Comparison of Wavelet Families with Application to WiMAX Traffic Forecasting

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***Abstract*- This paper deals with a wavelet based forecasting method for WiMAX traffic time series. It is based on an algorithm composed by a few steps. One of these steps is the computation of the Stationary Wavelet Transform, SWT. This transform has two parameters: the mother wavelet which generates the decomposition and the number of decomposition levels. The aim of this paper is to propose a strategy for the selection of the first parameter. Our work is centered on the following wavelet families Daubechies, Coiflet, Symmlet, Biorthogonal and Reverse Biorthogonal. Some simulation results prove the efficiency of the proposed selection method.**

***Keywords*: Wavelets, time-frequency localization, time series, forecasting, WiMAX traffic.**

I. INTRODUCTION

Traffic forecasting has always been a challenging issue for many researchers. Network traffic prediction plays a fundamental role in characterizing the network performance and it is of significant interest in many network applications (admission control, network management). Recently, many approaches involving time series models have been used for traffic forecasting, such as pure statistical or based on neural networks.

The SWT has been used for time series analysis and traffic forecasting in recent years. Wavelets provide a useful decomposition of the time series, in terms of both time and frequency, can effectively diagnose the main frequency component of the signal and also can abstract local information of the time series.

One of the main properties of wavelets is that they are localized in time (or space) which makes them suitable for analyzing non-stationary signals (signals containing transients and fractal structures). Sine and cosine functions, used in Fourier analysis are localized only in frequency. Therefore, small temporal changes in the observed signal can change almost all components of the corresponding Fourier expansion, [3]. The SWT has two parameters: the mother wavelet which generates the decomposition and the number of decomposition levels.

Applications of SWT in the field of WiMAX network traffic forecasting were reported in [5 - 8]. The goal is to predict traffic based on some historical data. Analyzing the traffic contained in the historical data in the WT domain, two of its parameters are extracted: its overall tendency and its variability. In [5], for each of these parameters, some statistical models based on first order ARIMA are built. Future events, like for example the saturation of a base station can be predicted using these models. Other prediction techniques, based on the use of neural networks in connection with the SWT, are proposed and compared with the pure statistical approach in [5-7]. The quality of prediction realized using the methods in [5-8] depends on the mother wavelets used for the computation of the SWT. These methods were optimized by the selection of the best mother wavelets. This optimization was done by brute force. The aim of this paper is to propose a mother wavelets selection method based on a solid theoretical explanation which implies the mother wavelets time-frequency localization.

The rest of the paper is organized as follows: Section II reminds the construction of the SWT in connection with the concept of multi-resolution analysis, the time-frequency localization of mother wavelets and the best known wavelet families. In Section III is presented the analyzed forecasting method. Section IV is dedicated to simulation details and results and, finally, the conclusions are presented in Section V.

II. THE SWT

The Continuous Wavelet Transform (W) is a relatively new mathematical tool, capable of providing time and frequency information simultaneously, hence giving a linear time-frequency representation of the signal.

The main difference between Fourier series and Wavelet series (which are obtained by the discretization of W in the frequency domain) is that the functions of a wavelet basis are double indexed (having a time index and a frequency index), while in the case of the Fourier basis there is a single frequency index.

A linear time-frequency transform correlates the signal with a family of waveforms that are well concentrated in time and in frequency. These waveforms are called time-frequency atoms, [1]. A family of time-frequency atoms (wavelet basis) $\psi_{u,s}(t)$ is generated by translating and dilating the mother wavelets:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-u}{s}) \quad (1)$$

which generates a set of functions, called daughter wavelets that can form a basis.

