On Using Turbo Codes Over Rice Flat Fading Channels

Maria Kovaci[1], Horia Balta[1], Miranda Nafornita[1]
[1] Department of Communications, University "Politehnica"of Timisoara,
Faculty of Electronics and Telecommunications, Timisoara, Romania
horia.balta@etc.upt.ro, maria.kovaci@etc.upt.ro, monica.nafornita@etc.upt.ro

***Abstract*—The turbo code performance over time-varying channels using QPSK modulation is evaluated in this paper. For this purpose we have considered a block random in line (BRL) interleaver. On the basis of this interleaver, we have further simulated the turbo code performance over Rice flat fading channel using 15 iterations. Also, we have studied the impact of the estimation error in the calculation of the channel log-likelihood ratios *Lc*.**

I. Introduction

Since turbo codes (TCs) were introduced in 1993 [1], numerous investigations have focused on their behavior over Rayleigh fading channels, [2], [3], [4], [5]. In this paper, based on the new type interleaver proposed by us in [6], block random in line interleaver (BRL), we will further study the turbo code (TC) performance over Rice flat fading channel. The channel estimation in the sight of the value construction of the *Lc* coefficient, necessary in the MAP algorithm, is also evaluated.

The Rice flat fading, [7], models the radio communications channels, for which the received signal has a direct component (assimilated to a non-fading channel, AWGN channel) and a fading component (assimilated to a Rayleigh channel) as is shown in Fig. 1. The power balance between the two components give a measure of the Rice fading, quantified in the following with the coefficient:

$$K = \frac{unbalanced\ \ power\ \ component}{total\ power}\,. \tag{1}$$

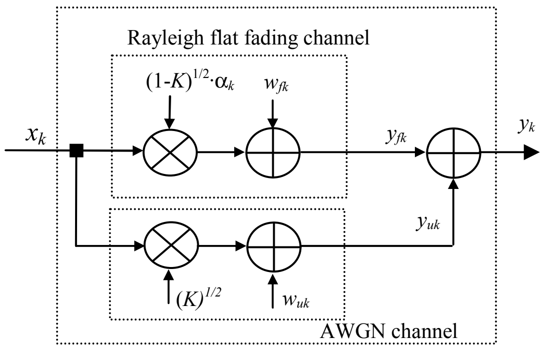

Figure 1. Rice flat fading channel –model for the simulations.

The variation of the BER function of the *K* coefficient, for three values of the signal to noise ratio (SNR), in the non-coding transmission case is presented in Fig.2. We can notice that the effect of the fading is essential even for small proportions of the fading component, given by 1-*K* difference. This effect is for small SNR even more obvious values. Using the notation from Fig.1, the input-output relation is the following:

$$y_k = y_{fk} + y_{uk} = r_k \cdot x_k + w_k\,, \tag{2}$$

where r_k is a Rice random variable, composed by a Rayleigh random variable, α_k, on $\sqrt{1-K}$ proportion, and a $\sqrt{K}$ constant. Thus, it results that $\overline{r_k^2}$ =1. Considering for x_k a unitary power, the SNR value can be:

$$\frac{E_b}{N_o} = \frac{1}{2}\cdot\frac{2\cdot E_b}{N_o} = \frac{1}{2}\cdot\frac{\overline{r_k^2}\cdot\overline{x_k^2}}{\overline{w_k^2}} = \frac{1}{2}\cdot\frac{1}{\overline{w_k^2}}\,, \tag{3}$$

thus, for simulations, the noise power value is:

