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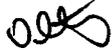
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Efficient Pulse Shaping and Robust Data Transmission Using Wavelets

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Abstract – **Recent work has shown that the use of wavelets in a data transmission chain brings several advantages. Thus, the wavelet mothers and the associated scale functions can generate orthogonal basis that obey Nyquist's first criterion and these basis can be used as pulse shaping waveforms instead of raised cosine filters. Furthermore, robust multicarrier data transmission can be performed using wavelet carriers instead of the classical sine waves proposed by OFDM. By associating these benefits, the road towards new powerful transmission techniques in data transmission systems is widely opened.**

Keywords – **wavelets, DWT, IDWT, Wavelet OFDM, pulse shaping.**

I. INTRODUCTION

Wavelets theory's applications have become very popular in the last decade in many areas of signal processing. These applications were widely used in fields as denoising, compression, segmentation or classification. Recently, some very desirable wavelet properties, as the orthonormality of a family generated by the translation and time dilatation of a wavelets mother function and their ability to split the time-frequency plane in a very flexible manner point towards interesting connections in data communication applications.

Thus, the author in [1] emphasized the relation between Meyer's wavelet and square-root raised cosine family filters, while in [2] it is stated that the wavelet mothers obey Nyquist's first criterion for zero Inter Symbol Interference (ISI) in a data communication system. Therefore, these functions could be used as pulse-shaping filters instead of the classical square-root raised-cosine family. Furthermore, recent research focused on multicarrier transmission techniques [3,4], highlighted that several drawbacks of Orthogonal Frequency Division Multiplexing (OFDM) can be steadily counteracted using wavelet carriers instead of the sine waveforms. Due to the fact that these wavelet carriers form an orthogonal family over the duration of a transmitted symbol, they can be separated at receiver's side by correlation techniques. In addition, they exhibit some advantages compared to classical OFDM in terms of complexity, flexibility and spectral efficiency.

The paper will provide an analysis of all benefits brought by the use of wavelet functions in different positions of a data transmission chain. A theoretical contribution shows that any Riesz basis forms an orthonormal family and classifies wavelet families as a particular case of Riesz basis. By simulation means we stress the advantages of wavelets in terms of implementation complexity and spectral efficiency.

In the next section, an overview of the wavelet transform (WT) is provided. The third section proposes a deeper analysis of Nyquist's first criterion and the way that wavelet families meet the requirements imposed by this criterion. A qualitative and quantitative evaluation of wavelet based OFDM (WOFDM) compared to classical OFDM is provided in section 4, while our final conclusions and future research directions in this field are drawn in the last section.

II. WAVELET TRANSFORM OVERVIEW

A modern approach of data communication research views the communication channel as a grid in a dual, time-frequency plane. In this context, a wavelets mother function has the ability to generate through its translation and scaling an orthonormal family. Using this basis, one can split the time-frequency plane in a flexible manner, respecting Heisenberg's incertitude criterion. The way that the time-frequency atoms are defined in this plane using wavelet basis is illustrated in fig.1c. A comparison with classical grids provided by other transforms is illustrated in fig. 1a and b. In the first case, the plane is split in vertical bands, as a result of an ideal sampling operation. The decomposition basis for the sampled signal is provided by a family of shifted deltas: $\{\delta(n-k)\}$, which have perfect time localization, but whose spectra span over the whole frequency axis. This approach can also be viewed as a mean to provide time Division Multiple Access. On the opposite situation, we have in fig. 1b the complex exponential basis used in Fourier decomposition $\{\exp(jk\omega_0 t)\}$. The signals that compose this basis have

