PSU-UNS
4th INTERNATIONAL CONFERENCE ON ENGINEERING TECHNOLOGIES - ICET 2009
Novi Sad, April 28-30, 2009

ON USING TURBO CODING OVER RAYLEIGH FLAT FADING CHANNELS

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Abstract: *In this paper we analyze the performance in terms of Bit Error Rate (BER) and Frame Error Rate (FER) of a turbo coded system over Rayleigh flat fading channels. A new type of interleaver, Block Random in Line (BRL) interleaver, is proposed and compared with the S-interleaver on the basis of the BER and FER performance of the turbo code. The simulation results show that turbo codes (TCs), using a BRL interleaver provide significant gain, similarly to the S-interleaver, over uncoded transmission in presence of Rayleigh fading.*

Key Words: - turbo codes, interleaver, fading channel, Rayleigh distribution

1 INTRODUCTION

The channel model considered in this paper is that of Rayleigh flat fading. Fading occurs in wireless communication systems in the form of a time-varying distortion of the transmitted signal. The term *flat* refers to the fact that the channel is not frequency selective, so that its function in the frequency domain is constant [1].

The transmission channel scheme considered in this work is shown in Fig.1, [2].

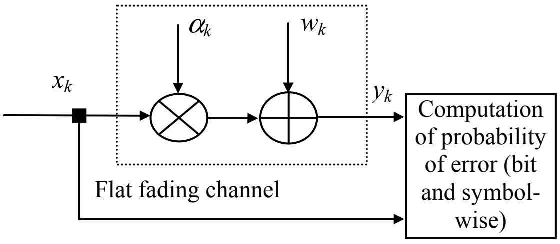


Fig. 1. Model of the fading channel used in simulations (uncoded case).

For Rayleigh flat fading channel the received sequence, for the time slot *k*, is:

$$y_k = \alpha_k \cdot x_k + w_k \,, \qquad (1)$$

where x_k is the transmitted data, w_k is a sample of Additive White Gaussian Noise (AWGN) and α_k is a Rayleigh random value which is characterized the time fluctuations.

The paper is organized as follows. Section II presents the *S*-interleaver, [3], and the new type interleaver, Block Random in Line Interleaver (BRL), [4]. Section III shows the turbo code (TC), implementation. Section IV presents the performances of these interleavers over flat Rayleigh fading channel. Finaly, section V is dedicated to some concluding remarks.

2 INTERLEAVER DESIGN

A. The S-interleaver

The *S*-interleaver is a random type interleaver. However, unlike the pure random interleaver, by construction a minimum interleaving distance equal with *S* is forced [3]. The interleaving mapping construction algorithm is as follows. We select a possible future position for the current bit. This position is compared to the last *S* positions already selected.

If the condition:

$$|\pi_s(i) - \pi_s(j)| > S \text{ for } i \text{ and } j \text{ with } |i - j| < S, \qquad (2)$$

is satisfied we go further. If the condition is not true, we select another position for the current bit, which will also be tested. The process will be repeated up to the moment when all the positions for the N_s bits will have been found. The simulations demonstrated that, if $S< \sqrt{N_s/2}$, then the process will converge. The design of this interleaver is difficult because the difficulty of the accomplishment of the condition increases with the rise in the number of bits already tested.

B. The Block Random in Line (BRL) Interleaver

This new interleaver type aims to continue the qualities of the block interleavers (high d_{min}) and of the random interleavers (a good spreading). This kind of interleaver represents an alternative to the *S*-type interleaver, which is difficult to be designed. In the following, the design of the block random in line interleaver is presented. We will suppose that the length of this interleaver is given by the relation:

$$N_{br}=X\times Y. \tag{3}$$

where X and Y are natural numbers.

