The Turbo-codes Performances in the (Radio) Rice Flat Fading Channels

Horia Balta*
Maria Kovaci*

***Abstract**: The received signal from (radio) Rice flat fading transmission channels has both unfading (like in the AWGN –Additive White Gaussian Noise channel) and fading components (like in the Rayleigh channel).*

In this paper the BER (Bit Error Rate) and FER (Frame Error Rate) TCs (turbo codes) performances provided by the simulations of the way of the TCs work in the Rice flat fading (frequency unselective) channel, for different unfading and fading power ratio values, are presented.

The 1/3 rate TCs, a S-type random interleaver with 1784 bits length, 15 iterations with a stop criterion, and the BPSK (Binary Phase Shift Keying) modulation have been used. The recursive systematic convolutional code with the constraint length K= 4 have been the implied component codes.

***Keywords**: Rice flat fading channels, turbo-codes*

I. INTRODUCTION

The fading phenomenon occurs in radio transmission channels. It is due to the presence of the multi-paths that varies during the transmission, [1]. Radio channel modeling still remains a challenging issue. A sufficient and acceptable model is to consider the input-output relation of the digital channel of the form:

$$y_k = \alpha_k \cdot x_k + w_k, \qquad (1)$$

where x_k and y_k are the transmitted and received data for the time slot k, respectively; the parameter α_k is a random value which is characterized the time fluctuations from symbol to symbol (fast fading) or from block to block (block fading). Its distribution determines the channel type: Rayleigh, Rice or Nakagami. The input sequence $\{x_k\}$ is binary, random, in NRZ bipolar format, i.e. with unitary variance. Finally, the samples w_k are zero-mean i.i.d., gaussian variables of variance σ^2.

The approach considered in this paper, shown in Fig.1, consists of two parts: the generation of the channel model and the BER computation.

In order to generate the transmitted sequence $\{x_k\}$ another sequence denoted $\{u_k\}$ is generated by random numbers with uniform distribution in the interval $[0,1)$. The targeted antipodal sequence is obtained after the following transformation:

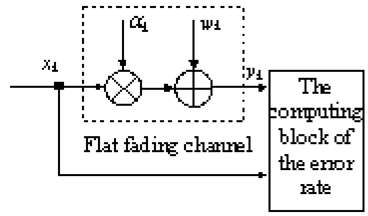

Fig. 1 Model considered through this paper in order to simulate the performance of a turbo-coded transmission over non frequency selective time-varying channels.

$$x_k=2\cdot[2\cdot u_k]-1 \tag{2}$$

where [ ] denotes the truncated operation at the integer part. It can be shown that [1]:

$$\overline{x_k^2} = 1 \tag{3}$$

The sequence $\{\alpha_k\}$ follows a Rice distribution. By normalizing α_k such that $\overline{\alpha_k^2} = 1$, the effective signal to noise ratio E_b/N_0 of the transmission is defined as follows [2]:

$$\frac{E_b}{N_o} = \frac{1}{2}\cdot\frac{2\cdot E_b}{N_o} = \frac{1}{2}\cdot\frac{\overline{\alpha_k^2}\cdot\overline{x_k^2}}{\overline{w_k^2}} = \frac{1}{2}\cdot\frac{1}{\overline{w_k^2}}, \tag{4}$$

where $E_b/N_0 = SNR\ (dB)$. Therefore, the variance of the additive noise can be expressed as:

$$\overline{w_k^2} = \frac{1}{2\cdot 10^{SNR/10}} \tag{5}$$

II. THE RANDOM NUMBERS GENERATION WITH RICE DISTRIBUTION

The Rice probability density has the relation:

$$p(\alpha) = \begin{cases} \frac{\alpha}{\sigma^2}\cdot\exp\left(-\frac{\alpha^2 + A^2}{2\cdot\sigma^2}\right)\cdot I_o\left(\frac{A\cdot\alpha}{\sigma^2}\right), & pentru \quad \alpha \ge 0 \\ 0, & pentru \quad \alpha < 0 \end{cases} \tag{6}$$

where:

$$I_0(x) = \frac{1}{2\pi} \cdot \int_0^{2\pi} e^{x \cdot \cos\psi} d\psi \tag{7}$$

is the modified Bessel function, of the first kind and zero order.

The Rice distributions for $\sigma = 1$ and for some particular values of the A parameter are drawn in Fig.2. In the case of $A = 0$, the Rice distribution is reduced to the Rayleigh distribution.