The Continuous Wavelet Transform of $f \in L^2(\Re)$ at time u and scale s (the frequency is the inverse of the scale) is a convolution of the wavelet $\psi_{u,s}(t) \in L^2(\Re)$ (obtained by dilation and translation of the mother wavelets ψ) with the signal $f \in L^2(\Re)$:

$$W_f(u,s) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\psi^*(\frac{t-u}{s})dt = f * \psi^*_{u,s} \quad (2)$$

Wavelet transform maps a raw data (observation of an underlying function, *f*) to a collection of coefficients which provide the information about the behavior of the function at a certain point of time, during a certain period around that point in time. The coefficients indicate what the function is doing, in which moment of time. More precisely, it measures the change of the local average at a specific scale around a specific moment of time.

The wavelet decomposition can be realized in different bases being more general than the Fourier decomposition, which uses a single basis. So, the wavelet analysis provides immediate access to information that could not be reachable using other methods such as Fourier analysis, [12].

A. Multi-resolution analysis

The multi-resolution analysis (MRA) was introduced by Stephane Mallat and Yves Meyer. Based on this concept, mother wavelets generating orthonormal bases of $L^2(R)$ can be built [2].

The motivation of MRA is to generate a sequence of embedded subspaces to approximate $L^2(R)$ for choosing a proper subspace for a specific application, in order to get a balance between accuracy and efficiency.

Mathematically, the MRA represents a decomposition of $L^2(R)$ into a sequence of closed subspaces, V_j, $j\in$ Z, which approximate $L^2(R)$ and the following conditions are satisfied:

1. $$\cdots V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \cdots, \quad (3)$$

2. $\overline{\bigcup_{j\in Z} V_j} = L^2(R)$, ($L^2(R)$ is the closure of the union of all V_j), and $\bigcap_{j\in Z} V_j = \{0\}$.

3. $$f\in V_j \Leftrightarrow f(2t)\in V_{j+1},\ \forall j \in \mathrm{Z} \quad (4)$$
$$f\in V_0 \Leftrightarrow f(t\text{-}k)\in V_0, \forall k \in \mathrm{Z}$$

4. There exists a function φ(*t*) such that its translates form an orthonormal basis for V_0. It can be proved that {φ(2*t*-*k*)} is an orthogonal basis for V_1.

Similarly, if $\varphi_{jk}(t) = 2^{j/2}\varphi(2^j t - k)$ then $\{\varphi_{jk}(t)\}$ forms an orthonormal basis for V_j. The function φ, which generates the bases of spaces V_j, is called *scaling function* of the multiresolution analysis. Its expression determines the analytical expression of the corresponding mother wavelets, ψ.

A direct application of MRA is the fast Discrete Wavelet Transform (DWT) algorithm. The DWT represents the discretization of a wavelet series in the time domain (or the discretization of a Continuous Wavelet Transform both in frequency and time). It decomposes discrete time signals into low-pass and high-pass components, sub-sampled by 2, while the inverse transform performs the reconstruction, [1].

The MRA is a procedure of analysis of a signal *s*(*t*) that takes into account its representation at multiple time resolutions. Adapting the signal's resolution allows one to process only the relevant part for a particular task. When the original signal *s*(*t*) is involved, the maximal resolution is exploited. When a variant of the original signal, for example the signal *s*(2*t*), is used then a poorer resolution is exploited. Combining few resolutions a MRA is obtained. Generally, the MRA is implemented based on the algorithm proposed by S. Mallat [1], which corresponds to the computation of the DWT. The signal is passed through a series of high pass filters and through a series of low pass filters. At each level, the high-pass filter (associated with mother wavelets) produces detail information *d*[*n*], while the low-pass filter (associated with the corresponding scaling function) produces coarse approximations, *a*[*n*]. Filtering operations determine the signal's resolution, meaning the quantity of detail information in the signal, while the scale is determined by up-sampling and sub-sampling operations.

The reconstruction operation is the inverse process of decomposition. The DWT of the original signal is obtained by concatenating all coefficients *a*[*n*] and *d*[*n*], starting from the last level of decomposition. Due to successive sub-sampling by 2, the length of the signal must be a power of 2, or at least a multiple of power of 2 and it determines the number of levels the signal can be decomposed to. The disadvantage of Mallat's algorithm is the decreasing of the length of the coefficient sequences with the increasing of the iteration index due to the use of decimators.

