Performance of Multi Binary Turbo Codes on Rayleigh Flat Fading Transmission Channels

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Abstract ***– In this paper we analyze the performance in terms of Bit Error Rate (BER) and Frame Error Rate (FER) of a multi-binary turbo coded system over Rayleigh flat fading multipaths channels. The turbo codes (TCs) proposed here calls for the parallel concatenation of two identical rate r/(r+1) recursive systematic convolutional RSC codes. It results rate r/(r+2) Multi-Binary Turbo Code (MBTC). Data block length has k=m·N=2·752 bits. The interleavers presented in 2005 by C. Douillard and C. Berrou are used. The results are illustrated with two examples based on double-binary (r=2) 8-state and 16-state TCs, for (6 7 1 5) RSC codes and (11 11 1 12) RSC codes. QPSK modulation is employed.***

Keywords: ***flat fading channel, Rayleigh distribution, multi binary turbo code.***

I. INTRODUCTION

Channel coding, for wireless applications on fading channels, is an important tool for improving communications reliability. Many researchers have study the performance of the coding techniques like: block codes [1], turbo codes [2], [3],[4], over Rayleigh fading channel. Multi binary turbo codes, presented in [5], have been shown to perform near the capacity limit on the additive white Gaussian noise (AWGN) channel.

In this paper we investigate performance of the MBTC codes over Rayleigh fading channel. The fading phenomenon occurs in radio transmission channels. It is due to the presence of multipaths that vary during the transmission [6]. The transmission channel scheme considered in this paper is shown in Fig.1, [7]:

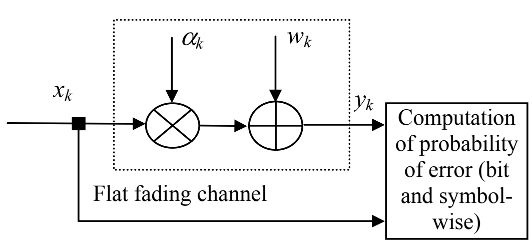


Fig.1 Model of the fading channel used in simulations (uncoded case).

For Rayleigh flat fading channel the received sequence, for the time slot k, is:

$$y_k = \alpha_k \cdot x_k + w_k \ , \qquad (1)$$

where x_k is the transmitted data, w_k is a sample of Additive White Gaussian Noise (AWGN) and α_k is a Rayleigh random value which is characterized the time fluctuations.

Applying multi-binary turbo encoding, the model of the system has the configuration from Fig.2. The BER computation is made by comparing the sequences from the input on multi binary turbo encoder and after the multi-binary turbo decoder. The relation between y and x is given by relation (1).

In Fig. 3 it results gains of tens dB, in the BER performance of the MBTCs compared with the uncoded case, over Rayleigh flat fading transmission channel.

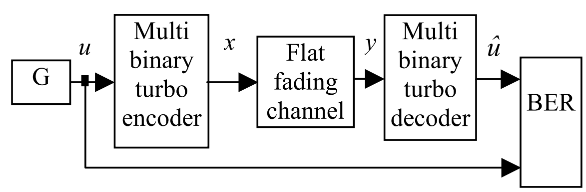


Fig. 2 Model of the multi binary turbo encoded transmission system.

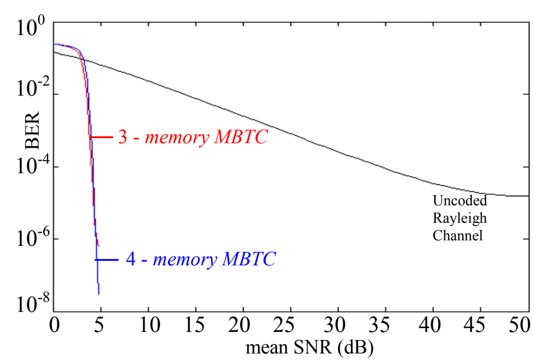


Fig. 3 BER performance for uncoded and 3-memory, 4-memory multi binary turbo coded transmission system over Rayleigh flat fading channel. BER is plotted as function of mean SNR (*dB*). Transmission of 188-byte data blocks. QPSK turbo coded modulation. MaxLogMAP algorithm (15 iterations).