Steps of the interleaver design:
- first, we build the following matrix:

$$c(i,j)=1+i+j\cdot X,\ i\in I=\{0,1\ldots X\text{-}1\}.\quad j\in J=\{0,1\ldots Y\text{-}1\} \tag{4}$$

- second, each line of this matrix, $c(i,J)$, is permuted using the relation:

$$\pi_r(i) = rand(i),\ \forall\ i\in I=\{1,2\ \ldots N_r\}, \tag{5}$$

is obtained the matrix b:

$$b(i,J)= c(i,\pi_r(J))\ \forall\ i\in I. \tag{6}$$

- finally, to increase the minimum distance of the interleaver, a reorder of the lines is done so that on the first X/2 positions can be found the odd lines. After reordering of the lines a read out on columns-size is made:

$$\begin{gathered}\pi_{BRL}(j+k\cdot Y+1)=b(i,j),\ i=(2\cdot k)\in I,\ j\in J.\\ \pi_{BRL}(j+k\cdot Y+Y_2+1)=b(i,j),\ i=(2\cdot k-1)\in I,\ j\in J.\end{gathered} \tag{7}$$

where $Y_2 = floor[(Y-1)/2]$.

In Fig.2, the scheme of the BRL interleaver is briefly presented:

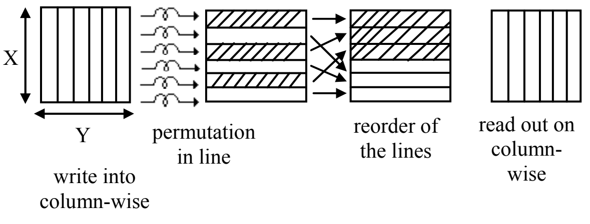


Fig. 2. Scheme of the Block Random in Line Interleaver.

The minimum interleaver distance is $d_{min}\geq Y/2$ for the BRL interleaver. For a good spreading, the dimension X must be chosen the greatest possible. A good compromise is obtained choosing $X \cong Y^2$.

These interleavers are used in the turbo code structure, presented in the following section.

3 TURBO CODE IMPLEMENTATION

In our implementation we considered the classical TC [5], shown in Fig. 3:

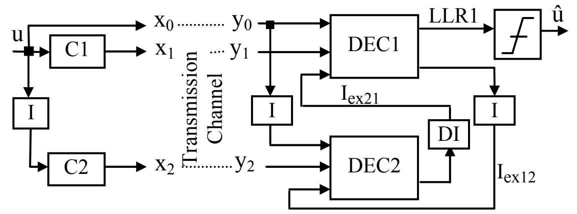


Fig. 3. Scheme of the classical turbo code

where, blocks I and DI realize interleaving and de-interleaving functions, using the interleavers presented in previous section.

The encoders are denoted with C1 and C2 and the notation for decoders are DEC1 and DEC2, respectively. There are also the following notations: - u for the input sequence; - x_0, x_1, x_2 for the encoder outputs; - y_0, y_1, y_2 for channel outputs; - I_{ex21} and I_{ex12} for extrinsic information; - *LLR* for Logarithm of Likelihood Ratio; – $\hat{u}$ for decoder output.

The component decoders use a MAP iterative algorithm. The different steps of the MAP decoding algorithm are summarized in Fig. 4 [6].

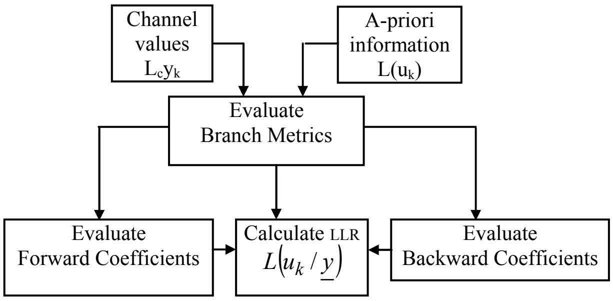


Fig. 4. Summary of the key operations in the MAP algorithm.

To decoding the received sequences from flat fading channel (whit his input-output relation done by (1)), the MAP algorithm must be adapted to L_c coefficients (channel reliability), so:

$$L_c = 4\cdot R\cdot B\cdot \overline{\alpha} \tag{8}$$

where B is the absolute value of the SNR , R is the turbo-coding rate and $\overline{\alpha}$ is the mean of the α random variable. If A=0, $\overline{\alpha}=\sqrt{\frac{\pi}{4}}=0.8862$.