perfect frequency localization, but they span the whole time axis. The grid in 1b) illustrates the principle of Frequency Division Multiple Access. Unlike for the cases above, wavelet signals provide good time and poor frequency resolution at high frequencies and good frequency and poor time resolution at lower frequencies (fig. 1c).
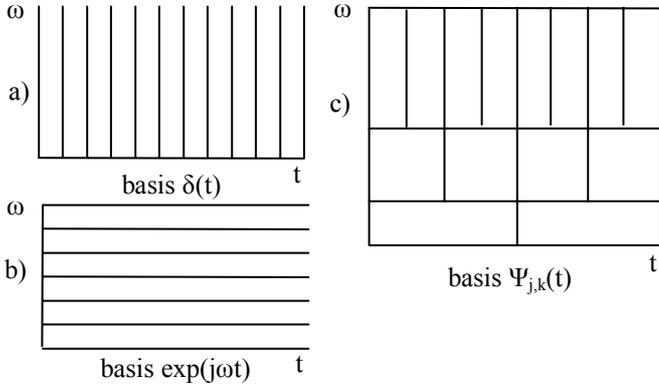

Fig.1: Communication channel viewed as a time-frequency plane.

Note that even if the widths and heights of the cells composing the plane in fig.1c change from cell to cell, the area of an atom remains constant. This way of splitting the time-frequency plane can be obtained using translated and scaled versions of a unique signal called "mother wavelets" and denoted by $\psi(t)$:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi(\frac{t-\tau}{s}) \qquad (1)$$

If we sample in equation 1 the continuous variables scale (s) and time offset (τ), we obtain a discrete version of this wavelets family, denoted by $\psi_{j,k}(t)$:

$$\psi_{j,k}(t) = s_0^{-j/2} \psi(s_0^{-j} \cdot t - k\tau_0) \qquad (2)$$

In order to obtain this way to define the wavelet family $\{\psi_{j,k}(t)\}$, the discretization relations used were: $s = s_0^j$ and $\tau = k s_0^j \tau_0$. A widely used choice for s_0 is $s_0 = 2$. If we consider now a continuous-time signal x(t), its discretized WT is:

$$DWT_x(j,k) = \int_{-\infty}^{\infty} x(t) \cdot \psi_{j,k}^*(t) \cdot dt \qquad (3)$$

Relation 3 is similar to the computation of Fourier coefficients of a signal. That's why the transform defined in (3) is sometimes called Wavelet Series Transform. Daubechies showed that for a wavelets mother $\psi(t)$ to exist, there should be another related function $\varphi(t)$ whose scaled versions are generated as: $\varphi_j(t) = \varphi(2^{-j}t)$. This function is

called scale function. If the scale function at j=-1 is known, the corresponding wavelet at scale j=0 can be obtained as a linear combination of scale functions from j=-1, which are "compressed" versions of $\varphi_0(t)$ (and this result can be generalized for any scale j) :

$$\psi_0(t) = \sum_k a_k \varphi(2t - k) \qquad (4)$$

III. NYQUIST'S FIRST CRITERION AND WAVELET BASIS

For a better understanding of our paper, we will first emphasize the main principle of Nyquist's first criterion for zero ISI, based on a classical baseband transmission chain drawn in fig. 2:
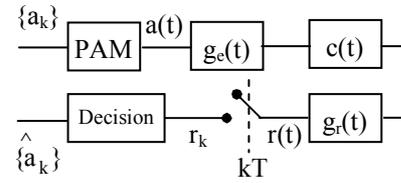

Fig. 2: A baseband transmission chain.

In fig. 2, data symbols $\{a_k\}$ are first passed through a Pulse Amplitude Modulator, obtaining the signal:

$$a(t) = \sum_k a_k \delta(t - kT) \qquad (5)$$

Note that T stands for the total duration allocated for transmitting one data symbol, and it is usually identical to the sampling step used at receiver's side. If we consider that the assembly transmission filter-physical channel-reception filter has the equivalent impulse response g(t), then the signal arrived at receiver will be:

$$r(t) = a(t) * g(t) = \sum_k a_k g(t - kT) \qquad (6)$$

The receiver samples this signal every T seconds, to identify the transmitted symbols. The n-th sample at receiver's side (corresponding to the n transmitted data symbol) will be:

$$\begin{aligned} r_n = r(nT) &= \sum_k a_k g((n-k)T) = \\ &= .. + a_{n-1}g(T) + a_n g(0) + a_{n+1}g(-T) + ... \end{aligned} \qquad (7)$$

