Performance Comparison of Punctured Turbo Codes and Multi Binary Turbo Codes

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***Abstract*—In this paper we analyze the performance of punctured turbo codes (TCs) and punctured multi binary turbo codes (MBTCs) over additive white Gaussian noise (AWGN) channel in terms of Bit Error Rate (BER) and Frame Error Rate (FER). We consider the original TCs that consist of the parallel concatenation of two identical rate 1/2 recursive systematic convolutional (RSC) codes. We also consider MBTCs that consist of the parallel concatenation of two identical rate 2/3 RSC codes. In order to achieve high spectral efficiency, we focus on codes with higher coding rates, i.e., rates 1/2, 3/5 and 2/3 for both TCs and MBTCs by using puncturing operations. For sake of compatibility with standards IEEE 802.11n and DVD-T, we consider only codes with moderate data block length, i.e., 1504 bits in all cases. The results are illustrated with examples based binary and duo-binary 16-state TCs.**

I. Introduction

The turbo-codes [1, 2] are characterized by their powerful error correcting capability while maintaining reasonable complexity and flexibility in terms of coding rates. Recently, C. Douillard and C. Berrou have proposed a new family of TC with multiple inputs in [3]. Particularly, they show by means of simulations that parallel concatenation of r-input binary RSC codes provides better overall performance than TCs with single input over AWGN channel.

In this paper we extend this analysis to the case of TCs and MBTCs with puncturing. The technique of puncturing is a simple method in order to achieve almost any coding rate with minimal increase of the computational complexity. This technique is particularly interesting in order to increase the spectral efficiency of the transmission scheme. For this purpose we consider the following three cases: 1/2, 3/5 and 2/3 rates. Although we limit our analysis to transmissions over AWGN channel, this approach might be extended to time-varying channels like Nagakami channels [4]. Finally, we use S-interleavers proposed in [5] in all our simulations. S-random interleavers are semi-random interleavers that exhibit excellent performance since they have very high minimum distances even for moderate block sizes. E.g., for block sizes of 752 and 1504 bits, the S-interleavers designed in [6] (see also [7] for a more detailed analysis) yields minimum distances of 19 and 27, respectively. The minimum distance can even be further increased as in [5]. It is based on a random selection with the following constraint:

$$d(i, j) = |\pi(i) - \pi(j)| + |i - j| \geq S$$

where π represents the fully random permutation function and $d(i,j)$, the interleaving distance between the positions i and j, $i,j=1, \ldots, N$ with N codeword size. Based on this method, the interleavers that we design have a minimum distance equal to 22 for block size of 752 and a minimum distance equal with 31 for block size of 1504.

The rest of the paper is structured as follows. Section II describes the constituent RSC encoders for the both TCs and MBTCs. Section III details the puncturing operations for TCs and the punctured MBTCs. The simulation results are provided and analyzed in Section IV. Section V presents some concluding remarks.

II. The Constituent RSC Encoders

The turbo-encoder proposed in [5, 8] consists of the parallel concatenation of two identical rate-1/2 recursive systematic convolutional (RSC) codes with single input and memory 4. Fig. 1a) hows their general structure. We choose $H=[11\ 1\ 12]_{10}$ as generator matrix for its excellent performance as shown in [5, 8]. The constituent encoders for MBTCs are the rate-2/3 RSCs encoders proposed in [3]. Their generator polynomial is equal to $H=[11\ 11\ 1\ 12]_{10}$. More details may be found in [4]. The memory of this encoder, m, is equal to 4, i.e., the encoder can be realized with 4 shift registers as depicted in Fig. 1 b). The feedback and forward polynomials used in this paper are $(23, 35)_8$, respectively. Denote the encoder state as S_m, with $m=1,\ldots,4$; u, u_1 and u_2 represents the input sequence bits, and c the sequence of redundant bits. The state diagrams of the constituent RSC encoder for TCs and MBTCs contain 2^m nodes with $m=4$ at each time lag, which yields to $2^m=16$ possible states; 2^r branches are connected to the each node that correspond to the feasible 2^r inputs vectors that verify the equations of the encoder. Thus, the diagram contains 2^{m+r} branches or transitions. Each transition is associated to one symbol duration. At a given time lag, r inputs bits are considered in order to generate r+1 bit at the output of the encoder.

