n=0}^{N} \left| \cos \omega_B^* - F^*(\omega_B^{n+1}, \omega_{12}^{n+1}) \right|^2 \right]^{1/2} = \left( \left( CT^* \right)^T W_B \left( CT^* \right) \right)^{1/2} \tag{16}
\]

\[
D_S(T^*) = \left[ \sum_{n=0}^{N} \left| \cos \omega_S^* - F^*(\omega_B^{n+1}, \omega_{12}^{n+1}) \right|^2 \right]^{1/2} = \left( \left( CT^* \right)^T W_S \left( CT^* \right) \right)^{1/2}
\]

where \( F^* \) is the transformation calculated using the optimal coefficients given by \( T^* \).

III New Conditions for the Optimal Second Order Transformation

The least squares design procedure developed in the previous Section showed to be very effective in the first order McClellan transformation design [5], [7]. In this case, the number of independent transformation coefficients is very low and the procedure can be applied efficiently only for one ideal contour, usually the edge of the pass-band of the 2-D filter. For optimal symmetric contours filters design, the constraints are in number of three: the first of them express the symmetry (5), the other two impose the low-pass character of the filter. It leaves practically a single transformation coefficient to be determined by the least squares procedure. A two-contour design of the transformation demands a greater flexibility and the second order transformation represents a normal answer for these challenge. For the nine coefficients of these transformation there are, in the case of a low-pass 2-D eightfold symmetric low-pass filter, five constraints, leaving four independent coefficients to solve the problem. Therefore, in spite of leading to increased filter dimensions, the second order McClellan transformation offer a better solution for a more accurate design, especially for two-contour cases.

A first attempt to use the designing procedure developed in the previous Section, revealed a major drawback. Namely, unlike the case of the first order optimal transformation design, the second order optimal transformation design does not ensure automatically a bijective mapping of points on the frequency axis of 1-D filter in corresponding unique isopotential contours in 2-D filter frequency plane. Fig. 1 presents the first quadrant isopotential contours of a 2-D circular symmetric filter obtained by a second order McClellan transformation. The transformation coefficients are calculated by the optimal procedure for minimum errors on passband and stopband circles of radius \( \omega_B = 0.4 \pi \), respectively \( \omega_S = 0.85 \pi \). The optimization is made on \( N = 50 \) points taken on each of two contours and the weighting coefficient is \( w = 1 \). As it is obvious from the picture, despite the use of low-
Fig. 1. The isopotential contours for a two-contours circular symmetric filter designed by the optimal procedure introduced in Section II.

The first inequality in (17) is written also as
\[ s(\omega_1, \omega_2) \leq 0, \quad \forall \omega_1 \in [0, \pi] \text{ and } \forall \omega_2 \in [0, \pi] \]  
(19)

An important simplification is given by the properties of frequency response in 2-D frequency plane. As equations (5) stating symmetry are used as constraints in (13), the second inequality in (17) is automatically fulfilled if equation (17) is carried out. In conclusion, in the case of eightfold symmetric 2-D filters, the bijectivity of the second order McClellan transformation is achieved by the use of equations (5), (12) and (15). At last, notice must be made that (19) holds for low-pass 2-D filters. For high-pass octal symmetric 2-D filters, the last inequality will reverse.

IV Iterative Design of Second Order McClellan Transformation

The use of inequality (19) in the design of 1-D to 2-D second order FIR filter transformation is made on an iterative base. The design rely on the optimal constrained least-squares procedure introduced in Section II and adds iteratively supplementary constraints based on inequality (19). It can be shown that the so-called Kuhn-Tucker (KT) conditions are necessary and sufficient for the optimality of the solution of this constrained problem [9], [10]. The use of KT conditions generalizes the Lagrangian approach made in Section II for finding the minimum of a function subject to equality constraints to minimization with equality and inequality constraints.