From all the terms in (7) it is only the middle term that is useful, because it contains the information corresponding to the n-th transmitted symbol a_n. All the other terms contribute in an undesirable manner to the value of the sample r_n and they are called ISI terms. If we want zero value for this ISI terms, Nyquist's first criterion imposes the following condition on the equivalent channel response:

$$g(nT) = \begin{cases} 1, \text{if } n = 0 \\ 0, \text{if } n \neq 0 \end{cases} \qquad (8)$$

Remember that $g(t)$ is the equivalent response of the sequence transmission filter-channel-reception filter. If we consider now that $g_e(t)$ acts as a pulse shaping filter, then, generally $g_r(t)$ will be a matched filter, having the response $g_r(t)=g_e(T-t)$. In the following, we consider an ideal transmission environment, with $c(t)=\delta(t)$. The receiver examines the received signal and outputs a result every T seconds. Under these assumptions, the equivalent response $g(t)$ will be equal to the autocorrelation function of $g_e(t)$:

$$g(t) = g_e(t) * g_e(T-t) = \int_0^T g_e(\tau) \cdot g_e(t-T+\tau)d\tau =$$
$$= R_{ge}(t+T) \qquad (9)$$

Taking into account (8) and (9), it results that Nyquist first criterion can be reformulated as:

$$R_{ge}[n] = R_{ge}(nT) = \delta[n] \qquad (10)$$

In the following, we will provide a theoretical prove of the fact that any function that generates orthonormal basis by translations with integers, meets Nyquist'a first criterion for zero ISI. The result will be then particularized for mother wavelets functions.

A. Riesz Basis

A family of functions $\{g(x-k)\}_{k \in Z}$ composes a Riesz basis if and only if there are two positive numbers A and B so that:

$$A \sum_{k=-\infty}^{\infty} a_k^2 \leq \int_{-\infty}^{\infty} \left| \sum_{k=-\infty}^{\infty} a_k g(x-k) \right|^2 dx \leq B \sum_{k=-\infty}^{\infty} a_k^2 \qquad (11)$$

This condition can be rewritten as:

$$0 \leq A \leq \sum_{k=-\infty}^{\infty} \left| G(\xi + 2k\pi) \right|^2 \leq B \qquad (12)$$

where $G(\xi)$ has the form of a Fourier transform of $g(x)$ [5]. If we use notation $A(\xi) = |G(\xi)|^2$, then the middle term in relation (12) can be rewritten as:

$$\sum_{k=-\infty}^{\infty} \left| G(\xi + 2k\pi) \right|^2 = \sum_{k=-\infty}^{\infty} A(\xi + 2k\pi) \qquad (13)$$

The right term in (13) can be seen as the spectrum of an ideally sampled signal $a(x)$, the sampling step being equal to 1. $A(\xi)$ represents a power spectral density and thus $a(x)$ can be interpreted as the autocorrelation function of g, according to Wiener-Hinchin theorem.

If we particularize equation (11) for A=B=1 [5], the family $\{g(x-k)\}_{k \in Z}$ becomes an orthonormal basis. In this case, relation (12) will transform in:

$$\sum_{k=-\infty}^{\infty} \left| G(\xi + 2k\pi) \right|^2 = 1 \qquad (14)$$

Since $|G(\xi)|^2$ stands for the spectra of the sampled autocorrelation function of $g(x)$, applying the Inverse Fourier Transform to (14) will lead to a relation similar to (10), which meets the zero ISI criteria. This allows for a very important conclusion to be drawn: any function generating orthonormal basis by translation with integers meets Nyquist's first criterion. In signal processing terms, the signals composing this family can be used as pulse shaping waveforms in a data transmission system. This result provides a high degree of generality and opens new possibilities regarding the way that the pulse shaping filters must be chosen.