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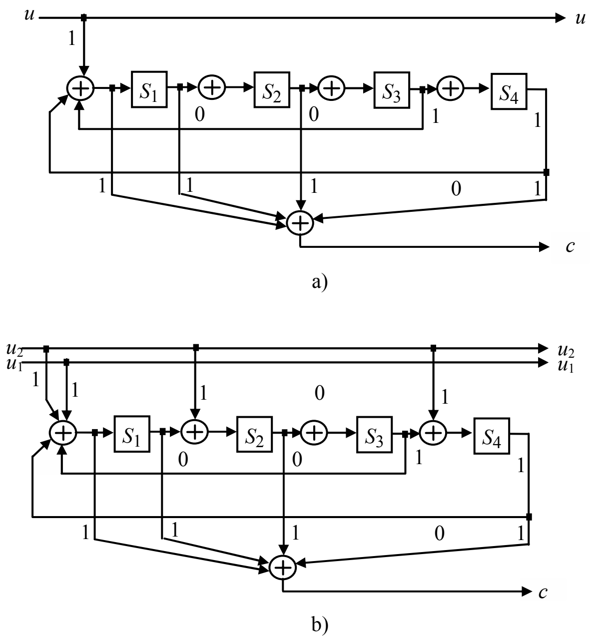

Figure 1. a) The constituent RSC encoder for TC; b) The constituent RSC encoder for MBTC. Both encoders have feedback=$(23)_8=(10011)_2$ and redundancy=$(35)_8=(11101)_2$ polynomials.

In the case of TCs, we have only one binary input, i.e., r=1 as shown in Fig. 1a). For MBTCs, there are two inputs, r=2 as depicted in Fig. 1b). The state diagram has 4 branches in the case of MBTCs instead of 2 branches for TCs.

In the next section, we describe the puncturing operations for both TCs and MBTCs in order to achieve higher rate.

III. PUNCTURED TURBO CODES AND MULTI BINARY TURBO CODES

As previously mentioned, TC and MBTC encoders that we considered in Section II consist of two 16-state, rate r/r+1 RSC encoders with parallel structure. The information bits are interleaved before entering into the second encoder as shown in Fig. 2 a) and Fig. 2b). Without puncturing, the overall code rate of TCs is 1/3 (one input u and three outputs) and the overall code rate is 1/2 for MBTCs (two inputs u_1 and u_2 and four outputs). In order to increase the spectral efficiency of the transmission, higher coding rates can be achieved by puncturing the parity bits c_1 and c_2. The optimal punctured matrices for code rates 1/2, 3/5 and 2/3 are presented in the Table 1 for TCs and MBTCs [9, 10]

TABLE I. OPTIMAL PUNCTURING MATRICES FOR RATES 1/2, 3/5 AND 2/3.

Correcting codes	The code rates		
	***R*=1/2**	***R*=2/3**	***R*=3/5**
TC	$P=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$P=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}$	$P=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$
MBTC	$P=\begin{bmatrix}1\\1\end{bmatrix}$	$P=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$P=\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\end{bmatrix}$

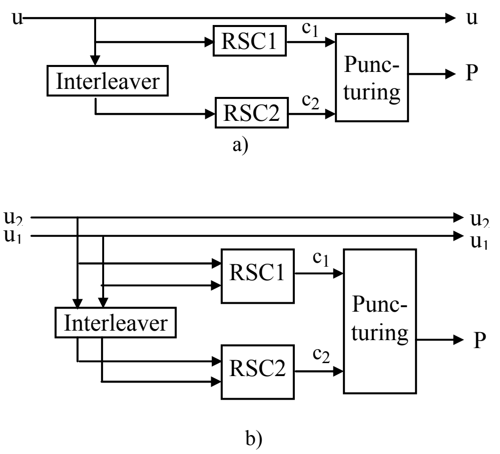

Figure 2. a) Turbo code encoder with 1/2 RSC encoders; b) Multi binary turbo code with 2/3 RSC encoders.

