Wavelet OFDM Performance in Frequency Selective Fading Channels

Marius Oltean, Miranda Naforinita,
Faculty of Electronics and Telecommunications, Timisoara, Romania
Agenda

- WOFDM overview
- DWT-based implementation of WOFDM
- Daubechies wavelets in the time-frequency plane
- Simulation scenario
- Results & Conclusions
Objectives

- To evaluate the performance of the WOFDM transmission through FSF channels
- To highlight the influence of the wavelets “carriers” on the WOFDM performance
  - Time-frequency localization of the carriers
  - Time & frequency characteristics of the channel
  - BER performance
WOFDM Overview

- WOFDM = Wavelets based OFDM
- It can be seen as an orthogonal, “multi-carrier” modulation
- OFDM’s complex exponentials replaced by wavelet “multiple carriers”
- WOFDM symbol can be expressed as:

\[
s(t) = \sum_{j=-J}^{0} \sum_{k} w_{j,k} \psi_j(t-k) + \sum_{k} a_{-j,k} \varphi_{-j}(t-k) \quad (1)
\]

- \(\{\psi_j(t-k)\}\) defines an orthogonal family of carriers
- Data symbols to be sent may be interpreted as a sequence of approximation (\(a_{j,k}\)) and wavelet coefficients (\(w_{j,k}\))
- Numerical implementation (Mallat’s IDWT)
DWT based implementation

Fig. 1: IDWT (Mallat) based implementation of WOFDM.
Daubechies wavelets in the time-frequency plane

- Excellent time localization
- Very poor frequency localization
- Good frequency localization
- Poor time localization

Number of vanishing moments
Time and frequency response depends on the power/delay of the second path and on the Doppler shift.

T-F properties of the wavelet carriers adapted to the T-F properties of the channel.

Background: in flat, time variant channels, wavelets that have good time localization (e.g., Haar) lead to the best results.
Focus on transmitter/receiver

- Equally likely bipolar symbols (BPSK)
- Daubechies wavelets tested in the IDWT modulator
  - 4 iterations (4 transmission scales)
- BER performance is computed for the overall block, as well as on a per-scale basis
  - Lower transmission scales stronger affected by ISI
  - Higher transmission scales stronger affected by Doppler

Fig. 2: Simulation scheme for the WOFDM transmission in a two-ray FSF channel.
Focus on the channel

Two ray channel
Both paths are random (fluctuant)
Time variability
  - $\text{ray}_k[n]$: Rayleigh pdf, Jakes’ PSD, $f_m$: maximum normalized Doppler shift
Frequency selectivity
  - $\tau_1$: relative delay of the second path
  - $P_1$: the relative power of the delayed path
  - Multipath delay spread:
    \[ \sigma_\tau = \sqrt{\tau^2 - (\bar{\tau})^2} \] (2)
Overall BER performance

Fig. 3: Overall BER performance of a WOFDM/OFDM transmission using Daubechies-8, in a two ray FSF channel, $\tau_1=1$.

- **Poor BER results because**: NO equalization, NO channel coding, NO compensation for the Doppler shift
- **No difference made by the Doppler shift when the ISI component is strong**
  - A difference only occurs when $P_0/P_1=10$
- **WOFDM slightly better than OFDM**
Results
Haar vs Daub20: some time, frequency and scale considerations

Some transmission scales are stronger affected by ISI than others
- No BER difference among different wavelets

When ISI is less critical, wavelets with good frequency localization achieve better results
- The result is the opposite of the flat fading channel case

At the same scale, Daub20 performs much better than Haar

Fig. 4: Haar vs Daubechies 20: the BER performance at the third and the fourth transmission scale, $P1=P2$, $fm=0.005$. 

![Graph showing BER performance comparison between Haar and Daub20 wavelets at different transmission scales.](image)
Results

BER performance (Daubechies wavelets)

When ISI and Doppler are “balanced”, wavelets that achieve good T-F compromise lead to better results

- Best result: Daubechies-12, worst result: Daubechies-4
- at the fourth scale, we have the strongest impact of Doppler, and the less important ISI influence

Fig. 5: BER performance at the fourth scale of a WOFDM transmission, for all the Daubechies wavelets, $P1=P2$, $fm=0.05$. 
Conclusions

- For the considered channel model, ISI strongly degrades the BER
  - No overall BER difference among the tested wavelets, at the scales dramatically impacted
  - Interesting effects may be however shown on the transmission scales
- At some scales, less distorted by ISI, a good frequency localization of the carriers transforms into an advantage
  - In some particular cases, wavelets with a good time-frequency localization led to the best results
Thank You!

Marius Oltean & Miranda Naforina