B. Pulse Shaping With Wavelets

A well-known property of wavelets families states that for any wavelets mother $\psi(t)$, the family composed of its translated versions $\{\psi(t-k)\}_{k \in Z}$ represents an orthonormal family. Thus, taking into account equation (14), the spectrum of these functions meets the relation:

$$\sum_{k=-\infty}^{\infty} \left| \Psi(\xi + 2k\pi) \right|^2 = 1 \qquad (15)$$

If we replace in (9) $g_e(t)$ by $\psi(t)$, and considering the meaning of (15) then it results that the sampled autocorrelation of $\psi(t)$ leads to $\delta[n]$ as in (10):

$$R_\psi[n] = R_\psi(nT) = \delta[n] \qquad (16)$$

The same conclusion is true for the scale functions $\varphi(t)$ too. Consequently, in the data transmission chain illustrated in figure 2, we can replace the pulse shaping filter impulse response $g_e(t)$ by $\psi(t)$ or $\varphi(t)$ and we can design the matched filters $g_r(t)$ in an optimized manner using the formula:

$$g_r(t) = \psi(T-t) \qquad (17)$$

IV. WAVELET-BASED OFDM VERSUS FOURIER-BASED OFDM

A. WOFDM: an overview

Largely used in the modern communication systems, Orthogonal Frequency Division Multiplexing (OFDM) relies

on a multicarrier approach, where data is transmitted using several parallel substreams. Every stream modulates a different complex exponential subcarrier, the used subcarriers being orthogonal to each other. The orthogonality is the key point that allows subcarrier separation at receiver. This modulation is implemented computing an Inverse Discrete Fourier Transform, (IDFT). In the following the Fourier based OFDM will be noted FOFDM. In the same manner that the complex exponentials define an orthonormal basis for any periodic signal, the wavelet family as defined in (2) forms a complete orthonormal basis for $L^2(\Re)$:

$$< \psi_{j,k}(t), \Psi_{m,n}(t) > = \begin{cases} 1, \text{if } j = m \text{ and } k = n \\ 0, \text{otherwise} \end{cases} \quad (18)$$

Equation (18) indicates that all the members of the wavelet family $\{\Psi_{j,k}(t) = 2^{-j/2}\Psi(2^{-j}t - k)\}_{k \in Z}$ (we considered $s_0 = 2$ and $\tau_0 = 1$) are orthogonal to each other. Consequently, if instead of complex exponential waveforms we use wavelet carriers, we will still be able to separate these subcarriers at receiver, due to their orthogonality. This is the main idea, which lies behind the wavelet-based OFDM (WOFDM) techniques [3,6]. As for the classical OFDM, the WOFDM symbol can be generated by digital signal processing techniques, such as the Inverse Discrete Wavelet Transform (IDWT). In this case, the transmitted signal is synthesized from the wavelet coefficients $w_{j,k} = < s(t), \psi_{j,k}(t) >$ located at the k-th position from scale j ($j = 1, \ldots, J$) and approximation coefficients $a_{J,k} = < s(t), \phi_{J,k}(t) >$, located at the k-th position from the coarsest scale J:

$$s(t) = \sum_{j=1}^{J} \sum_{k=-\infty}^{\infty} w_{j,k}\psi_{j,k}(t) + \sum_{k=-\infty}^{\infty} a_{J,k}\phi_{J,k}(t) \quad (19)$$

Note that, in practice, a sampled version of the sygnal s(t) is generated by means of Malat's Fast Wavelet Transform (FWT) algorithm [5]. In this case, the data vector is composed as follows: data = $[\{a_{J,k}\}, \{w_{J,k}\}, \{w_{J-1,k}\}, \ldots, \{w1,k\}]$ and is modulated onto a contiguous finite set of dyadic frequency bands [6] and onto a finite number of time positions k within each scale. A practical baseband implementation of wavelet OFDM is shown in figure 3.
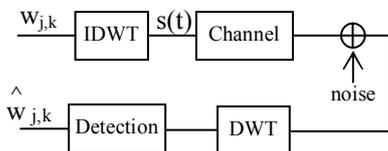

Fig. 3: Baseband scheme of a WOFDM system.