The information bits are not punctured. The puncturing matrices concern only the parity bit(s), i.e., c_1 and c_2 for TCs and for MBTCs.

Remark 1: In order to obtain coding rates higher than $r/(r+1)$ with MBTCs, fewer redundant symbols have to be discarded compared to TCs. Consequently, the correcting ability of the constituent code would be less degraded.

In the section IV we show the simulation results for all rates proposed in Table 1 in terms of bit error rate (BER) and frame error rate (FER).

IV. SIMULATION RESULTS

The tail-biting trellis terminations are used in [3, 5]. We consider the following setup. For both TCs and MBTCs, the trellis of the first encoder is closed to 0 and the trellis of the second encoder is unclosed. To obtain rates 1/2, 3/5 and 2/3 for both TCs and MBTCs, we use the puncturing matrix in Table I. The redundant bits are interleaved by an S-interleaver described in the introduction. For a block length equal with 752 bits, the minimum distance of the interleaver is equal to 22, and for a block length equal with 1504, the minimum distance of the interleaver is equal to 31. The length of the coded sequence is equal to 188 bytes=2·752 bits.

In our simulation we assumed binary input signaling over additive white Gaussian channel (AWGN) and we used the Max-Log-MAP version of the decoding algorithm [11]. This suboptimal version is preferred in practice to optimal MAP [12], due to its low computational complexity while keeping near-optimal performance The scaling factor of the extrinsic information is equal with 0.75 [4]. 15 iterations are assumed at the decoder.

In Figs. 3-8, BER and FER performance of the MBTCs and binary TCs, BTCs, are plotted for rates 1/2, 2/3 and 3/5.

In Fig. 3, for BER<10^{-2}, MBTC exhibit a gain of about 0.1dB of MBTC compared to TC. A similar gain is observed for FER<10^{-4} as shown in Fig. 4. For a FER=5.10^{-5}, MBTC perform as close as 0.55 decibels from the Shannon limit.

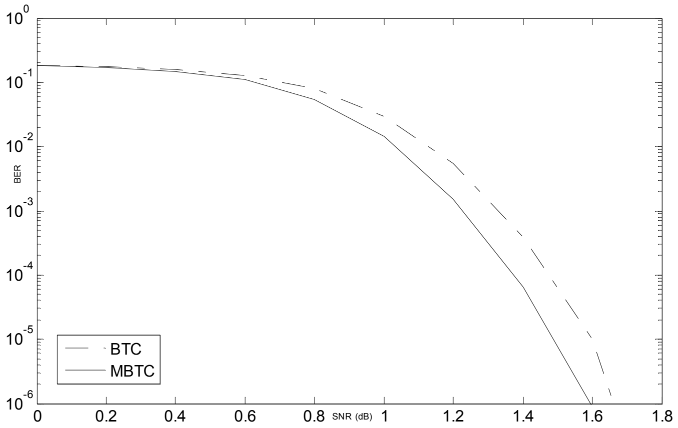

Figure 3. Bit Error Rate (BER) performance for 1/2 rate Turbo Coded, H=[11 1 12] and Multi Binary Turbo Coded, H=[11 11 1 12] transmission over AWGN channel, is plotted as function of SNR.