$$\overline{w_k^2} = \frac{1}{2\cdot 10^{SNR/10}}\,, \tag{4}$$

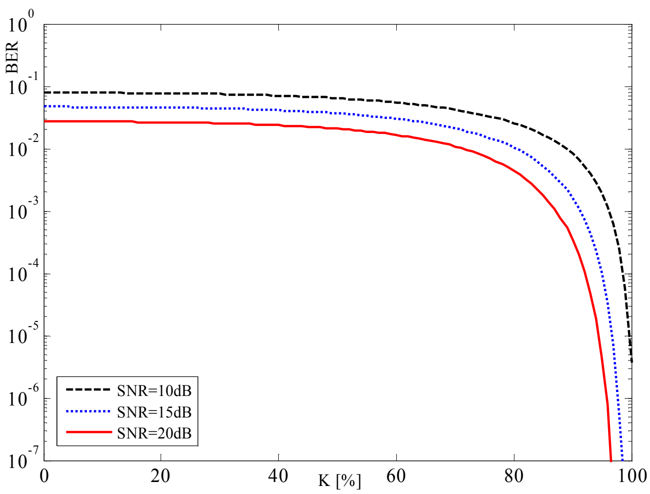

Figure 2. BER versus *K* coefficient

The rest of the paper is organized as follows. Section II gives the description of the BRL interleaver. In Section III the TC design is shown. In Section IV we present our results. Section V concludes the paper.

II. Block Random in Line Interleaver Design

This interleaver type aims to continue the qualities of the block interleavers (high d_{min}) and of the random interleavers (a good spreading). This kind of interleaver represents an alternative to the *S*-type interleaver, which is difficult to be designed. In the following, the design of the BRL interleaver is presented. We will suppose that the length of this interleaver is given by the relation:

$$N_{br} = X \times Y \, , \tag{5}$$

where *X* and *Y* are natural numbers.

First, we build the matrix:

$$\mathbf{c}(i, j) = 1 + i + j \cdot X, \text{ with } i \in I = \{0,1,\ldots,X-1\} \text{ and } j \in J = \{0,1,\ldots,Y-1\}. \tag{6}$$

Second, each line of this matrix, $\mathbf{c}(i,J)$, is permuted using the relation:

$$\pi_r(i) = rand(i), \forall i \in I = \{1,2,\ldots,N_r\}, \tag{7}$$

is obtained the matrix **b**:

$$\mathbf{b}(i, J) = \mathbf{c}(i, \pi_r(J)), \forall i \in I \, , \tag{8}$$

Finally, to increase the minimum distance of the interleaver, a reorder of the lines is done so that on the first *X*/2 positions can be found the odd lines. After reordering of the lines a read out on columns-size is made:

$$\pi_{BRL}(j + k \cdot Y + 1) = \mathbf{b}(i, j), \quad i = (2 \cdot k) \in I, \quad j \in J;$$
$$\pi_{BRL}(j + k \cdot Y + Y_2 + 1) = \mathbf{b}(i, j), \quad i = (2 \cdot k - 1) \in I, \quad j \in J \tag{9}$$

where $Y_2 = floor[(Y-1)/2]$.

III. Turbo Code Design

In this paper we consider the classical TC [1] shown in Fig. 3.

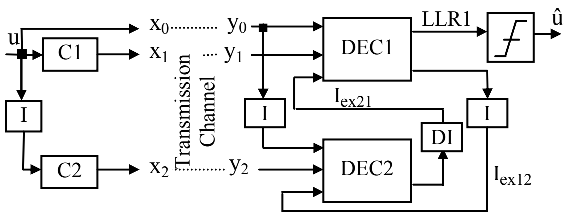

Figure 3. Scheme of the classical turbo code.

where, blocks I and DI realize interleaving and de-interleaving functions, using the interleavers presented in previous section.

The encoders are denoted with C1 and C2 and the notation for decoders are DEC1 and DEC2, respectively. There are also the following notations: - *u* for the input sequence; - x_0, x_1, x_2 for the encoder outputs; - y_0, y_1, y_2 for channel outputs; - I_{ex21} and I_{ex12} for extrinsic information; - *LLR* for Logarithm of Likelihood Ratio; $-\hat{u}$ for decoder output.

The component decoders use a MAP iterative algorithm. The different steps of the MAP decoding algorithm are summarized in Fig. 4 [8].

