A NEW DISCRETE WAVELET TRANSFORM

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The Discrete Wavelet Transform (DWT) has two parameters: the mother of wavelets and the number of iterations. Selecting different parameters for different DWT of the same signal, different energy concentrations in the wavelet domain are obtained. So, for a particular signal there is a better pair of parameters that realizes the best energy concentration in the wavelet transform domain. In applications is difficult to find this best pair of parameters. This is the reason why the aim of this paper is to introduce a new DWT less sensitive to the parameter selection. This transform is built using a technique very modern in telecommunications, the diversity enhancement. The diversity is enhanced in the wavelet transform domain computing different DWT, of the same signal, with different parameters. So, the input signal, represented by a vector, is transformed into a matrix. Each column of this matrix represents the DWT of the input signal, computed with a different pair of parameters. This matrix represents the result of the new discrete wavelet transform, named the Diversity Enhanced Discrete Wavelet Transform, (DEDWT). The new transform can be used with good results in denoising applications, especially for low SNR signals.

1. INTRODUCTION

The discrete wavelet transform, DWT, realizes a concentration of the energy of the input signal in a small number of coefficients. This concentration's enhancement is useful for the reduction of the number of operations in the application considered. For a given signal, using different wavelet's mothers, different energy concentrations are obtained. So, for a given input signal there is a best wavelet's mother, that realizes the higher energy concentration. The aim of this paper is to propose a new DWT less sensitive to the selection of the wavelet’s mother. The construction is based on the diversity enhancement’s principle. Such a transformation is useful for the denoising of low signal to noise ratio, SNR, signals. In the second section of this paper is presented the construction of the new transform, the DEDWT. The computation of its inverse is also described. The central result of this paper, described in the third section, is the application of the new transformation in denoising applications. In the forth section some simulation results are presented. The last section is dedicated to conclusions.

2. THE DEDWT, A NEW DISCRETE WAVELET TRANSFORM

The parameters of the DWT are: the wavelet's mother $\psi(t)$ and the number of iterations, $M$. An excellent environment for the simulation of signal processing methods based on wavelets, is the Matlab toolbox WaveLab, [1]. This is the reason why this toolbox will be used in the simulations reported in this paper.

A very modern signal processing method, developed in communications, is based on the enhancement of the diversity of the signal to be processed. In our case the diversity enhancement can be realized in the DWT domain. The parameters of the DWT are the wavelet's mother and the number of iterations. So the diversity can be enhanced computing for the same signal, $x[n]$, some different discrete wavelet transforms. For each of them a different pair of parameters must be used. In this way a new DWT is obtained. This transform will be called in the following the Diversity Enhanced Discrete Wavelet Transform (DEDWT). It is a redundant discrete wavelet transform. This new transform realizes the correspondence between the vector $x[n]$ and a matrix $\text{DEDWT}[n,m]$. Every column of this matrix represents one of the DWT of the signal $x[n]$. This transform can be inverted. Its inverse will be called IDEDWT. For every column of the matrix $\text{DEDWT}[n,m]$ the corresponding IDWT is computed. A new matrix, $E[n,m]$, is obtained. Every column of this matrix contains the signal $x[n]$. Computing the mean of the columns of the matrix $E[n,m]$ the vector $x_0[n] = x[n]$, is obtained.

Another source for the enhancement of the diversity can be the circular translation of the samples of the signal $x[n]$. Using this source another redundant DWT, named translation-invariant DWT, was proposed in [2]. For a better enhancement of the diversity in the wavelets transform domain this transform can be associated with the transform proposed in this paper.

3. USING THE DEDWT FOR DENOISING

We will consider in this paper the case of additive white gaussian noise with zero mean channels. We deal with the signal $x[n] = x[n] + n[n]$, where $n[n]$ is a noise. To estimate the signal $x[n]$, Donoho, [3], proposed the following method:

1. The Discrete Wavelet Transform (DWT) of the signal $x[n]$ is computed obtaining the signal $y[n]$. 
2. A non-linear filtering procedure, called soft thresholding, is applied in the wavelet transform domain:

\[
y_o[n] = \begin{cases} 
\text{sgn}(y_i[n])\left(|y_i[n]| - t\right), & i|y_i[n]| \geq t, \\
0, & |y_i[n]| < t,
\end{cases}
\]  

(1)

where \(t\) is a threshold.