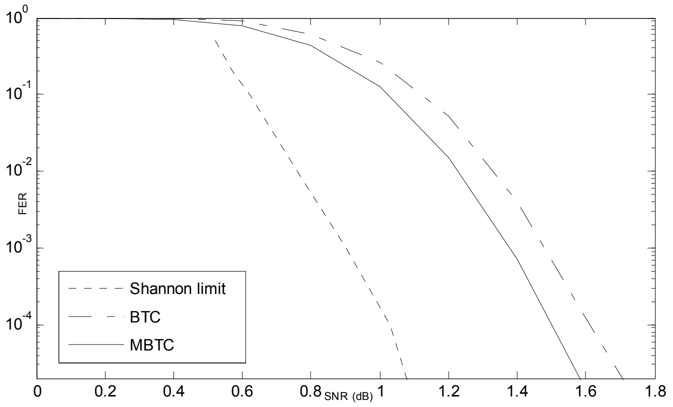

Figure 4. Frame Error Rate (FER) performance for 1/2 rate Turbo Coded, H=[11 1 12] and Multi Binary Turbo Coded, H=[11 11 1 12] transmission over AWGN channel and the Shannon limit are plotted as functions of SNR.

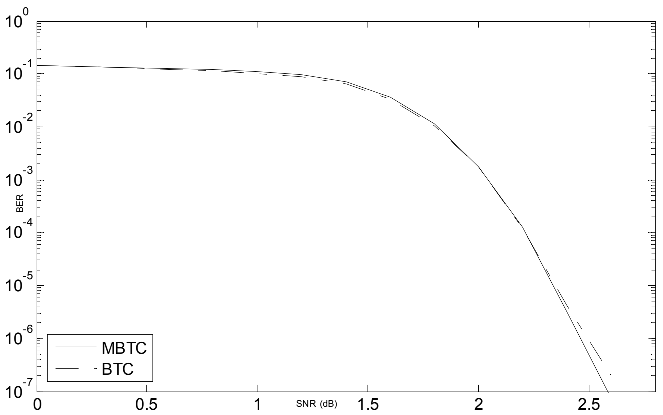

Figure 5. BER performance for 2/3 rate Turbo Coded, H=[11 1 12] and Multi Binary Turbo Coded, H=[11 11 1 12] transmission over AWGN channel is plotted as function of SNR

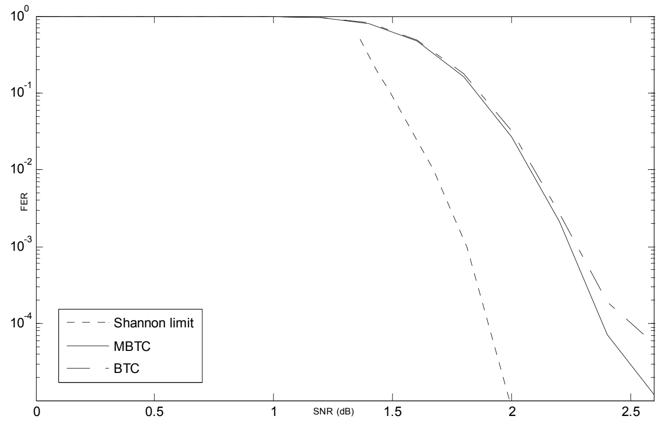

Figure 6. FER performance for 2/3 rate Turbo Coded, H=[11 1 12] and Multi Binary Turbo Coded, H=[11 11 1 12] transmission over AWGN channel is plotted as function of SNR. There is also plotted the Shannon limit.

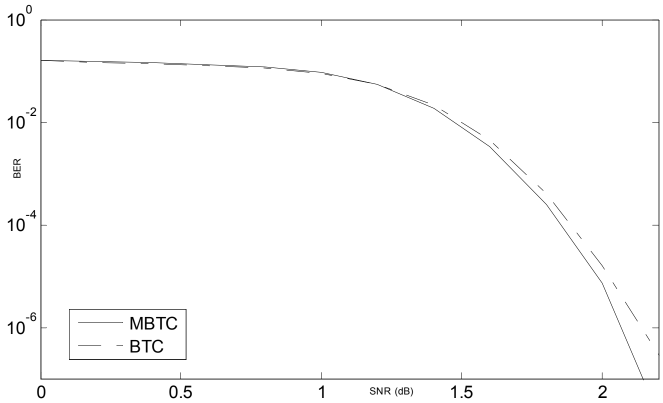

Figure 7. BER performance for 3/5 rate Turbo Coded, H=[11 1 12] and Multi Binary Turbo Coded, H=[11 11 1 12] transmission over AWGN channel is plotted as function of SNR.

