Wavelet OFDM Performance in Frequency Selective Fading Channels

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***Abstract*—The BER performance of a wavelet-based multi-carrier modulation scheme is investigated in this paper. The channel is considered both frequency-selective and time-variant. It is shown that the time-frequency localization of the wavelet carriers has a significant influence on the BER performance, especially at certain transmission scales.**

Keywords- wavelet OFDM, time-frequency localization, fading

I. INTRODUCTION

Wavelets represent a successful story of the last decade in signal processing. They are widely used in various applications as compression, denoising, segmentation or classification. By the other hand, in data communications, the same successful story can be assigned to multi-carrier modulation techniques. Thus, most of the wireless data transmission standards propose Orthogonal Frequency Division Multiplexing (OFDM) as transmission technique at their physical layer. We can mention here WiFi (IEEE 802.11), WiMAX (IEEE 802.16) or LTE (3 GPP rel. 8). It is mainly the very good resilience of OFDM to the inter-symbol interference (ISI) that propels OFDM as a highly reliable choice.

The Wavelet OFDM (WOFDM) technique, sometimes referred to as wavelet modulation, is the point where the above concepts meet with each-other. Although they are widely used in signal processing, few wavelets applications are known in data transmission. The idea that gathers the two concepts is to use wavelet signals as carriers in an orthogonal, multi-carrier data transmission.

Despite its undoubted advantages, OFDM has some drawbacks too. Some of them can be counteracted by associating the multi-carrier concept and the wavelet signals. Thus, some wavelet based multi-carrier schemes may provide significant out-of-band side-lobes rejection, by comparison to OFDM [1]. Furthermore, WOFDM proved to be more robust to the time variability of radio the channel [2]. Due to their time-scale nature and to their finite duration, wavelet carriers provide a better "time-frequency" compromise than OFDM's complex exponentials, which must be windowed by a time gate in order to be localized on the time axis [3].

An important common feature of the two multi-carrier modulation techniques is that, exactly as for OFDM, the WOFDM modulation/demodulation can be simply implemented by digital signal processing. Thus, Mallat's filter bank algorithm for the Discrete Wavelet Transform (DWT) computation [4] can be used to modulate/demodulate data. The DWT has two important parameters: the wavelets mother and the number of iterations used in computation. Our previous research was focused on the influence of the two parameters, in the context of a flat and time-variant channel, with Rayleigh distributed fading envelope [2].

In most of the practical cases, the radio channel exhibits not only variability in time, but frequency selectivity too [5]. These kinds of channels are commonly referred to as Frequency Selective Fading (FSF) channels. This paper investigates the BER performance of a WOFDM transmission in FSF channels, mainly focusing on the influence that the wavelets mother, which generates the carriers, has on the BER performance of the system. We show that the best results are brought by the wavelets which are well adapted to the time-frequency characteristics of the channel.

In section II, we outline the basic WOFDM concepts. The chain used for simulation purposes is described in section III. The results are displayed and commented in section IV. In the last section, the most important conclusions are highlighted.

II. WOFDM OVERVIEW

WOFDM implementation is related to the two fundamental concepts of the wavelet theory: the orthogonal decomposition and the multi-resolution analysis [6]. According to these concepts, any signal from $L^2(\mathbb{R})$ can be decomposed into the sum of its projection on a subspace V_J, and on the subspaces W_j, j=0,...,J . The orthonormal basis of the subspace V_J is represented by the array of functions $\left\{\varphi_J\left(t-k\right)=2^{-J/2}\cdot\varphi\left(2^{-J}t-k\right)\right\}_{k\in Z}$,where $\varphi(t)$ is known as "scale function". The subspaces W_j are generated by translating in time and scaling a function called "wavelets mother", denoted by $\psi(t)$. The basis so obtained are: $\left\{\psi_j\left(t-k\right)=2^{-j/2}\cdot\psi\left(2^{-j}t-k\right)\right\}_{k\in Z}$. The indexes j and k are usually referred to as "scale" and "position" respectively. The result of the wavelet decomposition is shown in (1):

$$s(t) = \sum_{j \le J} \sum_{k} w_{j,k} \Psi_{j,k}(t) + \sum_{k} a_{J,k} \varphi_{J,k}(t) \qquad (1)$$

In (1), J is the coarsest level (the poorest time resolution) used for signal decomposition, $w_{j,k}$ are the wavelet (detail) coefficients and $a_{J,k}$ are called approximation coefficients. In the context of an orthogonal modulation, the coefficient sets $[w]$ and $[a]$ represent the data to be transmitted, the signals $\varphi(t)$ and $\Psi(t)$ are the "subcarriers" and $s(t)$ can be referred to as "WOFDM symbol". The demodulation can be done due to the fact that the subcarriers belong to orthogonal families. Equation (1) is close to the computation of an inverse wavelet transform, the signal being synthesized from some "decomposition" coefficients. However, in practice, the WOFDM signal is not directly computed using (1). Rather, a discrete version of the WOFDM symbol is generated by applying the Inverse Discrete Wavelet Transform (IDWT) on the input set of coefficients [2].