IV. Simulation Results

We consider the following setup for our simulations in Fig.5. The considered TC consists of the parallel concatenation of two identical rate 1/2 recursive, systematic, convolutional code (RSC) with a memory of 3, *M*=3, (15/13). The trellis of the first encoder is terminated at zero and the trellis of the second encoder is unterminated. The rate of this turbo code is equal to 1/3 (more precisely equal to $\frac{1}{3} \cdot \frac{N-M}{N}$ due to the termination of the first encoder). It is worth noting that no puncturing is needed. We used two interleavers. One is the interleaver described in the previous paragraph, with *X*=119 and *Y*=15, and the other is an *S*-interleaver, [9], with *S*=29. The data block length, *N*, is equal to 1785 bits, and 1784, respectively. In our simulation we assume QPSK signaling with perfect channel phase recovery at the receiver. As mentioned in the previous section, we used the MAP decoding algorithm [10]. A maximal number of 15 iterations with a stopping criterion are employed.

The transmission channel that we have considered in our simulations is the Rice time selective channel. In our diagrams we have drawn five curves corresponding to the five values of *K* coefficient: 0, 0.25, 0.5, 0.75 and 1.

We can observe that, using both interleavers, the performance of TC for Rice flat fading channels with *K*>0 are upper bounded by the performances over Rayleigh channel (which corresponds to *K*=0, i.e. the most time-selective channel) and lower bounded by static channels (with *K*=1, i.e. AWGN channel).

Comparing the curves from Fig.5 a) with the curves from Fig.5 b) we observe that we have obtained with the BRL interleaver the similar BER performance like in the case of using the *S*-interleaver, but the design of the BRL interleaver is simpler [6]. Similar BER performances for these two interleavers had obtained for TC over AWGN channels in [6].

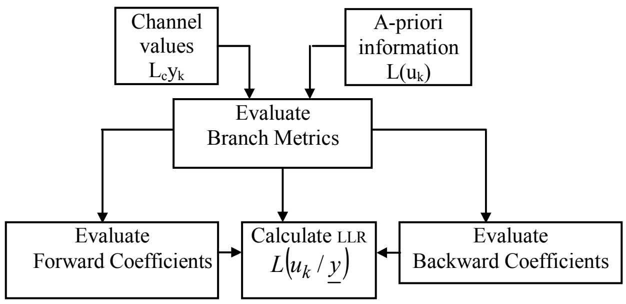

Figure 4. Summary of the key operations in the MAP algorithm

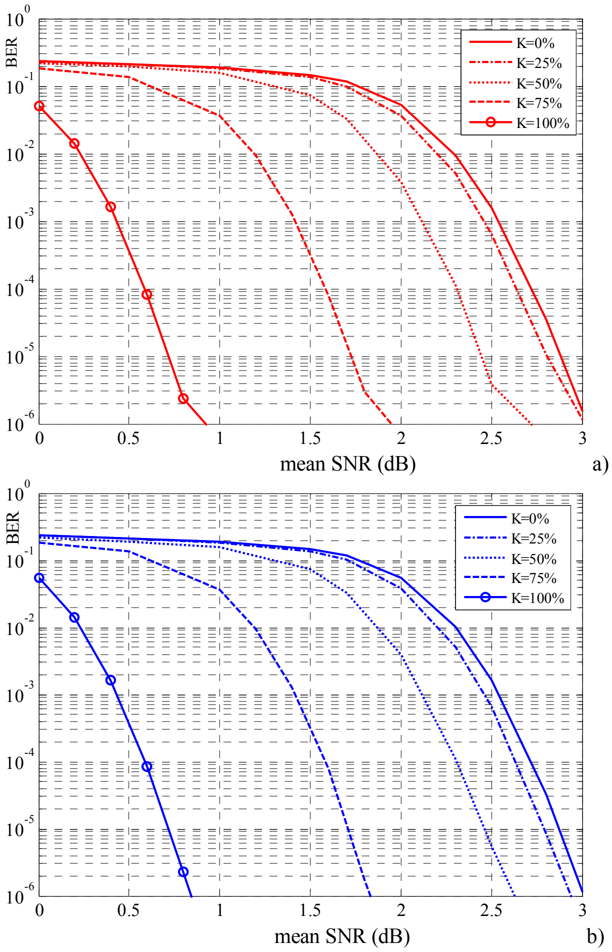

Figure 5. BER performance over Rice flat fading channel for K=0, 25, 50, 75, and 100 [%], considering 15/13 RSC code and using: a) the BRL interleaver and b) the *S*-interleaver.