II. MULTI BINARY CONVOLUTIONAL ENCODER

The general scheme of a multi input encoder (with r/(r+1) rate) and a 16-state duo-binary encoder (with 2/3 rate) with generator polynomials [11 11 1 12] are presented in [8]. The scheme of the 8-state duo binary encoder with generator polynomials [6 7 1 5] is in Fig.4.

Let $S_t = \begin{bmatrix} s_3^t & s_2^t & s_1^t \end{bmatrix}^T$ and $U_t = \begin{bmatrix} u_2^t & u_1^t \end{bmatrix}^T$ denote the encoder state. The full generator matrix of the 3 – memory multi input encoder has the following form:

$$H = \begin{bmatrix} h_{3,3} & h_{2,3} & h_{1,3} & h_{0,3} \\ h_{3,2} & h_{2,2} & h_{1,2} & h_{0,2} \\ h_{3,1} & h_{2,1} & h_{1,1} & h_{0,1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 7 & 1 & 5 \end{bmatrix}_{10} \quad (2)$$

By skipping the weights for the redundant bit and the weights for the recursive part, we obtain the matrix H_0 as follows:

$$H_0 = \begin{bmatrix} h_{2,3} & h_{1,3} \\ h_{2,2} & h_{1,2} \\ h_{2,1} & h_{1,1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \quad (3)$$

The feedback vector and the output vector have the form:

$$\begin{aligned} H_F &= \begin{bmatrix} h_{0,3} & h_{0,2} & h_{0,1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \\ H_{out} &= \begin{bmatrix} h_{3,3} & h_{3,2} & h_{3,1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \end{aligned} \quad (4)$$

where the operator $(.)^T$ denotes the transpose of a vector.

The main equation of the circuit in Fig.3 is:

$$(S_{t+1})_{3\times1} = (H_0)_{3\times2} \cdot (U_t)_{2\times1} + (T)_{3\times3} \cdot (S_t)_{3\times1} \quad (5)$$

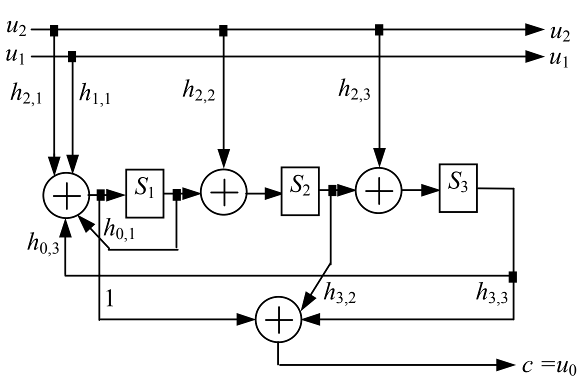


Fig. 4 The scheme of the 8-state duo binary encoder with *H*=[6 7 1 5].

where the matrix T is defined as:

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0_{2\times1} & I_2 \\ & H_F \end{bmatrix} \quad (6)$$

The redundant output is equal to:

$$c^t = H_{out} \cdot S_t + W \cdot S_{t+1} \quad (7)$$

where the vector W is the all-zero vector save its last entrie which is equal to 1:W=[0 0 1].
In the following section these particular duo binary convolutional encoders are used for both constituent codes of the multi binary turbo codes that we proposed in the next section.

III. MULTI BINARY TURBO CODES

The novelty brought by MBTC consists in the usage of the multi input encoders as component codes [5]. The main advantage of the multi-binary turbo-codes versus Turbo Codes (TCs) [9] is their minimum distance, which is in general larger than classical TC and. A parallel concatenation of two identical *r*-ary RSC encoders with an interleaver (∏) is presented in Fig. 5.
In this paper we used the interleavers, ∏, proposed in [5]. Here, for both considered interleavers the permutation have made in two levels. First, an intrasymbol permutation was performed. Second, an intersymbol permutation has done after particular forms given in [5].
For example, for the following sequence with 8 words:

1	3	5	7	9	11	13	15
2	4	6	8	10	12	14	16

applying the intrasymbol interleaver (the first level), we obtain the sequence:

1	4	5	8	9	12	13	16
2	3	6	7	10	11	14	15

and, after the second level of the permutation, for 16-state duo-binaryTC, it results:

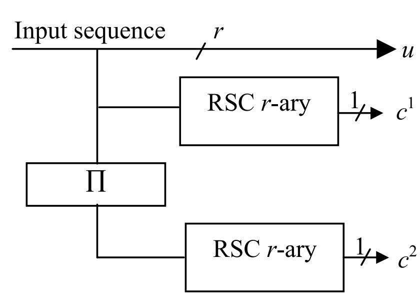


Fig. 5 The r-ary turbo-encoder.