The practical implementation of such a system requires finite-length dyadic data sequences at system input. If we denote by N the length of our input data sequence (which must be a power of 2), then the maximum number of decomposition levels for the DWT equals L=$\log_2(N)$, and equation (19) becomes:

$$s(t) = \sum_{j=1}^{J} \sum_{k=1}^{2^{L-j}} w_{j,k}\psi_{j,k}(t) + \sum_{k=1}^{2^{L-J}} a_{J,k}\phi_{J,k}(t) \quad (20)$$

where J stands for the number of decomposition levels used, (whose the maximum value is L).

As we can remark from the description above, from the conceptual point of view WOFDM and FOFDM look quite similar. Both can be implemented in the baseband, using fast processing algorithms. In the following, we will perform a deeper analysis of both techniques, focusing on the advantages provided by the wavelet-based technique compared to the classical FOFDM. Some of these benefits were at the base of a standardization proposal for the physical layer of IEEE 802.16 (WiMAX) networks [4].

B. Benefits of WOFDM

The goal of this section is to prove that the implementation of WOFDM is simpler than the implementation of the classical OFDM.

As a first point, the practical implementation of a transmission system based on FOFDM requires the use of Nyquist filters, to suppress the ISI. It is already proved in this paper that this kind of filters is not required in the architecture of a transmission system based on WOFDM, since wavelet functions which intervene in IDWT computation act, themselves, as pulse-shaping filters.

On the other hand, a cyclic prefix (CP) is used in order to eliminate the interference between two successive FOFDM symbols and to ease the equalization. The CP is obtained by copying the last samples of the FOFDM symbols in front of it. Since practical implementations of FOFDM use an IDFT modulator, the data at the input of FOFDM system can be viewed as frequency domain samples. Consequently, the CP, which doesn't contain useful information, occupies part of the available bandwidth, leading to a reduction of the spectral efficiency, which is proportional to the length of the CP. For example, the WiMAX standard (IEEE 802.16e) imposes a maximal CP duration of one fourth of the useful symbol, diminishing the spectral efficiency significantly.

A comparison of the power spectral densities (PSD) of WOFDM and FOFDM (with a cyclic prefix ratio of 1/4) is illustrated in figure 4. In order to obtain the spectra of the two modulations, 256 equally likely bipolar symbols (+1 and -1) were passed to an IDFT and IDWT modulator respectively. In the case of FOFDM, a cyclic prefix of 64 symbols was added to form the prefixed FOFDM frame. We achieve the a CP duration of one fourth of the useful symbol. Both types of symbols (FOFDM and WOFDM) were then interpolated to simulate the analog waveforms corresponding to the data sequence to be transmitted.

As fig. 4 illustrates, WOFDM provides a 20 dB extra rejection of adjacent frequency bands compared to FOFDM,
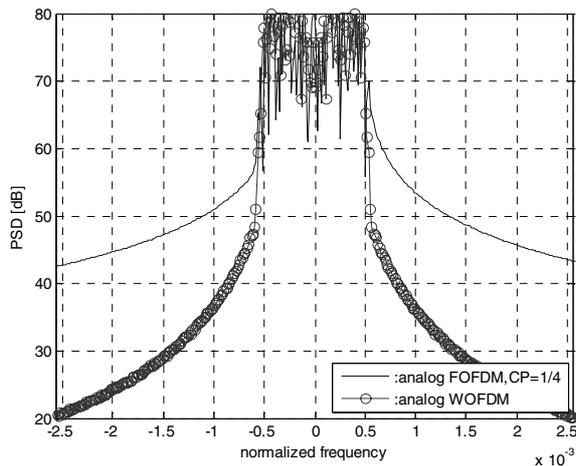
Fig.4: Spectral efficiency of FOFDM and WOFDM: a straightforward comparison

difference mainly caused by the cyclic prefix and by the fact that we consider rectangular waveforms for the input data. In this context, the need for pulse shaping filters (square-root raised cosine) strongly imposes in order to increase the FOFDM spectral efficiency and to provide satisfactory decay of out-of-band sidelobes in practical implementations.

On the other hand, as it is highlighted in the previous section, the use of IDWT inherently leads to a pulse-shaping operation which meets the zero-ISI criterion. Furthermore, the cyclic prefix is meaningless in the case of the WOFDM modulation, and thus the spectral efficiency is not decreased by this overhead.