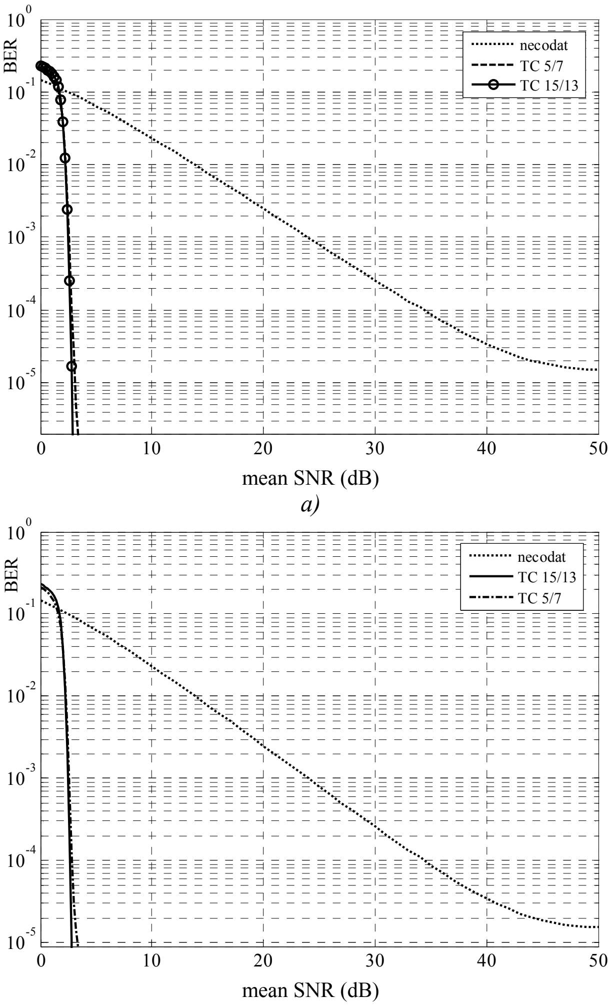
The next section shows the BER and FER performance of TCs over Rayleigh flat fading channel.

4 SIMULATION RESULTS

In [4] we show that the performances of the proposed BRL interleaver are closed to the performances of the *S* interleaver, with maximum *S*, using TCs over AWGN channel. Many researchers had studied the performance of the turbo coding techniques, over Rayleigh flat fading channel, [7, 8].

In this paper we studied the behavior of the BRL interleaver versus *S*-interleaver, when TCs are used over Rayleigh flat fading channel.

In Fig. 5. a) and Fig. 5. b) it results gains of tens *dB*, in the BER performance of the TCs compared with the uncoded case, over Rayleigh flat fading transmission channel, using both interleavers: *S*-interleaver and BRL interleaver. The interleaver length is equal to 1784 bits, for the *S*-interleaver (with *S*=29) and 1785 (with X=119 and Y=15), for the BRL interleaver. These interleavers were introduced into an unpuncturated turbo code at the rate of 1/3, in which two identical recursive systematic convolutional (RSC) 15/13 and 5/7 codes, having the constraint length *K*=4, respectively *K*=3, are connected in parallel. Also, in the simulations we considered the QPSK modulation and the MAP decoding algorithm. 15 iterations are assumed at the decoder.

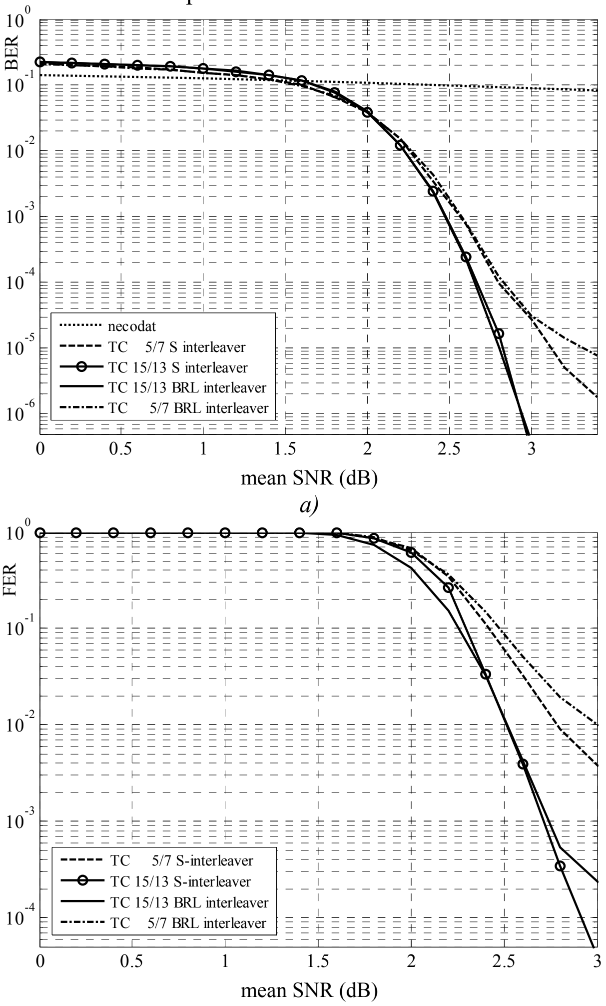


a)

b)