Another way to implement a MRA is to use the *à trous* algorithm, proposed by Shensa [9], which corresponds to the computation of the Stationary Wavelet Transform (SWT). The SWT decomposition tree is presented in Figure 1. In this case the use of decimators is avoided but at each iteration different low-pass and high-pass filters are used. At each level the filters are up-sampled versions of the corresponding filters from the previous level. So the differences between SWT and DWT are that the signal is not sub-sampled in the SWT case but the filters are up-sampled at each level. The SWT is an inherently redundant scheme as each set of coefficients contains the same number of samples as the

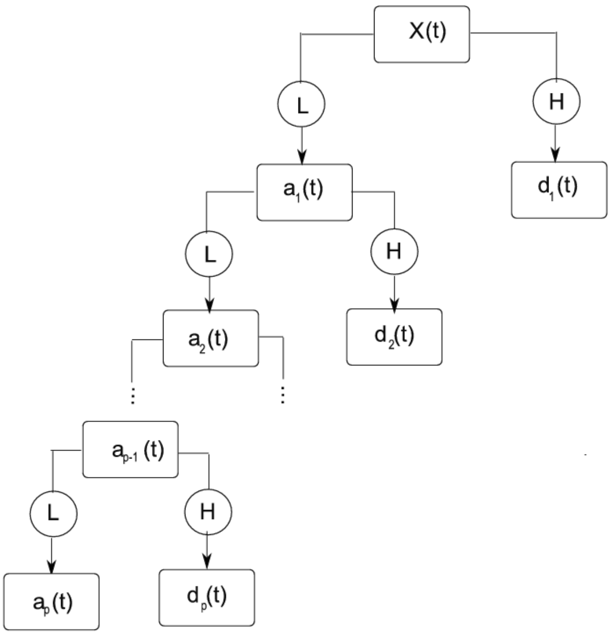


Figure 1: The Stationary Wavelet Transform decomposition tree.

input – so, for a decomposition of N levels, there is a redundancy of $2N$. Because no sub-sampling is performed, sequences $a_1[n]$ and $d_1[n]$ are of length N (instead of $N/2$ as in the DWT case). At the next level of the SWT, $a_1[n]$ is split into two using modified filters obtained by dyadically up-sampling the filters from the previous level, as presented in Figure 1. This process is continued recursively.

B. Wavelet families

There is a variety of wavelet families [1] such as: Daubechies, Symmlet or Coiflet, etc. The qualities of their elements vary according to several criteria: the length of the support of the mother wavelet, the number of vanishing moments, the symmetry or the regularity. Another two criteria are of high importance: the existence of a corresponding scaling function and the orthogonality or the bi-orthogonality of the resulting analysis. Since the mother wavelet produces through translation and scaling all wavelet functions used in the transformation, it determines the characteristics of the resulting Wavelet Transform. Therefore, details of a particular application should be taken into account and the appropriate mother wavelet should be chosen in order to use the Wavelet Transform effectively.

1. Orthogonal Wavelet Families

The Daubechies wavelet family is named in the honor of its inventor, the Belgian physicist and mathematician Ingrid Daubechies and represents a collection of orthogonal mother wavelets characterized by a maximal number of vanishing moments for some given length of the support. Corresponding to each mother wavelets from this class, there is a scaling function (also called father wavelet) which generates an orthogonal MRA. The elements of the Daubechies' family mostly used in practice are db2÷db20. The index refers to the number of vanishing moments. The number of vanishing moments is equal to half of the length of the support in the case of Daubechies family of mother wavelets. For example, db1 (the Haar wavelet) has one vanishing moment, db2 has two, etc. The Daubechies mother wavelets are not symmetric.