3. Taking the inverse DWT (IDWT), the de-noised version, \(x_o[n]\), is obtained.

There are some recent papers dealing with this denoising method, [4-7], but the case of low SNR signals isn’t analyzed.

The value of the threshold \(t\), from the second step of the Donoho’s denoising method, recommended in [3], is:

\[t = \sqrt{N \ln N} \sigma\]

where \(N\) represents the number of samples of the signal \(x_i[n]\) and \(\sigma^2\) represents the power of \(n_i[n]\). Using this value the minimax mean square approximation error of the signal \(x[n]\) with the signal \(x_o[n]\) is obtained, for a large class of input signals with different regularities, [3]. The first disadvantage of this denoising method is the fact that it is not adaptive. In [8] is proved that the Donoho's denoising method don't works well in the case of low SNR signals. When the SNR of the input signal decreases, the distortion in the recovered signal increases. For very low SNR the output signal becomes equal with zero (the entire noise is suppressed but the entire signal, \(x[n]\), is suppressed too) because the value of the threshold required becomes superior to any wavelet coefficient of the signal \(x[n]\). So, for low SNR signals the use of a threshold with the value prescribed by the Donoho's denoising method produces unacceptable high distortions. These are the reasons why in this paper is proposed the substitution of the DWT and IDWT with DEDWT and IDEDWT in the Donoho’s algorithm, already presented.

4. SIMULATION RESULTS

The results are presented in Fig. 1.

In the top (Fig. 1 a) is presented the signal \(x[n]\). Under this waveform (Fig. 1 b) is represented the acquired signal \(x_i[n]\). Its SNR is of 10.8. Under this waveform (Fig. 1 c) is represented the result of the proposed denoising method, the signal \(x_o[n]\). Finally, in the bottom of Fig. 1, (Fig. 1 d) is represented the result obtained applying the Donoho's denoising method with the Haar's wavelet's mother and a number of 16 iterations. The sources for the enhancement of the diversity were the classical nine Daubechies wavelet's mothers the Haar's wavelet’s mother and the iterations numbers 1, 2, 4 and 8. The matrix DEDWT has 36 columns. The
signal to distortion ratio, SDR, enhancement of the proposed denoising method is of 1.7573. The SDR enhancement of the classical Donoho’s denoising method is only of 1.5215. So the proposed denoising method realizes a better SDR enhancement. This conclusion can be also obtained analyzing the Fig. 1 c and 1 d. The transition distortions from Fig. 1 d can’t be observed in Fig. 1 c.

![Fig. 1 – Simulation results.](image)

5. CONCLUSIONS

The utility of the new discrete wavelet transform, proposed in this paper, was proved. Its use in denoising applications has good results, outperforming the results obtained using the DWT. The increasing of the SDR can be explained on the basis of the use of the mediator, implicitly introduced for the computation of the IDEDWT. The classical Donoho’s denoising method can be further improved if another procedure for the selection of the threshold $t$ is used. This procedure is reported in [9]. Combining this selection procedure and the new transform, proposed in this paper, signals with low SNR ($\geq 1$) can be well reconstructed on
the basis of the denoising procedure. The principle of the construction of the new transform, the diversity enhancement, is a powerful tool in telecommunications. Its application realizes an increasing of the redundancy of the signal to be processed. This redundancy’s increasing can be exploited also in other applications of the wavelet’s theory. An example is the images watermarking. Because the image obtained in the DEDWT domain has greater dimensions versus the image to be watermarked, there is more room in the transformed image to embed the watermark. This is the reason why the use of the new wavelet transform, proposed in this paper, in watermarking applications, permits the transparent inclusion of an increased amount of information in the watermarked images.

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REFERENCES