As for the *S*-interleaver case, for the BRL interleaver the TC performance is not linear improved with the *K* parameter. Thus, if the power of the continuous component is under 25% from total power (*K*< 0.25), the Rice channel behavior is the same like the Rayleigh channel. For values over 50% of the *K*, the performances are significant improved.

The next results present how is influenced the BER performance by the estimation of the SNR value, estimation given by *Lc* factor. Thus, we used values for *Lc* given by the following relation: $Lc=4 \cdot R \cdot B \cdot f$, where *R* is the turbo-coding rate and *B* is the absolute value of the SNR.

For the simulations in Table I and Table II we have considered two more interleavers, pseudo-random interleaver (with *N*=1784 bits) defined in [11] and Takeshita-Costello interleaver (with *N*=2048 bits) defined in [12]. Moreover, we also have used 25/23 convolutional code (RSC) with a memory of 4. In all Tables we considered the value of SNR equal to: 2.8dB for *K*=0% and *K*=25%, 2.5dB for *K*=50%, 1.8 dB for *K*=75% and 0.8dB for *K*=100%.

In Table 1 a), when a BRL interleaver and a 15/13 RSC code are used, we have obtained good BER values for $0\% \le K \le 50\%$ when $f = 0.6 \div 0.8$ (theoretically, for *K*=0, *f* is

TABLE I. BER·10^{-4}, FOR TCs WITH 15/13 RSC ENCODERS

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	369,6	0,184	**0,038**	**0,044**	**0,061**	0,160	0,350	0,700
25	2.8	10,04	0,042	**0,015**	**0,016**	**0,022**	0,053	0,107	0,182
50	2.5	643,1	0,151	**0,012**	**0,015**	**0,018**	0,025	0,037	0.098
75	1.8	1274,5	34,51	0,025	**0,015**	**0,013**	**0,018**	0,029	0,046
100	0.8	1569,1	1131,7	27,25	0,077	**0,016**	**0,014**	**0,023**	0,037

a) BRL interleaver, *N*=1785 bits

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	384,03	0,184	**0,021**	**0,031**	**0,060**	0,115	0,332	0,678
25	2.8	247,4	0,044	**0,011**	**0,008**	**0,017**	0,054	0,084	0,189
50	2.5	657,8	0,172	**0,005**	**0,003**	**0,006**	0,023	0,054	0,098
75	1.8	1275,9	36,45	0,031	**0,004**	**0,002**	**0,007**	0,017	0,052
100	0.8	1568,4	1134,3	28,767	0,090	**0,015**	**0,011**	**0,022**	0,032

b) *S*-interleaver, *N*=1784 bits

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	363,4	0,196	**0,034**	**0,046**	**0,066**	0,190	0,382	0,382
25	2.8	235,2	0,046	**0,009**	**0,009**	**0,026**	0,046	0,102	0,169
50	2.5	638,2	0,137	**0,011**	**0,015**	**0,015**	0,024	0,053	0,097
75	1.8	1275,9	32,63	0,024	**0,013**	**0,013**	**0,016**	0,028	0,051
100	0.8	1568,7	1132,2	27,73	0,078	**0,020**	**0,021**	**0,031**	0,040

c) *Pseudo*-random interleaver, *N*=1784 bits

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	344,8	0,141	**0,108**	**0,112**	**0,127**	0,171	0,257	0,405
25	2.8	210,1	0,082	**0,077**	**0,078**	**0,075**	0,092	0,101	0,160
50	2.5	633,5	0,120	**0,084**	**0,085**	**0,088**	**0,085**	0,106	0,109
75	1.8	1275,2	23,55	**0,086**	**0,084**	**0,087**	**0,081**	0,097	0,106
100	0.8	1569,8	1133,3	17,57	**0,113**	**0,122**	**0,094**	**0,107**	**0,101**