The Haar wavelet was the first mother wavelets proposed by Alfred Haar in 1909 and has the shortest support among all orthogonal wavelets. It is not well adapted to approximating smooth functions because it has only one vanishing moment. Haar wavelet transform has some advantages: it is conceptually simple and fast, it is memory efficient, and it is a good choice to detect time localized information. Symmlets, also known as Daubechies least asymmetric mother wavelets, are compact, orthogonal, continuous, but only nearly symmetric mother wavelets. Their construction is very similar to the construction of Daubechies wavelets, but the symmetry of Symmlets is stronger than the symmetry of Daubechies mother wavelets. Coiflets are mother wavelets designed by Ingrid Daubechies and named in the honor of Ronald Coifman (another researcher in the field of wavelets theory) to be more symmetrical than Daubechies mother wavelet and to have a support of size $3p$ - 1 instead of $2p$ - 1 (like in the case of Daubechies mother wavelets).

2. Biorthogonal Wavelet Families

Biorthogonal wavelets exhibit the property of linear phase, which is needed for signal and image reconstruction. If, instead of a single wavelet, two wavelets are used (one for decomposition and the other for reconstruction), interesting properties are derived. Designing biorthogonal wavelets allows additional degrees of freedom than orthogonal wavelets, for example the possibility of constructing symmetric wavelet functions, [1]. An example of several wavelets mother, generated in Matlab, is presented in Figure 2. The number which follows the wavelet name represents the number of vanishing moments. Reverse biorthogonal wavelets family is obtained from biorthogonal wavelet pairs. Both biorthogonal and reverse biorthogonal wavelet families are compactly supported biorthogonal spline wavelets for which symmetry and exact reconstruction are possible using Finite Impulse Response (FIR) filters.

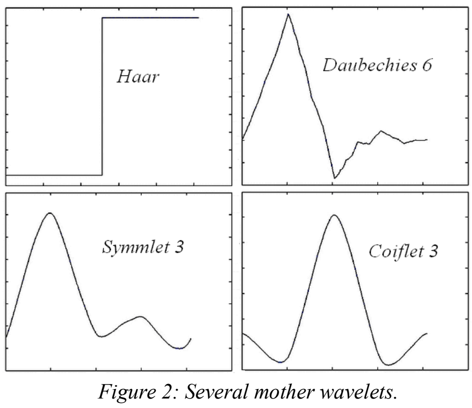


Figure 2: Several mother wavelets.

A. Time-frequency localization

In [2] it is highlighted, based on the duality of the Fourier transform, that signals perfectly localized in time have an unlimited bandwidth, meaning that they are not localized in frequency. As well, band limited signals have an infinite duration. Therefore, to measure these quantities two concepts are used: the effective duration, σ_t and the effective frequency band σ_ω. A measure of the time-frequency localization of a given signal can be obtained by the product $\sigma_\omega^2 \cdot \sigma_t^2$, [4].

The Heisenberg uncertainty principle states that the following inequality is true:

$$\sigma_t^2 \sigma_\omega^2 \geq \frac{\pi}{2} \quad . \qquad (5)$$

The shorter is the effective duration of a signal the wider is its effective frequency band.

In the case of the Wavelet transform (DWT or SWT), both time and frequency localizations depend on the scale factor s, [2]. The Continuous Wavelet Transform can be stated as a scalar product for every value of the scale factor s:

$$W_x(s,t) = \langle x(\tau), \psi_s(\tau - t) \rangle, \quad \psi_s(\tau) = \sqrt{s}\psi(s\tau) \qquad (6)$$

If $\psi(\tau) \in \Re$ we will have:

$$W_x(u,s) = x(u) * \breve{\psi}_s(u), \qquad \breve{\psi}_s(t) = \psi_s(-t) \qquad (7)$$