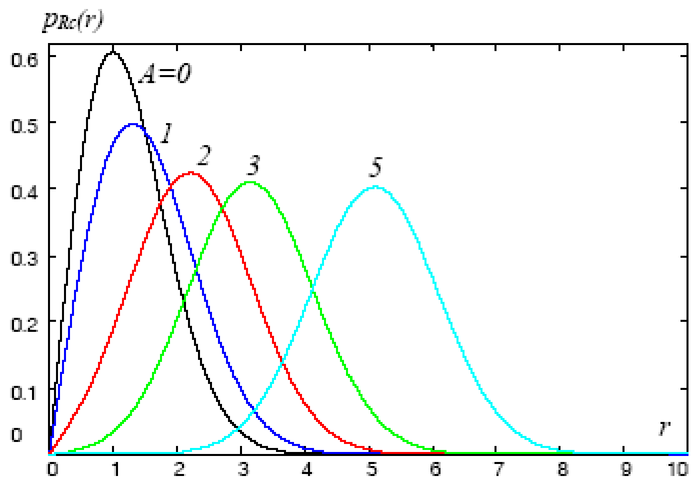

Fig.2 Rice distribution, for $A = 0$; 1; 2; 3 and 5, and $\sigma = 1$.

An α random variable, with Rice distribution, is obtained by combining of the two normal random variables x_1 and x_2, with the same dispersion σ^2, but only one with null mean:

$$\alpha = \sqrt{x_1^2 + x_2^2} \tag{8}$$

The x_1 variable can be thought as the following:

$$x_1 = x_3 + A, \tag{9}$$

where x_3 is a normal variable with null mean and σ^2 dispersion. It results the relation:

$$\alpha = \sqrt{(x_3 + A)^2 + x_2^2} = \sqrt{r^2 + 2 \cdot A \cdot r \cdot \cos\Phi + A^2} \ , \tag{10}$$

where:

$$r = \sqrt{x_2^2 + x_3^2} \ , \tag{11}$$

is a random variable with Rayleigh distribution. If u is a random variable with constant distribution on $[0, 1)$, then by the following transformation:

$$r = F_{Ry}^{-1}(u) = \sqrt{-2 \cdot \sigma^2 \cdot \ln(u)} \ , \qquad (12$$

it can be obtained a Rayleigh distributed random variable. Using, in the following, the generating way of the Rayleigh distributed random variable, gives by (12), we can obtain:

$$\alpha = \sqrt{-2 \cdot \sigma^2 \cdot \ln(u_1) + 2 \cdot A \cdot \sqrt{-2 \cdot \sigma^2 \cdot \ln(u_1)} \cdot \cos(\pi \cdot u_2) + A^2} \tag{13}$$

In contrast with the generating of the random variable with the Rayleigh distribution, the relation (12), the relation (13) can generate a random variable with Rice distribution and implies two variables with uniform distribution.

III. THE TURBO-CODED TRANSMISSION SYSTEM IN RICE FLAT FADING

The Turbo-Coded (TC) scheme that we considered in our simulations is presented in Fig.3.

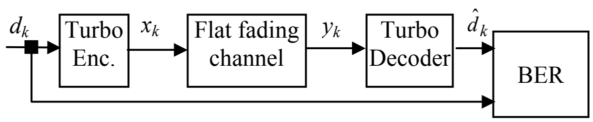

Fig. 3 The turbo-coded transmission system through flat fading channel.

The component decoders use a MAP or SOVA iterative algorithm. The different steps of the MAP decoding algorithm are summarized in Fig. 4 [4].

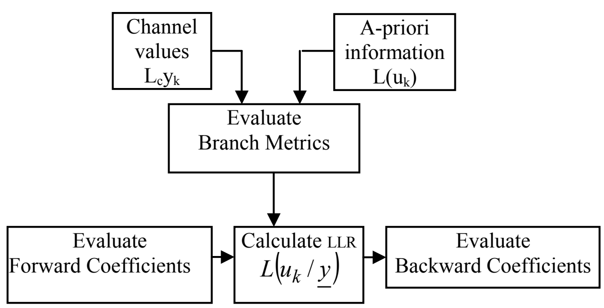

Fig. 4. Summary of the key operations in the MAP algorithm.

To decoding the received sequences from flat fading channel (whit his input-output relation done by (1), where α is done by (13)), the MAP algorithm must be adapted to L_c coefficients (channel reliability), so:

$$L_c = 4 \cdot R \cdot B \cdot \overline{\alpha} \tag{14}$$

where B is the absolute value of the SNR , R is the turbo-coding rate and $\overline{\alpha}$ is the mean of the α random variable. If $A=0$, $\overline{\alpha} = \sqrt{\frac{\pi}{4}} = 0.8862$.

IV. EXPERIMENTAL RESULTS

We considered the following setup for our simulations. A Recursive Systematic Convolutional Code, 1/3 rate is employed, [6]. A S-interleaver is used. The simulations are made for BPSK signaling. A stop criterion after each iteration is applied when the decoded sequence matches with a codeword. However, a maximal number of 15 iterations is tolerated. The transmission channel is the Rice flat fading channel described in the previous sections.