Fig. 5. BER Performance of TCs on Rayleigh flat fading channels for 5/7 and 15/13 RSC codes, using: a) the S-interleaver and b) block random in line interleaver, compared with the uncoded case

In Fig. 6 the BER and FER performance of TCs are shown. From Fig. 6 a) it results that the curves obtained using the BRL interleaver are similarly with the curves obtained with *S*-interleaver. There are some small differences on Fig. 6 b), for a FER=$2\cdot10^{-4}$, where *S*-interleaver behaves better versus BRL interleaver, with approximate 0.15 *dB*, but the design of the new interleaver is simpler.



a)

b)

Fig. 6. Performance of: a) BER and b) FER, of TCs on Rayleigh flat fading channels for 5/7 and 15/13 RSC codes, using the S-interleaver and block random in line interleaver

The Table 1 and Table 2 presents how are influenced the BER and FER performance by the estimation of the SNR value, estimation given by *Lc* factor. Thus, we used values for *Lc* given by the following relation: $Lc=4\cdot R\cdot B\cdot f$, where *R* is the turbo-coding rate and *B* is the absolute value of the SNR.

Table 1. BER$\cdot10^{-9}$, for TCs with 15/13 RSC encoders in function of f and the interleaver (ilv) type.

Ilv	**SNR (dB)**	**f=Lc/(4·R·B)**				
		0.4	**0.5**	**0.6**	**0.7**	**0.8**
S-ilv	2.8	38403775	18411	2130	3175	6047
BRL	2.8	36961840	18434	3814	4412	6142
Ilv	**SNR (dB)**	**f=Lc/(4·R·B)**				
		0.9	**1**	**1.1**	**1.2**	
S-ilv	2.8	11504	33293	67827	135499	
BRL	2.8	16023	35052	70013	114227	

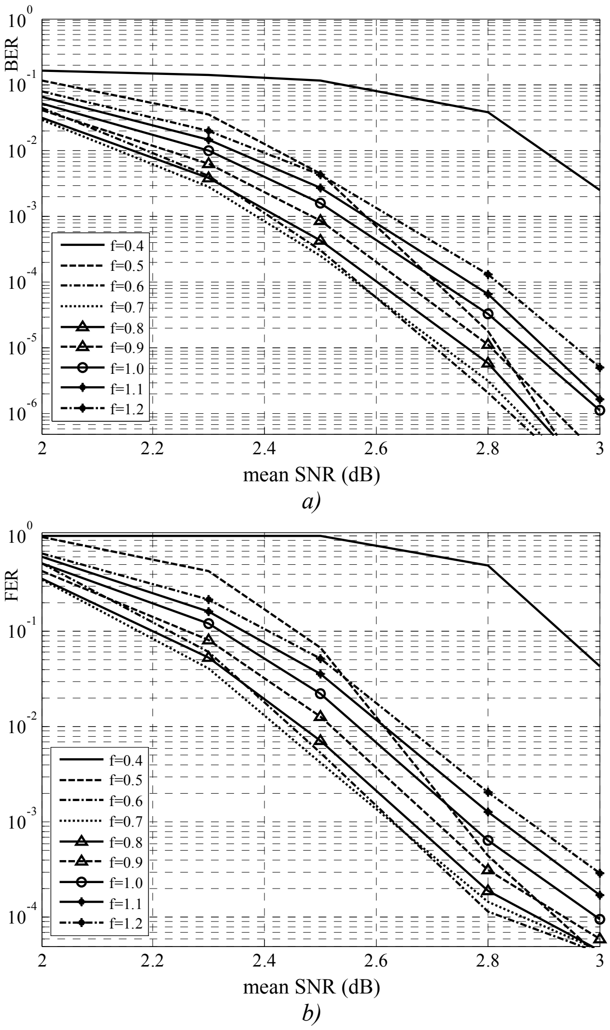
Table 2. FER·10^{-6}, for TCs with 15/13 RSC encoders in function of f and the interleaver (ilv) type.