Therefore, for every s>0 the wavelet transform of a signal $x(t)$ represents the response of a linear time-invariant system at $x(t)$, having the impulse response $\breve{\psi}_s(t)$. The system has the frequency response:

$$\mathcal{F}\{\breve{\psi}(t)\}(\omega) = \mathcal{F}\{\psi_s(t)\}(-\omega) = \mathcal{F}\{\sqrt{s}\psi(-st)\}(\omega) =$$
$$= \frac{1}{\sqrt{s}} \mathcal{F}\{\psi(t)\}(-\frac{\omega}{s}) \qquad (8)$$

So, the temporal "window" $\psi_s(t)$ is "responsible" for the temporal localization of the signal $x(t)$, while the frequency "window" $\mathcal{F}\{\psi_s(t)\}(-\frac{\omega}{s})$ is "responsible" for the localization in frequency. The effective duration and the effective frequency band are:

$$\sigma_t^2 = \frac{{}_t\sigma^2}{s^2} \text{ and } \sigma_\omega^2 = s_\omega^2 \sigma^2, \qquad (9)$$

where ${}_t\sigma^2$ and ${}_\omega\sigma^2$ represent the duration of the temporal "window", respective the bandwidth of the frequency "window". More theoretical details are given in [2].

It is noticed that the time localization is getting worse with the increasing of the factor s, while frequency localization improves with the increasing of s. Also,

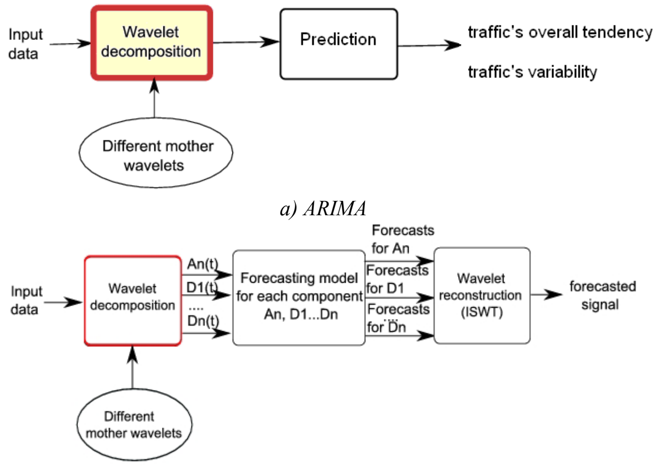
$$\sigma_t^2 \sigma_\omega^2 = {}_t\sigma^2 {}_\omega\sigma^2 \quad . \qquad (10)$$

Regardless of the value of s, the time-frequency localization determined by $\psi_s(\tau)$ *is identical with the one realized by the generating "window"* $\psi(t)$. In [4] it is stated that Haar functions have good time localization, but they have an infinite effective bandwidth, meaning that they are not localized in frequency. Contrary, cardinal sinus functions have good frequency localization, but they have an infinite duration. These two examples represent extreme cases, but between them there are mother wavelets (for example the elements of the Daubechies family) for which the product gives finite values. These functions have poorer time localization than Haar functions and poorer frequency localization than the cardinal sinus, but they provide a better time-frequency „compromise" than Haar or cardinal sinus.

Some conclusions can be drawn from [4]. First, the effective duration of the wavelets functions is stronger influenced by the number of vanishing moments than their effective bandwidth. Then, the effective duration of the Daubechies wavelets increases monotonically with the number of vanishing moments; an opposite evolution is observed for the effective bandwidth. Finally, the time-frequency localization of wavelets from the Daubechies family monotonically increases with the number of vanishing moments. The aim of the proposed paper is to apply the conclusions presented in [4] to the forecasting methodology.