Mallat [4] gave an ingenious solution for the IDWT computation, using an algorithm based on filter banks. Because it is of particular interest for the understanding of the paper, this algorithm will be briefly presented in the following. At each iteration, an up-sampling and filtering operation is performed. Figure 1 describes the IDWT algorithm for three iterations (decomposition levels). g_1 and h_1 are the impulse responses of the synthesis low-pass and high-pass filter respectively. Their coefficients depend on the wavelets mother which is used.

For a deeper understanding of this process, let's consider the "complementary" scheme (the signal processing from right to left, corresponding to the DWT computation): after each iteration, the number of wavelet coefficients halves (due to the down-sampling operator) and the bandwidth halves too (by filtering). Thus, if the input data vector has N samples, then half of them will be stored in the vector $[w_1]$, and they will be transmitted through the channel using the upper half of the dedicated bandwidth (due to the high-pass filtering). Next, $[w_2]$ contains one quarter of the symbols to be transmitted. Finally, considering the last iteration (J), we will have 2^{L-J} symbols grouped in the approximations vector and an equal number stored in the details vector, where L stands for the maximum number of possible iterations and it equals $\log_2 N$. The different bands (scales) used in transmission are shown in fig. 2, along with the data vectors transmitted through these bands.

Due to the time-scale nature of the wavelet transform, the total duration of the data symbols transmitted at each scale ($[a_J]$ and $[w_j]$, $j = 1,..., J$) remains constant for all iterations. Thus, considering that the time allocated for the transmission of each data symbol is T_S, then the duration of the symbols after the j-th iteration (at the j-the scale from fig. 1) is:

$$T_S(j) = 2^j T_S \qquad (2)$$

III. TRANSMISSION CHAIN

The BER performance of a WOFDM system is investigated by simulation means. For comparison purposes, an Inverse Fast Fourier Transform (IFFT) based OFDM system is simulated too. The channel exhibits frequency selectivity and variability in time.

The input data block is composed of $N = 1024$ samples, which are equally likely bipolar symbols (1 and -1).

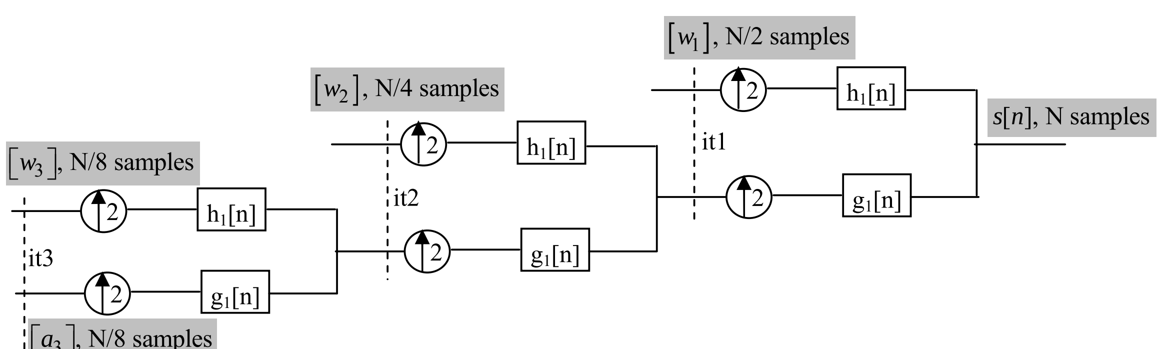

Fig. 1: IDWT implementation using filter banks.

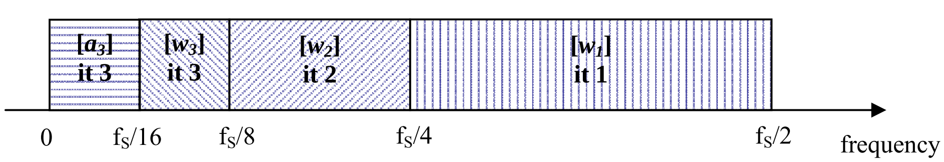

Fig. 2: WOFDM transmission bands.