Even if the first derivative of the transformation must be, in the case of a low-pass 2-D filter, negative over the entire frequency plane, it's enough to impose this condition in some points of the plane, that it will be carried out over the entire plane. Let's denote by \((\omega_{11}, \omega_{21}), \ldots, (\omega_{1p}, \omega_{2p})\) the 2-D points where the condition (19) is imposed. In a compact form, the inequalities that \(s(\omega_1, \omega_2)\) fulfills in this points can be written as
\[ s_p T \leq 0 \]
(20)

where \(s_p\) represents the \(P \times (M + 2)\) coefficients matrix of (19) calculated in \(P\) imposed points. With these new inequality constraints, the Lagrangian function (14) takes the more generalized form
\[ \Lambda(T, \lambda, \mu) = E(T) + \lambda^T (ST - K) + \mu^T (s_p T) \]
(21)

where \(\mu\) is a \(P \times 1\) Lagrange multiplier vector. The solution of the constrained minimization problem is given in this case by the matrix equation
\[
\begin{bmatrix}
2C & W C & S \quad s_p \\
S & 0 & 0 & T \quad 0 \\
0 & 0 & \lambda & K \\
0 & 0 & \mu & 0
\end{bmatrix}
\]
(22)

In order that (22) to be validated, the necessary and sufficient KT optimality condition [8], [9] for this specific minimization problem must be satisfied. The condition is
\[ \mu \geq 0 \]
(23)

and means that only those elements of the vector \(\mu\) are positive where the inequality constraints are active. It is important to observe also that the signs of elements of the first Lagrange multiplier \(\lambda\), which corresponds to equality constraints are unimportant.

The problem of course is that the active constraints are not known, i.e., the corresponding \(P\) points in the frequency plane are not known in advance. To break the deadlock, an intermediate solution is assumed to be known. On this basis the function \(s(\omega_1, \omega_2)\) is calculated on a grid over the first quadrant of the frequency plane. The
maximum positive violation of condition (19) is chosen as an indicator for an active constraint in the final solution.

Fig. 2 Flowgraph of the iterative algorithm for the constrained least square design of second order McClellan Transformation

Based on the observations made above, the main iterative algorithm was constructed as follows:

1. Compute the least-square equality-constrained solution according to (15).
2. Compute the function \( s(o_1, o_2) \) over the first quadrant of the frequency plane.
3. Find the frequency pair \((o_{1p}, o_{2p})\) where the maximum violation of condition (19) occurs.
4. If \( s(o_{1p}, o_{2p}) \leq 0 \) stop; else continue.

5. Append the inequality constraint \( s(o_{1}, o_{2}^p) \leq 0 \) to the actual set of inequality constraints \( s_{p_1, T} \leq 0 \).
6. Solve equation (22) according to KT condition (23). If the smallest element of \( \mu \) is negative, remove the corresponding constraint and repeat 6; else go to 2.

The algorithm presented above resumes, with necessary differences for the case of optimal second order McClellan transformations, the work of Lang, Selesnick and Buruss (1980) on constrained LS design of 2-D filters.

Although this basic algorithm converges for most reasonable design situations, there are some common cases where it lacks the convergence. In these cases, the failure of the algorithm to converge takes a specific form. Instead of converging to a single set of transform coefficients, the algorithm will end up cycling between two different sets, neither of which satisfy condition (19). Selesnick in [11] met a similar behavior in the case of constrained LS design of multiband 1-D FIR filters. To avoid cycling, he introduced an additional inner-loop.

After each iteration, the modified algorithm checks the values of \( s(o_{1}, o_{2}) \) over the previous \( \mathcal{B} \) constraint set of frequencies. If \( s(o_{1}, o_{2}) \) is negative over these frequencies, then the algorithm proceeds exactly as does the original algorithm. However, if it is found that \( s(o_{1}, o_{2}) \) violates the constraint at some frequency belonging to the previous constraint set, then (i) that frequency is appended to the current constraint set \( \mathcal{A} \) and (ii) the same frequency is removed from the record of previous constraint set frequencies \( \mathcal{B} \).

Although the appliance of the modified iterative procedure improved the performance of the constrained least square design of McClellan transformations, the lack of convergence persisted in some situations. Some explanations on this lower efficiency of the algorithm in McClellan transformations designs stay firstly in the different updating process used in our case. If the design of 1-D FIR filters rely on a multiple exchange of constraint set \( \mathcal{A} \) at each iteration, this operation is more difficult to realize in the case of 2-D functions, so like in [10], at each iteration only one 2-D frequency \((o_{11}, o_{21})\) is appended to \( \mathcal{A} \). Difficulties arise also from the very flat behaviour of \( s(o_{1}, o_{2}) \) in a large region around the origin of frequency plane. These considerations led to a modified iterative constrained least square algorithm, which showed convergence in all designing trials made.