III. Forecasting Methodology

The forecasting methods we proposed in this paper are based on the SWT and several prediction methods. In Figure 3 are shown the main steps followed in our work. The part we refer to in this paper is the SWT. We used the SWT to decompose the original signal into a range of frequency bands. The level of decomposition (n), depends on the length of the original signal. For a discrete signal, in order to be able to apply the SWT, if decomposition at level n is needed, 2^n must divide evenly the length of the signal. The n^{th} level of decomposition, gives us n + 1 signals for processing: one approximation signal and n detail sequences. The value of n gives the maximal number of resolutions which can be used in the MRA. It corresponds to the poorer time resolution. The decision of choosing 16 samples per day is argued in [11]. There is shown that WiMAX traffic exhibits some periodicities which are better noticed if we modify the sampling interval from 15 minutes to 90 minutes. This represents the highest time resolution which is used in the proposed MRA. By preliminary analyses of our data, we have decided to use six decomposition levels for the MRA of WiMAX traffic, starting with a time resolution of 90 minutes. So, the second parameter of the SWT (the number of decomposition levels) is selected. The aim of this paper is the selection of the first parameter of the SWT, the mother wavelets.



a) ARIMA

b) ANN and RW

Figure 3: The forecasting framework.

High frequency components can be used to predict the near future (short-term dependencies which describe the traffic variability), while low frequency components can capture long-range dependencies (the overall tendency of the traffic).

The aim of [5] was to find statistical models for the overall tendency and for the variability of the traffic. These models can be found neglecting some resolutions from the corresponding MRA. Selecting only approximation coefficients at the sixth decomposition level of the SWT, a_6, we can predict the overall tendency of WiMAX traffic. Hence, the overall tendency of the traffic is a very low frequency signal, requiring mother wavelets with good frequency localization. The traffic variability can be predicted by selecting only detail coefficients at the third and fourth decomposition levels, d_3 and d_4. Hence, the traffic's variability is a relative high frequency signal, requiring mother wavelets with good time localization. In consequence the proposed forecasting methodology requires mother wavelets with both time and frequency good localization. So, we consider that **the best results will be obtained using mother wavelets with a good time-frequency localization which corresponds to a reduced number of vanishing moments.** Considering that the prediction accuracy of the traffic variability is more important than the prediction accuracy of the traffic overall tendency, it results that time localization is more important than the time-frequency localization. So, **the best mother wavelet seems to be the Haar wavelet in our case.**

After the computation of SWT with six decomposition levels, we set to zero all detail coefficients d_1, d_2, d_5 and d_6 and we compute a linear combination of the rest of coefficients, which will serve in the next step. In references [5-7] were used the following prediction methods: AutoRegressive Integrated Moving Average (ARIMA), described in [4, 5] (which is a pure statistical prediction method), Artificial Neural Network (ANN), [5, 6] and Random Walk (RW), [5]. In the case of the pure statistical prediction method, considered in Figure 3a, we have obtained separately the first order linear models of the traffic overall tendency by processing the sequence a_6 and of the traffic variability by processing the couple of sequences d_3 and d_4. The trajectory of each of these models is represented through a sloping line corresponding to the weekly increase.

In the case of other prediction methods based on ANN or on RW at the output of the system in Figure 3b) the future traffic predicted is directly obtained. These predictions are made for every sequence obtained after decomposition. The final forecasted signal is obtained after applying the Inverse SWT. In this case, the level n of decomposition is chosen in function of the selected number of samples per day: so for 16 samples we will have 3 levels of decomposition. The forecasting is done by using $n + 1$ Artificial Neural Networks (ANN), one for the approximation and n for the details, [7].

IV. SIMULATION DETAILS AND RESULTS

All the simulations were made using Matlab® software, and Wavelab850 toolbox [10], a library containing Matlab functions, very useful for implementing a variety of algorithms related to wavelet analysis.

A. Data sets

To evaluate our method we used data obtained by monitoring the traffic from 67 Base Stations (BS) composing a WiMAX network. The period of collection is of eight weeks, from March 17^{th} till May 11^{th}, 2008.