It is proved in this section that there are several practical advantages of WOFDM modulation compared to its Fourier-based pair. Through simulation means we will now show that, despite its increased simplicity, WOFDM provides similar performance to FOFDM in AWGN channels (figure 5).
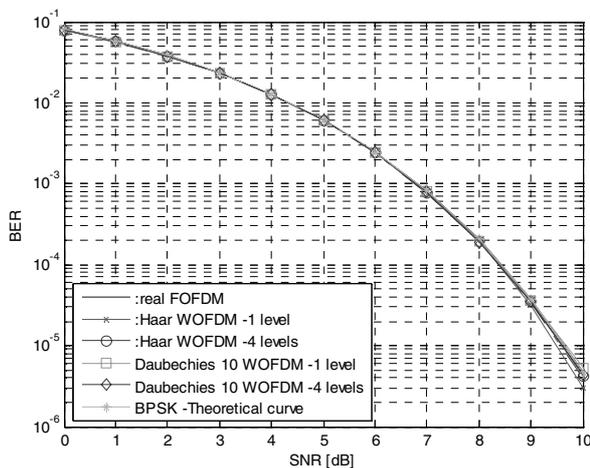

Fig.5: BER performance of FOFDM and WOFDM with different wavelet mothers in AWGN conditions

As an additional conclusion, figure 5 clearly shows that in such channels, BER performance doesn't depend neither on the type of wavelets mother, nor on the number of decomposition levels used in DWT computation. For this

simulation, we considered data blocks of 1024 bits at the modulator input and the BER was computed for the transmission of 10000 such blocks.

The result of these simulations proves to be similar to the theoretical error probability for BPSK transmission over AWGN channels. This is a normal result, since the real gain of such multicarrier techniques in terms of BER performance is to be demonstrated mainly in frequency selective and time variant environments.

A simplified FOFDM transmission scheme can be obtained if, instead of the IDWT/DWT blocks from fig. 3, we use DFT/IDFT computation algorithms. Note that, in general DFT provides a result that is complex. If we want our FOFDM symbol to be real, then certain symmetry of the input signal X[k] is required:

$$X[k] = X^*[N - k + 2], \quad k = 2,...,N/2 \qquad (21)$$

In practical implementations using FFT, the data block length N must be a power of 2. X[1] and X[N/2+1] must be real values. This constraint imposes some supplementary operations to be performed. Thus, if we want a real output, it is possible to compose the data sequence at the IFFT input X[k] from a N/2 length useful data sequence using (21), but this will reduce the useful throughput to 1/2. Another choice is to compute N/2 complex numbers from the N real values (half of the initial sequence will constitute the real part, whereas the other half will compose the imaginary part). Then, the remaining N/2 values will be the conjugate mirror values of the first half of our sequence, computed with respect to (21). All these supplementary operations increase the complexity of the FOFDM modulator. If any importance is paid to this constraint regarding the real form of the signal, then, because of the noise introduced in the channel, the DFT demodulator at reception will output itself a complex data signal, even if the input is real. This will pose supplementary problems at detector side.

On the other hand, if, instead of DFT, the DWT algorithm will be used (thus obtaining a wavelet modulated symbol), then the output signal will be real no matter what is the input signal sequence (if we consider that this input sequence is, itself, real). This is due to the fact that DWT and IDWT are real transforms. The conclusion is that from the point of view of practical implementation of the two techniques, WOFDM outperforms FOFDM. A quantitative measure supporting this conclusion is displayed in figure 6. For all the simulations shown in fig. 5, an adjacent parameter is computed, measuring the simulation time required for every scenario. For the completeness of the result, note that all simulations were executed under Matlab-6 on a Intel Celeron 1.7 GHz processor with 640MB of RAM. The bars shown in figure 6 clearly highlight that the highest computation time is required for FOFDM implementation. This is mainly due to the supplementary operations needed in order to generate a real OFDM symbol, explained above, and to the fact that, for short-time wavelet mothers, Mallat's computation algorithm works faster than FFT [5]. Finally, admitting that some