7	5	3	9	15	13	11	1
8	6	4	10	16	14	12	2

Blocks of k bits (k being a multiple of r) are encoded twice by the bi-dimensional code, whose rate is $r/(r+2)$ (for r=2, it results 1/2 rate), to obtain a multi binary turbo encoder scheme in Fig.5.
The MAP [10] and MaxLoMAP [11] decoding algorithms, to obtain the results in the next section we used. For MaxLogMAP decoding algorithm the log-function is approximated by the max function with some properly chosen offset coefficient.

IV. SIMULTION RESULTS

The simulations presented in this section were made with MBTCs, having their parameters defined in Table 1. The chosen parameters were mainly the ones used in [5]. The trellis closing is the exception which wasn't made circular.
One of the decoding algorithms used in this paper is the MaxLogMAP algorithm. The extrinsic information is less reliable, especially at the beginning of the iterative process. A more robust approach consists in scaling the extrinsic information with a scaling factor smaller than 1.0. The best observed performance is obtained for a scaling factor of 0.75 [12].
The other decoding algorithm is MAP algorithm. The Table 2 and Table 3 presents how are influenced the BER and FER performance by the estimation of the SNR value, estimation given by Lc factor.

Table1 Parameters of the simulated turbo coded

Parameter	**Used variant**
The turbo code configuration	Parallel
The component code	RSC code with: 3 memory (H=[6 7 1 5]) and 4 memory (H=[11 11 1 12])
Turbo coding rate	1/2
Punctured	no
Modulation	QPSK
Channel	Rayleigh flat fading
Interleaving	Intersymbol and intrasymbol interleaving defined in [5]
Interleaver	Adapted to the component code, defined in [5]
Data block length	188*bytes* = 2 x 752 = 1504 *bits*
Decoding algorithms	MAP decoding algorithm; MaxLogMAP with decoding per symbol and with the scaling factor of the extrinsic information equal to 0,75.[12]
Iteration number	15 iterations with a stop criterion iteration based on APP (A Posteriori Probability) distribution.
Simulated block number	Inverse proportionally with the logarithm of the BER.

Table 2 BER·10^9 in function of f and the state of the MBTC

MBTC	SNR (dB)	$f= Lc/(4 \cdot R \cdot B)$				
		0.4	0.5	0.6	0.7	0.8
8-state	4.2	1201433	8525	6013	5097	6057
16-state	4.2	15727910	15780	2881	5836	28147

Table 3 FER·10^6 in function of f and the state of the MBTC

MBTC	SNR (dB)	$f= Lc/(4 \cdot R \cdot B)$				
		0.4	0.5	0.6	0.7	0.8
8-state	4.2	22044	577	555	577	688
16-state	4.2	200422	222	66	88	555

Thus, we used values for Lc given by the following relation: $Lc=4 \cdot R \cdot B \cdot f$, where R is the duo-binary turbo-coding rate and B is the absolute value of the SNR.
The results in term of BER and FER plotted in Table 2 and Table 3, for a SNR=4.2 dB and for 3-memory MBTC show that are better for the factor f=0.7. For 4-memory MBTC the best result is obtained for f=0.6.
This conclusion is verified in Fig. 6. It is evident that the best BER and FER performance are obtained for 4-memory MBTC, with f=0.6.

