On Using Turbo Codes Over Rice Flat Fading Channels

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Abstract—The turbo code performance over time-varying channels using QPSK modulation is evaluated in this paper. For this purpose we have considered a block random in line (BRL) interleaver. On the basis of this interleaver, we have further simulated the turbo code performance over Rice flat fading channel using 15 iterations. Also, we have studied the impact of the estimation error in the calculation of the channel log-likelihood ratios $L_c$.

I. INTRODUCTION

Since turbo codes (TCs) were introduced in 1993 [1], numerous investigations have focused on their behavior over Rayleigh fading channels, [2], [3], [4], [5]. In this paper, based on the new type interleaver proposed by us in [6], block random in line interleaver (BRL), we will further study the turbo code (TC) performance over Rice flat fading channel. The channel estimation in the sight of the value construction of the $L_c$ coefficient, necessary in the MAP algorithm, is also evaluated.

The Rice flat fading, [7], models the radio communications channels, for which the received signal has a direct component (assimilated to a non-fading channel, AWGN channel) and a fading component (assimilated to a Rayleigh channel) as is shown in Fig. 1. The power balance between the two components give a measure of the Rice fading, quantified in the following with the coefficient:

$$K = \frac{\text{unbalanced power component}}{\text{total power}}.$$  \hspace{1cm} (1)

The variation of the BER function of the $K$ coefficient, for three values of the signal to noise ratio (SNR), in the non-coding transmission case is presented in Fig.2. We can notice that the effect of the fading is essential even for small proportions of the fading component, given by $1-K$ difference. This effect is for small SNR even more obvious values. Using the notation from Fig.1, the input-output relation is the following:

$$y_k = y_{fk} + y_{uk} = r_k \cdot x_k + w_k,$$ \hspace{1cm} (2)

where $r_k$ is a Rice random variable, composed by a Rayleigh random variable, $\alpha_k$, on $\sqrt{1-K}$ proportion, and a $\sqrt{K}$ constant. Thus, it results that $r_k^2 = 1$. Considering for $x_k$ a unitary power, the SNR value can be:

$$\frac{E_b}{N_o} = 1 + \frac{2 \cdot E_b}{N_o} \cdot \frac{r_k^2 \cdot x_k^2}{w_k^2} = 1 + \frac{1}{w_k^2},$$ \hspace{1cm} (3)

thus, for simulations, the noise power value is:

$$\frac{1}{w_k^2} = \frac{1}{2 \cdot 10^{\text{SNR}/10}}.$$ \hspace{1cm} (4)
The rest of the paper is organized as follows. Section II gives the description of the BRL interleaver. In Section III the TC design is shown. In Section IV we present our results. Section V concludes the paper.

II. BLOCK RANDOM IN LINE INTERLEAVER DESIGN

This interleaver type aims to continue the qualities of the block interleavers (high $d_{min}$) and of the random interleavers (a good spreading). This kind of interleaver represents an alternative to the S-type interleaver, which is difficult to be designed. In the following, the design of the BRL interleaver is presented. We will suppose that the length of this interleaver is given by the relation:

$$N_{bt} = X \times Y,$$

where $X$ and $Y$ are natural numbers.

First, we build the matrix:

$$\mathbf{c}(i,j) = 1 + i + j \cdot X, \text{ with } i \in \{0,1,\ldots,X-1\} \text{ and } j \in \{0,1,\ldots,Y-1\}. $$

(6)

Second, each line of this matrix, $\mathbf{c}(i,j)$, is permuted using the relation:

$$\pi_r(i) = \text{rand}(i), \forall i \in I = \{1,2,\ldots,N_r\},$$

is obtained the matrix $\mathbf{b}$:

$$\mathbf{b}(i,J) = \mathbf{c}(i,\pi_r(J)), \forall i \in I,$$

(8)

Finally, to increase the minimum distance of the interleaver, a reorder of the lines is done so that on the first $X/2$ positions can be found the odd lines. After reordering of the lines a read out on columns-size is made:

$$\pi_{BRL}(j+k \cdot Y+1) = \mathbf{b}(i,j), \text{ with } i \in \{1,2,\ldots,N_r\} \text{ and } j \in J,$$

(9)

where $Y_2 = \text{floor}[(Y-1)/2]$.

III. TURBO CODE DESIGN

In this paper we consider the classical TC [1] shown in Fig. 3.

![Figure 3. Scheme of the classical turbo code.](image)

where, blocks I and DI realize interleaving and de-interleaving functions, using the interleavers presented in previous section.

The encoders are denoted with C1 and C2 and the notation for decoders are DEC1 and DEC2, respectively. There are also the following notations: - $\mathbf{u}$ for the input sequence; - $x_0, x_1, x_2$ for the encoder outputs; - $y_0, y_1, y_2$ for channel outputs; - $I_{ex12}$ and $I_{ex21}$ for extrinsic information; - $LLR$ for Logarithm of Likelihood Ratio; - $\hat{u}$ for decoder output.