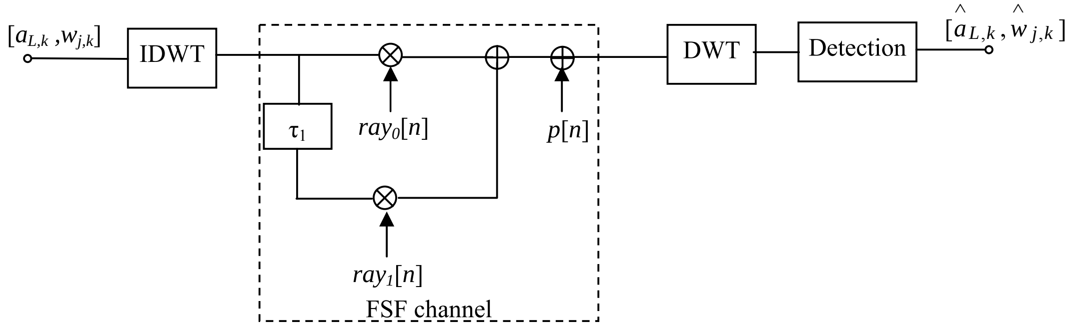

Fig. 3: Simulation chain for WOFDM transmission over FSF channels.

The transmission is done at four scales, that is, 4 IDWT iterations are performed by the modulator. Consequently, the input data block is "seen" by the IDWT modulator as:

$$data = \left[a_4(64), w_4(64), w_3(128), w_2(256), w_1(512)\right] \quad (3)$$

We are interested in the underlying link between the time-frequency localization of the wavelet carriers and the BER performance of the WOFDM scheme. Although we considered a wider portfolio of wavelets, we focused on some signals having specific time and frequency features: Haar (whose time localization is very good) and the Daubechies' wavelets, the time-frequency localization of which depends on their number of vanishing moments [3,6].

The channel is simulated using a "two-ray" model [7]. Its frequency selectivity is given by the two propagation paths, the second one having the relative delay τ_1 ($\tau_1 > T_S$) and thus introducing ISI. Both received components are random, because of the multiplicative Rayleigh processes, $ray_k[n]$. The later simulates the time variability of the channel, its power spectral density being referred to as "Jakes' Doppler spectrum" [5]. Two parameters of this model are of outmost importance. Firstly, the multipath delay spread is defined as:

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\overline{\tau})^2} \quad (4)$$

In equation (4), $\overline{\tau}$ and $\overline{\tau^2}$ represent the first and the second order of channel's power-delay profile [7, 8]. σ_τ is directly proportional to the power (P_1) and to the relative delay (τ_1) of the ISI component (the second ray). The second parameter is the maximum Doppler shift (f_m), which measures how fast is the channel changing in time. We consider two values for this parameter (0.05 and 0.005), featuring the fast and the slow fading scenario respectively [5]. These values, as well as the ones considered for σ_τ, correspond to the real values normalized by the sampling frequency, f_S .

The receiver is composed of a DWT "demodulator", which retrieves the coefficients modulated onto each scale, affected by the adverse effects of the channel. Finally, a threshold comparator evaluates the sign of every sample and gives an estimation of a transmitted symbol. The performance is evaluated using BER versus E_b / N_0 curves. The BER is computed independently at each transmission scale, as well as jointly, for the whole transmitted block.

IV. Experimental results

The overall BER results (fig. 4) are poor, for all simulation scenarios. ISI introduces important performance degradation, since it is not counteracted by any mean. Furthermore, this phenomenon is the main source of errors, impacting much more the BER performance than de time-variability of the channel. Thus, the Doppler shift makes a slight difference in fig. 4 only when the ISI component is severely attenuated with respect to the first path ($P_1 / P_2 = 10\,\text{dB}$).As a reference, the OFDM transmission leads to even poorer results. Thus, the joint effect of the time and frequency selectivity of the channel impacts more the orthogonality of its carriers, compared to WOFDM. In the later case, it results no significant dependency of the overall BER performance on the wavelets mother employed in the modulator. Yet, a deeper analysis carried out scale by scale, has revealed some interesting details.

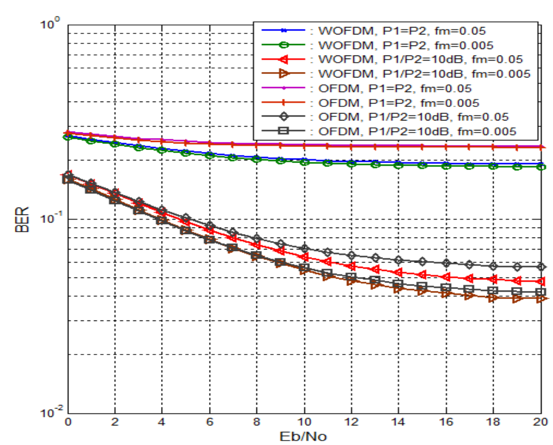

Fig. 4: Overall BER performance of a WOFDM/OFDM transmission using Daubechies-8, in a two ray FSF channel, τ_1=1.