d) *Takeshita-Costello* interleaver, *N*=2048 bits

TABLE II. BER·10^{-4}, FOR TCs WITH 25/23 RSC ENCODERS

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	891,3	0,955	**0,047**	**0,046**	**0,126**	0,340	0,997	2,541
25	2.8	720,5	0,215	**0,004**	**0,014**	**0,031**	**0,098**	0,283	0,689
50	2.5	1147,4	1,189	**0,012**	**0,012**	**0,014**	**0,043**	0,111	0,313
75	1.8	1499,6	188,08	0,106	**0,011**	**0,014**	**0,020**	**0,036**	0,125
100	0.8	1691,7	1409,1	150,5	0,309	**0,016**	**0,013**	**0,014**	0,038

a) BRL interleaver, *N*=1785 bits

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	896,06	0,839	**0,024**	**0,030**	**0,079**	0,362	0,992	2,484
25	2.8	723,1	0,202	**0,005**	**0,006**	**0,021**	0,063	0,247	0,648
50	2.5	1148,8	1,209	**0,008**	**0,007**	**0,011**	0,031	0,0807	0,3044
75	1.8	1499,5	192,4	0,105	**0,002**	**0,004**	**0,007**	0,0238	0,099
100	0.8	1691,7	1408	153,1	0,336	**0,015**	**0,010**	**0,012**	0,029

b) *S*-interleaver, *N*=1784 bits

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	877,4	0,773	**0,012**	**0,023**	**0,064**	0,287	0,932	2,219
25	2.8	698,01	0,163	**0,004**	**0,010**	**0,018**	0,073	0,211	0,615
50	2.5	1141,1	0,983	**0,007**	**0,007**	**0,008**	0,034	0,101	0,203
75	1.8	1499,1	177	0,067	**0,006**	**0,007**	**0,009**	0,023	0,092
100	0.8	1691,8	1407,8	139,88	0,290	**0,017**	**0,009**	**0,014**	0,031

c) *P*seudo-random interleaver, *N*=1784 bits

K [%]	SNR [dB]	f=Lc/(4·R·B)							
		0.4	***0.5***	***0.6***	***0.7***	***0.8***	***0.9***	***1***	***1.1***
0	2.8	873,05	0,317	**0,013**	**0,018**	**0,047**	0,111	0,395	1,041
25	2.8	687,3	0,069	**0,016**	**0,011**	**0,011**	0,029	0,088	0,221
50	2.5	1144,6	0,432	**0,020**	**0,016**	**0,010**	**0,016**	0,051	0,117
75	1.8	1500,2	150,1	0,055	**0,015**	**0,013**	**0,009**	**0,018**	0,040
100	0.8	1692,1	1408	112,9	0,131	**0,025**	**0,020**	**0,018**	**0,018**

d) *Takeshita-Costello* interleaver, *N*=2048 bits

equal to 0.8862 [2]), for $K = 75\%$ when $f = 0.7 \div 0.9$ and for $K = 100\%$ when $f = 0.8 \div 1.0$ (theoretically, f is equal to 1).

For same values of f similar BER performance are obtained when *S*-interleaver, pseudo-random interleaver and Takeshita-Costello interleaver are employed. Moreover, the results obtained in Table 1 are also confirmed when a code with a memory of 4, 25/23 RSC code, is used.

An example of BER performance on Rice flat fading channel, for 25/23 RSC code and *K*=50%, using BRL interleaver for $f = 0.4 \div 1.1$ is given in Fig. 6.

It is evident that the best BER performances are achieved for values of factor f mentioned already above, $f = 0.6 \div 0.8$. For example, considering BER=$4 \cdot 10^{-5}$, the performances given by the curves with $f = 0.6 \div 0.8$ are better with approximately 0.05 dB, 0.12 dB, 0.2 dB and more than 0.2 dB, versus the curves which correspond to f = 0.9, f = 1.0, f = 1.1 and f = 0.5, respectively. The worst values correspond to $f < 0.5$.