The component decoders use a MAP iterative algorithm. The different steps of the MAP decoding algorithm are summarized in Fig. 4 [8].

IV. SIMULATION RESULTS

We consider the following setup for our simulations in Fig.5. The considered TC consists of the parallel concatenation of two identical rate 1/2 recursive, systematic, convolutional code (RSC) with a memory of 3, $M=3$, $(15/13)$. The trellis of the first encoder is terminated at zero and the trellis of the second encoder is unterminated. The rate of this turbo code is equal to $1/3$ (more precisely equal to $\frac{N-M}{N}$ due to the termination of the first encoder). It is worth noting that no puncturing is needed. We used two interleavers. One is the interleaver described in the previous paragraph, with $X=119$ and $Y=15$, and the other is an S-type interleaver, [9], with $S=29$. The data block length, $N$, is equal to 1785 bits, and 1784, respectively. In our simulation we assume QPSK signaling with perfect channel phase recovery at the receiver. As mentioned in the previous section, we used the MAP decoding algorithm [10]. A maximal number of 15 iterations with a stopping criterion are employed.

The transmission channel that we have considered in our simulations is the Rice time selective channel. In our diagrams we have drawn five curves corresponding to the five values of $K$ coefficient: 0, 0.25, 0.5, 0.75 and 1.

We can observe that, using both interleavers, the performance of TC for Rice flat fading channels with $K>0$ are upper bounded by the performances over Rayleigh channel (which corresponds to $K=0$, i.e. the most time-selective channel) and lower bounded by static channels (with $K=1$, i.e. AWGN channel).

Comparing the curves from Fig.5 a) with the curves from Fig.5 b) we observe that we have obtained with the BRL interleaver the similar BER performance like in the case of using the S-interleaver, but the design of the BRL interleaver is simpler [6]. Similar BER performances for these two interleavers had obtained for TC over AWGN channels in [6].

![Figure 4. Summary of the key operations in the MAP algorithm](image)
Figure 5. BER performance over Rice flat fading channel for K=0, 25, 50, 75, and 100 [%], considering 15/13 RSC code and using: a) the BRL interleaver and b) the S-interleaver.

As for the S-interleaver case, for the BRL interleaver the TC performance is not linear improved with the K parameter. Thus, if the power of the continuous component is under 25% from total power (K< 0.25), the Rice channel behavior is the same like the Rayleigh channel. For values over 50% of the K, the performances are significant improved.

The next results present how is influenced the BER performance by the estimation of the SNR value, estimation given by Lc factor. Thus, we used values for Lc given by the following relation: Lc=4·R·B·f, where R is the turbo-coding rate and B is the absolute value of the SNR.

For the simulations in Table I and Table II we have considered two more interleavers, pseudo-random interleaver (with N=1784 bits) defined in [11] and Takeshita-Costello interleaver (with N=2048 bits) defined in [12]. Moreover, we also have used 25/23 convolutional code (RSC) with a memory of 4. In all Tables we considered the value of SNR equal to: 2.8dB for K=0% and K=25%, 2.5dB for K=50%, 1.8 dB for K=75% and 0.8dB for K=100%.

In Table 1 a), when a BRL interleaver and a 15/13 RSC code are used, we have obtained good BER values for 0% ≤ K ≤ 50% when \( f = 0.6 + 0.8 \) (theoretically, for \( K = 0, f \) is

### Table I. BER 10^-4, for TCs with 15/13 RSC Encoders

<table>
<thead>
<tr>
<th>K [%]</th>
<th>SNR [dB]</th>
<th>f=Lc/(4·R·B)</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.8</td>
<td>369.6</td>
<td>0.184</td>
<td>0.038</td>
<td>0.044</td>
<td>0.061</td>
<td>0.160</td>
<td>0.350</td>
<td>0.700</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>2.8</td>
<td>10.04</td>
<td>0.042</td>
<td>0.015</td>
<td>0.016</td>
<td>0.022</td>
<td>0.053</td>
<td>0.107</td>
<td>0.182</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2.5</td>
<td>644.1</td>
<td>0.151</td>
<td>0.012</td>
<td>0.015</td>
<td>0.018</td>
<td>0.025</td>
<td>0.037</td>
<td>0.098</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>1.8</td>
<td>1274.5</td>
<td>34.51</td>
<td>0.025</td>
<td>0.015</td>
<td>0.013</td>
<td>0.018</td>
<td>0.029</td>
<td>0.046</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.8</td>
<td>1569.1</td>
<td>1131.7</td>
<td>0.077</td>
<td>0.016</td>
<td>0.014</td>
<td>0.023</td>
<td>0.037</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) BRL interleaver, N=1785 bits