We have divided each data sequence into two parts, each corresponding to a specific interval of time. Data from the first interval were considered as historical and were used for prediction, while data from the second interval were used to evaluate the quality of prediction. Our data base is formed by numerical values representing the total number of packets from uplink and downlink channels, for each of the 67 BSs. The values were recorded every 15 minutes, so it can be easily deduced that for a given BS we have 96 samples/day, 672 samples/week, and a total number of 5376 samples. More details about the data bases used in this work are given in [5].

B. Quality Evaluation

To evaluate the performance of predictions, we considered the following statistical measures of error: the Mean absolute error (MAE), the Mean Square Error (MSE), the analysis of variance (ANOVA), the Symmetric Mean Absolute Percent Error (SMAPE), and the Root Mean Square Error (RMSE). We also computed SMAPE L, MAPE L and MAE L. These errors are calculated between the weekly mean of the original signal acquired after the historical interval and the weekly mean of the predicted signal for to the same interval. By applying ANN or RW we can obtain forecasts for each moment of time.

C. Results

Since our purpose was to compare the influence of different wavelet families on the prediction accuracy, we used the following wavelet families: Daubechies (db1, db2, db3, db4, and db5), Coiflet (coif1, coif2), Symlet (sym2), Biorthogonal (bior3.1), and Reverse Biorthogonal (rbio1.1, rbio2.2, rbio3.3). According to Table I, the 1st order Daubechies wavelet, db1, which is the simplest of the Daubechies family, and rbio 1.1 give the best prediction performance. Also, wavelet mother db3 provides good prediction results. The results represent the mean values for all the three forecasting methods (ARIMA, ANN and RW) and all the 67 BSs, with the observation that in the case of ARIMA only SMAPE L, MAPE L and MAE L could be calculated. The values obtained indicate that with the increase of the number of vanishing moments, the performance of the traffic prediction deteriorates.

V. CONCLUSIONS

In this paper we evaluated the WiMAX traffic prediction accuracy by using different types of mother wavelets. We have inferred that the mother wavelets selection must be realized searching the best time localization. This hypothesis was validated by simulations (see Table I). The best forecasting accuracy (the smallest prediction error) is obtained using Haar mother wavelets, db1. Good forecasting accuracy was obtained using mother wavelets with good time-frequency localization, which have a reduced number of vanishing moments, like rbio1.1 or db3. Therefore, we can conclude that in the case of WiMAX traffic, in order to obtain a good prediction, it is necessary to use Haar mother wavelets or mother wavelets having good time-frequency localization.

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TABLE I
A COMPARISON OF DIFFERENT QUALITY MEASURES FOR THE PROPOSED FORECASTING METHODS.

Mother wavelets	RSQ	SMAPE	MAPE	MSE	RMSE	MAE	SMAPE L	MAPE L	MAE L
coif 1	1.445	1.09	**0.2113**	11.72	2.80	1.0304	0.890	0.0020	0.9599
coif 2	1.493	1.22	0.2285	12.95	2.83	0.8748	0.837	0.0019	0.7191
db1	**1.168**	**1.08**	0.2367	**8.06**	**2.43**	**0.7685**	**0.812**	**0.0016**	0.7327
db2	1.364	1.15	0.2451	10.52	2.69	0.8408	0.855	0.0019	0.7768
db3	1.358	1.12	**0.2117**	**9.76**	**2.64**	0.8193	0.857	0.0018	0.7678
db4	1.490	1.11	0.2159	10.61	2.58	0.7985	0.834	0.0018	0.7563
db5	1.435	1.11	0.2190	12.56	2.75	0.8339	0.823	0.0019	0.7730
bior 3.1	0.695	1.13	0.3152	9.86	2.52	1.00	0.860	0.0018	**0.7071**
rbio1.1	**1.200**	**1.08**	0.2215	10.00	2.61	**0.7948**	**0.820**	**0.0017**	0.8947
rbio2.2	1.482	1.19	0.3202	10.29	2.71	0.8747	0.891	0.0018	0.7690
sym2	1.365	1.26	0.2146	13.20	2.89	0.8854	0.895	0.0019	0.7412