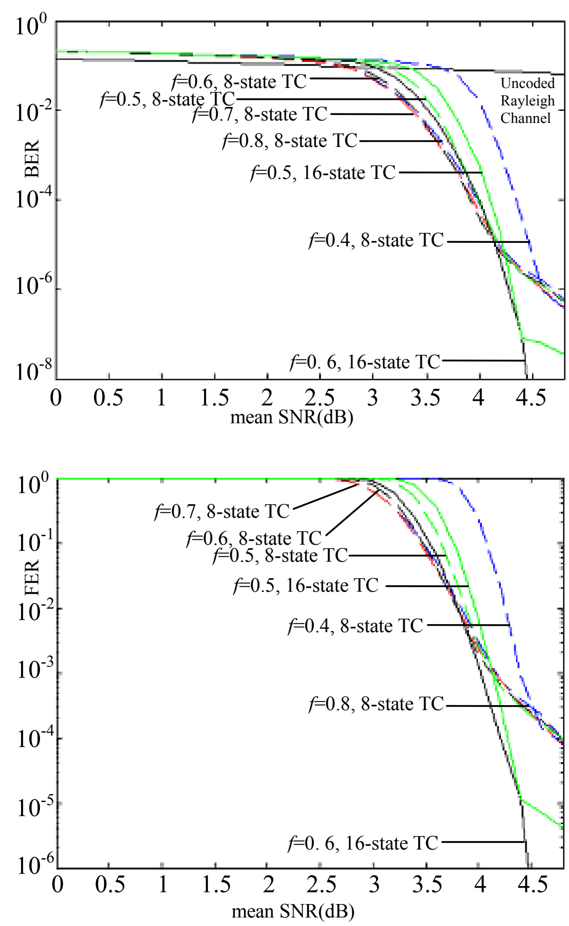
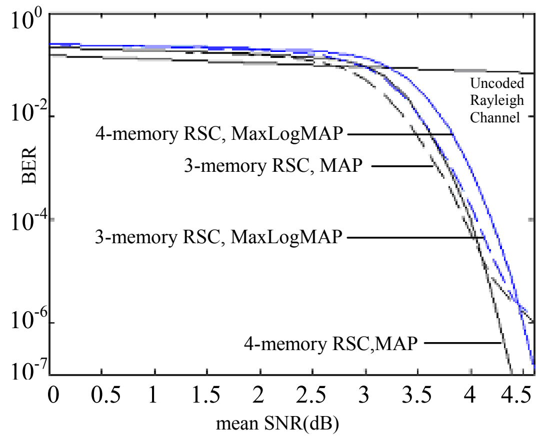


Fig.6 BER and FER performance for 8-state duo-binary TCs (dashed line) and for 16-state duo-binary TCs (solid line), for different values of factor f. MAP decoding algorithm. QPSK modulation.



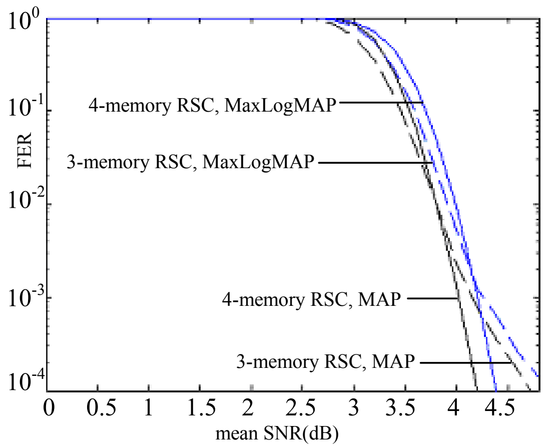


Fig.7 The performance in term of BER and FER.Comparison between two implementations for the decoding algorithm of MBTCs (with *r*=2) over Rayleigh fading channel: MAP algorithm and MaxLogMAP approximation. The MBTC that we used is described in Fig. 5, based on two 3-memory RSC codes (dashed line) and two 4-memory RSC codes (solid line). Codeword size is equal to 1504 bits with 1/2-rate.

Fig. 7 shows the BER and FER performance of the rate 1/2 duo-binary (8-state and 16 state) TCs as a function of SNR with MAP and MaxLogMAP decoding algorithms. For the 8-state duo-binary TC (curves in dashed line), for low SNR, the loss does not exceed 0.15 *dB*. For low BER, smaller than 10^{-5}, the loss becomes negligible.

III. CONCLUSIONS

In this paper we presented the BER and FER performance for multi binary turbo-coded transmission system over Rayleigh flat fading channel. The simulations results presented through this paper show that the MBTCs in the presence of the fading provides, as we expected, a tremendous performance gain of the order of tens *dB* even for moderate codeword length. For lower BER (10^{-6}) there is no noticeable error floor for the simulated SNR ranges, for 16-state MBTCs. The curves are very abrupt like in channel with no fading.

Moreover, the 1/2 rate coding of the MBTCs is accepted in most usually systems.

A study on *Lc* (reliability) coefficient in MAP decoding algorithm is made. The results show that this coefficient *Lc* is indicated to be decreased with a 0.7 factor for 8-state MBTCs, and with a 0.6 factor for 16-state MBTCs.

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