Ilv	**SNR (dB)**	**f=Lc/(4·R·B)**				
		0.4	**0.5**	**0.6**	**0.7**	**0.8**
S-ilv	2.8	485125	445	115	145	190
BRL	2.8	473333	1016	736	790	836

Ilv	**SNR (dB)**	**f=Lc/(4·R·B)**			
		0.9	**1**	**1.1**	**1.2**
S-ilv	2.8	315	645	1265	2070
BRL	2.8	1066	1403	1996	2723

The results in term of BER and FER plotted in Table 1 and Table 2, for a *SNR*=2.8 *dB*, for 3-memory TC and for bloc random in line interleaver and *S*-interleaver, show that they are better for the factors *f*=0.6÷0.8. The worst values are obtained for *f*<0.5.

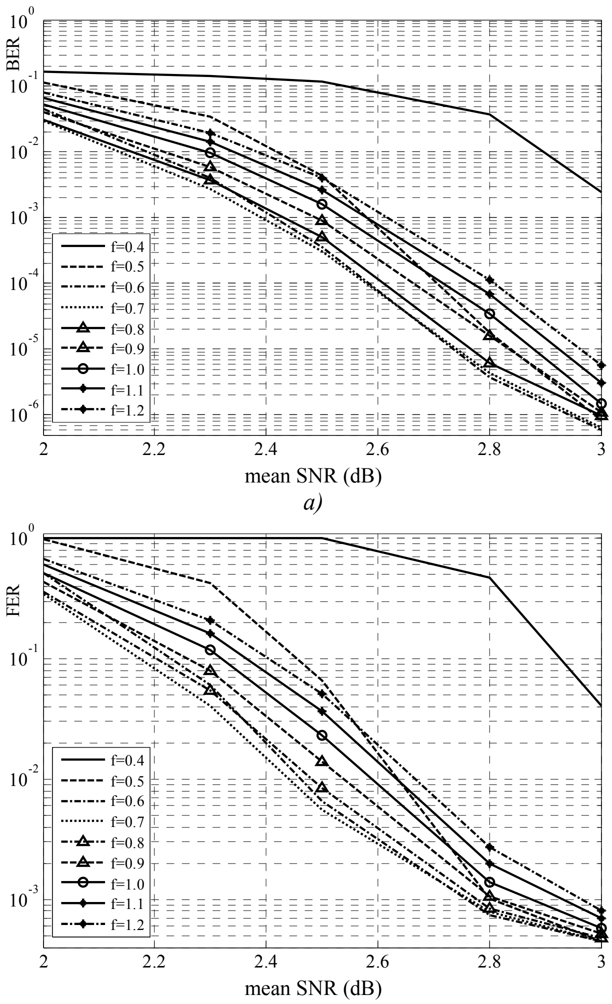
This conclusion is verified in Fig. 7. and Fig. 8. It is evident that the best BER and FER performance are obtained for the values mentioned already above, *f*=0.6÷0.8. Considering BER=10^{-5}, for example, the distance between the curves with *f*=0.6 and *f*=0.5 is approximately equal to 0.1 *dB*.



a)

b)

Fig.7. Performance of: a) BER and b) FER, of TCs on Rayleigh flat fading channels for 15/13 RSC codes, using the S-interleaver for different values of factor f



a)

b)

Fig.8. Performance of: a) BER and b) FER, of TCs on Rayleigh flat fading channels for 15/13 RSC codes, using the block random in line interleaver for different values of factor f

5 CONCLUSIONS

In this paper we presented the BER and FER performance for turbo-coded transmission system over Rayleigh flat fading channel. The simulations results, through this paper, show that the TCs in the presence of the fading provides, as we expected, a tremendous performance gain of the order of tens *dB* even for moderate codeword length, using both interleavers: the new proposed interleaver, block random in line interleaver, and the *S*-interleaver.

As in [4], when we analyzed the TCs over AWGN channel, for the same interleavers, in this paper we found that the BER performances of the proposed BRL interleaver are closed to the performances of the *S* interleaver, with maximum *S*, using TCs over Rayleigh flat fading channel, but the design of the new interleaver is simpler.

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