amount of this difference is caused by an eventually non-optimized software implementation of real OFDM, the difference remains noticeable. Thus, real OFDM requires a 3.8 longer computation time than 1-level WOFDM. This factor modifies to approximately 2.4 if we compare real FOFDM with WOFDM implemented with 4 decomposition levels.
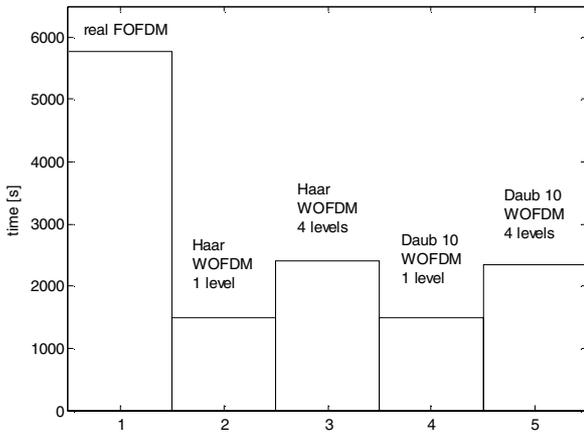

Fig.6: Computation time for FOFDM and WOFDM with different wavelet mothers for different numbers of decomposition levels

V. CONCLUSIONS AND FURTHER RESEARCH

In this paper we promote the use of wavelet functions in data transmissions. Thus, wavelets families obey Nyquist's first criterion and they can replace traditional square root raised cosine pulse shaping filters. A theoretical contribution shows that wavelets are only a particular case of a larger family of functions. Namely, any orthonormal basis (viewed itself as a particular case of Riesz basis) obeys Nyquist's zero ISI criterion and can be use for pulse-shaping purposes. Furthermore, the orthogonality of the signals composing a wavelet family allows to use wavelet carriers in multi-carrier transmission schemes. This guided us towards a deeper comparison of classical FOFDM and of the relatively new WOFDM. Whereas their BER performance is similar in AWGN conditions, WOFDM, which uses orthogonal wavelet carriers, has the advantage of simplicity and of an increased spectral efficiency. Thus, WOFDM implementation doesn't require pulse shaping filters, since IDWT itself (in collaboration with the necessary digital to analog converter which follows it) acts as a pulse-shaper, generating non-interfering waveforms. In addition, since IDWT and DWT are real transforms, the signal at their output is inherently real. Furthermore, Mallat's algorithm for DWT computation is at least as fast (depending on the wavelet mother used) as FFT algorithm.

The spectral efficiency of FOFDM is diminished in practical implementations by the use of a guard prefix. Under these conditions, WOFDM provides significant sidelobes attenuation gain, compared to cyclic-prefixed FOFDM. All

these gathered advantages impose wavelet based transmission schemes as a reliable and efficient alternative to classical OFDM.

It is already proved that WOFDM and FOFDM behave the same in an AWGN channel. It is not yet clear what are the differences between the two techniques in frequency-selective and time-variant environments. The authors in [7] show that there are such channels for which one of the two methods provide better BER performance. It is yet to investigate what are the concrete conditions where WOFDM outperforms FOFDM and vice versa, and this will be one of the author's research topic in the future.

On the other hand, the use of IDWT and DWT transforms in the modulation/demodulation process are not yet fully valued in terms of optimal detection. Thus, at receiver side, after the signal is demodulated by DWT means, we can view the data as coefficients in the domain of the wavelet transform. It is already known that in denoising-by-wavelets applications, there is a whole set of detection techniques which maximizes the signal-to-noise (SNR) ratio following a certain criteria. The application of such a strategy at receiver's side could further improve the BER performance of a WOFDM system compared to FOFDM or to single carrier techniques. This is another subject to be investigated.

Finally, if BER performance in AWGN channels doesn't depend neither on the wavelets mother used nor on the number of decomposition levels, the situation could be different in frequency-selective and time-variant channel respectively. The adaptation of these parameters to the concrete channel conditions is not yet extensively investigated and it is far to be fully exploited. This remark opens a new research field related to the use of wavelets in telecommunications.

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