### Table II. BER 10^-4, for TCs with 25/23 RSC Encoders

<table>
<thead>
<tr>
<th>K [%]</th>
<th>SNR [dB]</th>
<th>f=Lc/(4·R·B)</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.8</td>
<td>363.4</td>
<td>0.196</td>
<td>0.034</td>
<td>0.046</td>
<td>0.066</td>
<td>0.190</td>
<td>0.382</td>
<td>0.382</td>
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</tr>
<tr>
<td>25</td>
<td>2.8</td>
<td>247.4</td>
<td>0.044</td>
<td>0.011</td>
<td>0.008</td>
<td>0.017</td>
<td>0.054</td>
<td>0.084</td>
<td>0.189</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2.5</td>
<td>657.8</td>
<td>0.172</td>
<td>0.005</td>
<td>0.003</td>
<td>0.006</td>
<td>0.023</td>
<td>0.054</td>
<td>0.099</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>1.8</td>
<td>1275.9</td>
<td>36.45</td>
<td>0.031</td>
<td>0.004</td>
<td>0.002</td>
<td>0.007</td>
<td>0.017</td>
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<tr>
<td>100</td>
<td>0.8</td>
<td>1568.4</td>
<td>1134.3</td>
<td>0.090</td>
<td>0.015</td>
<td>0.011</td>
<td>0.022</td>
<td>0.032</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) S-interleaver, N=1784 bits

c) Pseudo-random interleaver, N=2048 bits

d) Takeshita-Costello interleaver, N=2048 bits

TABLE II. BER 10^-4, FOR TCs WITH 25/23 RSC ENCODERS

<table>
<thead>
<tr>
<th>K [%]</th>
<th>SNR [dB]</th>
<th>f=Lc/(4·R·B)</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
<th>1.1</th>
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<tr>
<td>0</td>
<td>2.8</td>
<td>989.3</td>
<td>0.955</td>
<td>0.047</td>
<td>0.046</td>
<td>0.126</td>
<td>0.340</td>
<td>0.997</td>
<td>2.541</td>
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</tr>
<tr>
<td>25</td>
<td>2.8</td>
<td>728.5</td>
<td>0.215</td>
<td>0.004</td>
<td>0.014</td>
<td>0.031</td>
<td>0.098</td>
<td>0.283</td>
<td>0.889</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2.5</td>
<td>1174.7</td>
<td>1.189</td>
<td>0.012</td>
<td>0.012</td>
<td>0.014</td>
<td>0.043</td>
<td>0.111</td>
<td>0.313</td>
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</tr>
<tr>
<td>75</td>
<td>1.8</td>
<td>1499.6</td>
<td>188.08</td>
<td>0.106</td>
<td>0.011</td>
<td>0.014</td>
<td>0.020</td>
<td>0.036</td>
<td>0.125</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.8</td>
<td>1691.7</td>
<td>1409.1</td>
<td>0.309</td>
<td>0.016</td>
<td>0.013</td>
<td>0.014</td>
<td>0.038</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) BRL interleaver, N=1785 bits
In this paper we show that the BER performances of the proposed BRL interleaver are closed to the performances of the S-interleaver, with maximum S, using TCs over Rice flat fading channel, but the design of the new interleaver is simpler.

Also, for factor f we found experimental values, at different values of K, which provide good BER performances.

equal to 0.8862 [2]), for K = 75% when f = 0.7 + 0.9 and for K = 100% when f = 0.8 + 1.0 (theoretically, f is equal to 1).

For same values of f similar BER performance are obtained when S-interleaver, pseudo-random interleaver and Takeshita-Costello interleaver are employed. Moreover, the results obtained in Table 1 are also confirmed when a code with a memory of 4, 25/23 RSC code, is used.

An example of BER performance on Rice flat fading channel, for 25/23 RSC code and K=50%, using BRL interleaver for f = 0.4 +1.1 is given in Fig. 6.

It is evident that the best BER performances are achieved for values of factor f mentioned already above, f = 0.6 - 0.8. For example, considering BER<4·10⁻², the performances given by the curves with f = 0.6 + 0.8 are better with approximately 0.05 dB, 0.12 dB, 0.2 dB and more than 0.2 dB, versus the curves which correspond to f = 0.9, f = 1.0, f = 1.1 and f = 0.5, respectively. The worst values correspond to f < 0.5.

V. CONCLUSIONS

In this paper we show that the BER performances of the proposed BRL interleaver are close to the performances of the S-interleaver, with maximum S, using TCs over Rice flat fading channel, but the design of the new interleaver is simpler.

Also, for factor f we found experimental values, at different values of K, which provide good BER performances.

![Figure 6. BER performance on Rice flat fading channel, for 25/23 RSC code and K=50%, using BRL interleaver for different values of factor f.](image-url)

The simulation had made considering four interleavers type and RSC codes with a memory of 3 and a memory of 4, respectively.

The obtained results show how is influenced the BER performance by the channel estimation, estimation given by Lc factor, by factor f, implicitly.

REFERENCES