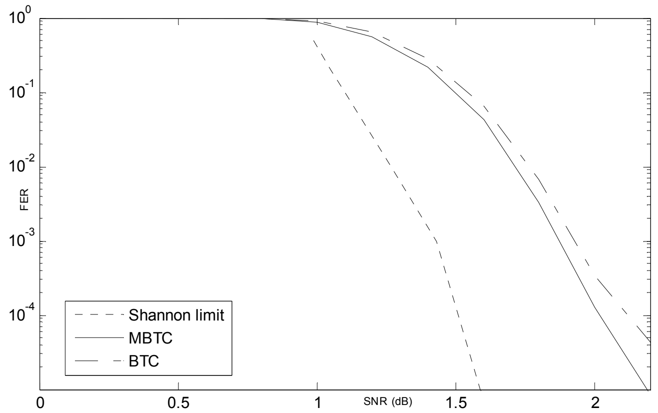

Figure 8. FER performance for 3/5 rate Turbo Coded, H=[11 1 12] and Multi Binary Turbo Coded, H=[11 11 1 12] transmission over AWGN channel and the Shannon limit are plotted as functions of SNR.

BER performance of the both rate-2/3 TCs and MBTCs are similar for BER greater than or equal to 10^{-6}. Beneath this value the MBTC have slightly better performance than TC as illustrated in Fig. 5. Interestingly, the performance gain with respect to FER is larger, i.e., for FER<10^{-4}, the gain is as large as 0.1dB using MBTC. For instance, at FER=$5\cdot10^{-5}$, this gain is equal to 0.15dB as depicted in Fig. 6. For this FER value, MBTC performs as close as 0.5dB of the Shannon limit. Another important feature which can be seen in Fig. 6 is that the error floor seems to be less important in the MBTC case than it is in the TC case.

In Fig. 7, we compare BER performance between MBTC and TC with rate 3/5 for both codes. Both codes perform similarly for BER larger than 10^{-4}. For lower values of BER, MBTC have slightly better performance than TC. For the FER<10^{-4}, we obtained a gain equal to or greater than 0.1dB. For instance, at FER=$5\cdot10^{-5}$ this gain is equal to 0.11dB as illustrated in Fig. 8. For this FER value, MBTC performs as close as 0.55dB of the Shannon limit. As for the rate-2/3, error floor seems to be less important in the MBTC case than it is in the TC case.

To summarize the comparison between punctured TCs and MBTCs, for FER equal to or greater than 10^{-4}, MBTC and TC behave similarly. For lower FER, the punctured MBTC have better performances than the punctured BTC mainly due to their lower error-floor. Note that a small gain of 0.2 decibels in our simulation results is observed in comparison with [3]. This is due to the fact that we assume 15 maximum iterations instead of 8 in [3] and that we use a S-interleaver different from the interleaver used in [3].

V. CONCLUSION

In this paper we compared punctured turbo-codes with single inputs to turbo-codes with multiple inputs. The performance loss caused by puncturing was quantified. By mean of exhaustive simulations, we show that turbo-codes with multiple inputs and with single input perform equivalently for FER equal to or greater than 10^{-4}. We obtained gains equal to 0.1 dB (for the 1/2 and 3/5 rates) and 0.15dB (for the 2/3 rate) at FER = $5\cdot10^{-5}$. Moreover, the punctured MBTCs perform as close as 0.6dB versus the Shannon limit for any rate.

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