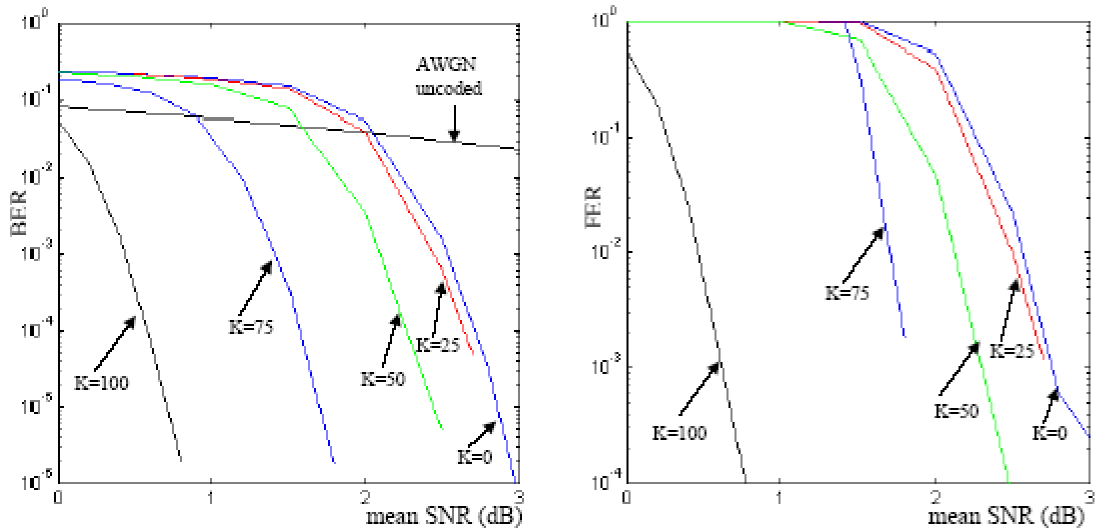

Fig.5 The performances of the TCs for different values of the K ration (%).

The component decoders use the MAP algorithm, described in the precedent paragraph. For the beginning, we used the *1* value for the mean of the α random, to compute L_c. The simulation results, for five values of the *K* coefficient, defined like a rapport between the power of the unfaded component (given by the continuous component, *A*, from (13)) and the total power of the Rice random variable, considerate unitary, are presented in Fig.5. Those:

$$\alpha = \sqrt{-(1-K)\cdot\ln(u_1) + 2\cdot\sqrt{K\cdot(1-K)}\cdot\sqrt{-\ln(u_1)}\cdot\cos(\pi\cdot u_2) + K}\ . \qquad (15)$$

The Table I presents how are influenced the BER and FER performances by the estimation of the signal to noise ration value, estimation given by L_c factor. Thus, we used values for L_c given by the following relation:

$$L_c = 4\cdot R\cdot B\cdot f. \qquad (16)$$

Table I. $BER\cdot 10^9$ in function of *f* and *K*

K (%)	*SNR* (dB)	$f = L_c / 4\cdot R\cdot B$						
		0.6	*0.7*	*0.8*	*0.9*	*1.0*	*1.1*	*1.2*
0	*2.8*	*2068*	*2641*	*8260*	*13941*	*31798*	*68399*	*119371*
25	*2.7*	*2322*	*3666*	*6603*	*17098*	*46235*	*84003*	*151791*
50	*2.5*		*104*	*553*	*2083*	*3941*	*8107*	*16198*
75	*1.8*	*3380*	*774*	*823*	*1503*	*1768*	*4598*	*10356*
100	*0.8*	*2590511*	*7040*	*1810*	*1341*	*1347*	*3335*	*4705*

V. CONCLUSIONS

In this paper the BER and FER TCs performances provided by the simulations of the way the TCs work in the Rice flat fading (frequency unselective) channel, for different unfading and fading power ratio values, are presented.

As the Fig.5 shows, the performance of TC for Rice flat fading channels are upper bounded by the Rayleigh ($K=0$) flat fading performances and lower bounded by static channels ($K=1$). The TC performance is not linear improved with the K parameter. Thus, if the power of the continuous component is under 25% from total power ($K< 0.25$), the Rice channel behavior is the same like the pure fluctuant channel (Rayleigh). For values over *50%* of the K, the performances are significant improved.

The channel estimation in the sight of the value construction of the L_c coefficient, necessary in the MAP algorithm, it is not a critical problem. So, as is shown in Table I, the errors until 20% in the estimation of the Lc factor, does not influences the TC performances with more than 0,2 dB.

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∗ Assistent engineer, Technical University, Timşioara, 2 Av. V. Pârvan, tel. 0040 256 403325, e-mail: horia.balta@etc.utt.ro;

∗ Assistent engineer, Technical University, Timşioara, 2 Av. V. Pârvan, tel. 0040 256 403308, e-mail: maria.kovaci@etc.utt.ro.