In figure 5, the BER results at scales 3 and 4, for Daubechies-20 and Haar wavelet are plotted. It can be noticed that, the Daubechies wavelet, with a good frequency localization, leads to important BER improvements, compared to the Haar-based WOFDM. The gain brought by Daubechies-20 is of approximately 16 dB at scale 3, and 12 dB at scale 4. This proofs that, when the channel is frequency selective, the frequency localization of the carrier is of outmost importance and it predominates over the time localization. Nevertheless, the later is critical for the flat, time-variant channel [2].

A second conclusion is that the BER performance clearly improves at higher scales. The explanation is two fold: at these scales, the transmitted symbols have longer duration (see equation 2), and they are consequently less afected by ISI. By the other hand, for the lower scales, data are transmitted through frequency bands which exhibit important attenuations, given the simulation model considered for the channel.

The general rule (as already highlighted), is that ISI predominates over channel's time-variant behavior. Under these conditions, the frequency localization of the wavelet carriers proves to be essential. However, when the two effects are balanced, different conclusions can be drawn. In order to "balance" ISI and the time-variance, we will "favor" the later effect against the former, by choosing $f_m = 0.05$ and by computing the BER only for that transmission scale which is less affected by ISI (the 4th). The results are shown in fig . 6. Even if the BER curves are close one another, an interesting pattern can be highlighted. Thus, the worst performance is clearly shown by the two "extreme" cases: Daubechies-20 and Daubechies-4. These signals excel in either the frequency, or the time localization, but they suffer to the complamentary feature. The best performance is obtained by using Daubechies-12, that provides a convenient time-frequency compromise. We may state, that, in our simulation scenario, this wavelet was better adapted to the time-frequency pattern of the channel.

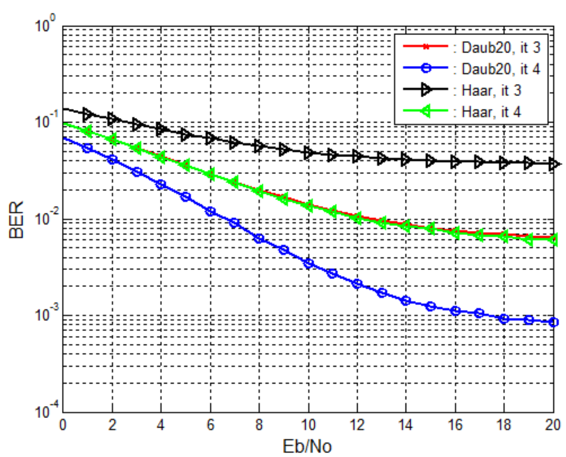

Fig. 5: The BER performance for Haar and Daubechies-20 WOFDM, at the third and the fourth transmission scale, $P_1=P_2$, $\sigma_{\tau n}=0.5$, $f_m=0.005$.

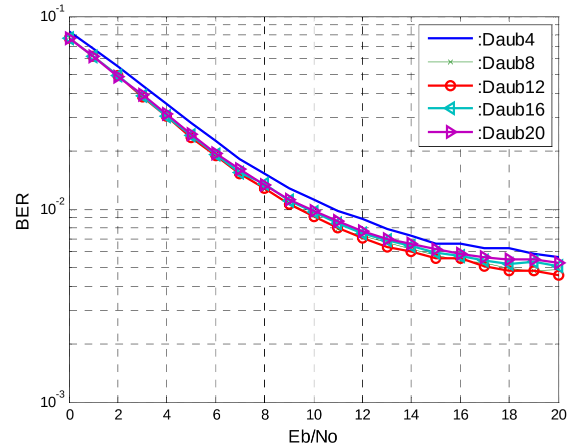

Fig. 6: BER performance at the fourth scale of a WOFDM transmission, for all the Daubechies wavelets, $P_1=P_2$. $\sigma_{\tau n}=0.5$, $f_m=0.05$.

V. CONCLUSIONS

BER performance of a WOFDM transmission through a FSF channel is investigated in this paper. Our main goal is to asses how is this performance infuenced by the choice of the IDWT parameters, and mainly by the wavelets mother.

Several conclusions can be derived from the simulations. As a general rule, ISI is the most important source of errors for the transmission. Under these conditions, the wavelets with good frequency localization provide the best results, whereas the wavelets well localized in time lead to poor performance. If ISI and the time variability of the channel (assessed by the maximum Doppler shift, f_m) are balanced, those wavelets that achieve a convenient time-frequency trade-off, adapt better to the channel's conditions and lead to superior performance.

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