V. CONCLUSIONS

In this paper we show that the BER performances of the proposed BRL interleaver are closed to the performances of the *S*-interleaver, with maximum *S*, using TCs over Rice flat fading channel, but the design of the new interleaver is simpler.

Also, for factor f we found experimental values, at different values of *K*, which provide good BER performances.

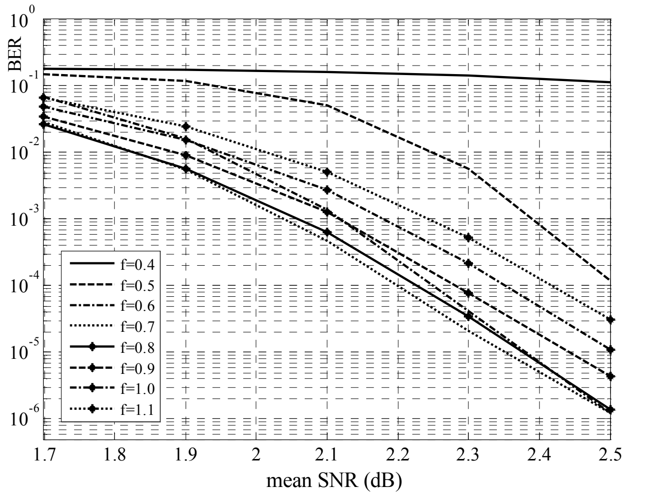

Figure 6. BER performance on Rice flat fading channel, for 25/23 RSC code and *K*=50%, using BRL interleaver for different values of factor *f*.

The simulation had made considering four interleavers type and RSC codes with a memory of 3 and a memory of 4, respectively.

The obtained results show how is influenced the BER performance by the channel estimation, estimation given by *Lc* factor, by factor *f*, implicitly.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit error-correcting coding and decoding: Turbo-codes”, Proc.ICC’93, Geneva, Switzerland, May 1993, pp. 1064 – 1070.

[2] E. K. Hall and S. G. Wilson, “Design and analysis of turbo codes on Rayleigh fading channels”, IEEE Journal on Selected Areas in Communications, Vol. 16, No. 2, February 1998, pp. 160-174.

[3] D. F. Yuan and X. F. Song, “Turbo code performance over Rayleigh fading channel using QPSK modulation”, Proceedings on IEEE TENCON’02, pp. 1056-1059.

[4] L. Conde Canencia and C. Douillard, “Performance estimation of 8-PSK turbo coded modulation over Rayleigh fading channels,” 3rd International Symposium on Turbo Codes & Related Topics, Brest, France, Sept. 2003, pp. 567-570.

[5] M. Kovaci, H. Balta, and M. Nafornita “Performance of multi binary turbo codes on Rayleigh flat fading transmission channels”, Analele Universitatii din Oradea, Fascicola Electrotehnica, Sectiunea Electronica, 2006, pp. 64-67.

[6] M. Kovaci, H. G. Balta, and M. M. Nafornita, “The performances of interleavers used in turbo-codes”, International Symposium on Signals, Circuits and Systems, 2005. ISSCS 2005, Volume 1, July 14-15, 2005 pp.363 – 366.

[7] John G. Proakis, Digital Communications, McGraw-Hill Series in Electrical and Computer Engineering Stephen W., 2001.

[8] L.Hanzo, T.H.Liew, and B.L.Yeap, Turbo Coding, Turbo Equalisation and Space-Time Coding for Transmission over Fading Channels, John Wiley & Sons Ltd, England, 2002.

[9] D. Divsalar and F.Pollara, “Turbo codes for PCS applications“, IEEE Int. Conf. Communications, 18-22 June, 1995, pp. 54-59.

[10] L.R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimising symbol error rate”, IEEE Transactions on Information Theory, Vol. 20, pp. 284-287, March 1974.

[11] Consultative Committee for Space Data Systems “Telemetry Channel Coding”, Blue Book, CCSDS 101.0-B-6, Oct 2002.

[12] O.Y. Takeshita and D.J. Costello, “New classes of algebraic interleavers for turbo-codes”, ISIT 1998, August 16-21, USA.