A flowgraph of the algorithm is shown in Fig. 2. The figure reveals its multi-loop iterative structure. The largest loop is used, in case of non-convergence (i.e., \( \max s(o_{1}, o_{2}) \neq s(o_{11}, o_{21}) > 0 \)), to append this 2-D frequency, \((o_{11}, o_{21})\) to the current constraint set \( \mathcal{A} \). The second iterative loop removes from \( \mathcal{A} \) that frequency which violates mostly Kuhn-Tucker conditions by having the most negative Lagrange multiplier \( \mu \) in the set. Since this loop is repeated until all multipliers \( \mu \) are positive, a case to avoid and which occurred at least once in design trials is that \( \mathcal{A} \) becomes an empty set. Obviously, it is a new case of cycling, which had not appeared in Selesnick work [11]. In Fig. 2, this case is removed by the insertion in the first decision block of a supplementary condition. The last loop in the flowgraph is the result of the test for constraint violation over the previous constraint set \( \mathcal{B} \) (i.e. \( \max s(o_{1}, o_{2}) \leq 0 \), \( \forall (o_{1}, o_{2}) \in \mathcal{B} \)). There are also differences by con S'mathcal'( parison with [11]. Unlike the Selesnick work where the frequency
maximum violation is transferred from $B$ to $A$ in an attempt to break up any possible mechanism of cycling, a maximum of $s(\omega_1, \omega_2)$ is calculated over a vicinity of the point of maximum violation from $B$. The frequency of this new maximum denoted by $s(\omega_1', \omega_2')$ is appended to set $A$, instead of maximum violation in set $B$, like in the previous work. Whenever such a transfer takes place, the secondary set $B$ is emptied.

The algorithm was implemented in a MATLAB program [12]. The contours defining the edges of passband and stopband that the transformation try to optimize are defined by a partition of $N$ points taken on each of them in the first quadrant of 2-D plane. A grid of $N$ by $N$ points is used to calculate in the same first quadrant the differential of the transformation, $s(\omega_1, \omega_2)$, after each iteration. Finally, a weighting coefficient $v$ is used to impose different weights of the squared error on two curves. At the limit when $v$ takes very low or very large values, the two contour design algorithm developed here reduces to the design of a second order transformation optimized for one imposed contour. The algorithm is constructed such that the minimal least square error on the imposed contours and conditions 5, 12 and 19 hold after convergence. Consequently, convergence is tantamount to the optimality of the solution. Although the convergence has not been proved, the algorithm converged in all design cases tried. Some results are presented in the second part of this work.

V Conclusions

The challenge of this first Part of the paper is the calculus of optimal coefficients of the second order 1-D to 2-D McClellan transformation $F(\omega_1, \omega_2)$ used in the design of eightfold symmetric 2-D FIR filters. As in the case of first order McClellan transformation [5], [7], a least squares criterion is used to minimize the total error between the ideal contours of the 2-D filter band edges and the isopotential curves described by the transformation. Equality constraints are used to express additional initial designing conditions.

Unlike the first order transformation design, the equality constraints are not enough to give a bijective mapping of 1-D filter frequency points in unique contours in the 2-D filter frequency plane. To overcome the difficulty, the paper proposes for the case of eightfold symmetric low-pass 2-D filters designs the supplementary condition (17) which states that the partial derivatives of $F(\omega_1, \omega_2)$ are nonpositive over the entire frequency plane. A new algorithm appends iteratively this condition as supplementary inequality constraints to the existing equality constrained least squares minimization problem. Using the Kuhn-Tucker criterion to validate the optimality of the Lagrangian problem solution, the algorithm converges in all examined applications. The algorithm is also appealing because it has been implemented with an especially simple MATLAB program.

The second Part of the paper reports for the new iterative algorithm a large range of octal symmetric 2-D FIR filters designs. The paper also estimates from the point of view of geometrical accuracy, the efficiency of using the second order transformation instead of a first order transform.

VI References