The Performances of Interleavers used in Turbo Codes

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Abstract— **A new type of interleaver, block-random interleaver, is proposed and compared with the principal types of interleavers on the basis of the BER (Bit Error Rate) performances of the turbo code. A Recursive Systematic Convolutional Code, 1/3 rate, unpunctured turbo code was used. The convolutional code has the constraint length, K=4 with generator matrix [1, 15/13], in octal representation. The MAP algorithm, 12 iterations were used. A Log Likelihood Ratio (LLR) stop criterion was selected. The BER and FER (Frame Error Rate) for different interleaver lengths were obtained.**

***Keywords*—Turbo-Code, Interleaver, AWGN channel, BPSK modulation**

I. Introduction

An interleaver makes a permutation of a sequence of numbers, so it implements a permutation function [1]:

$$\pi : I \rightarrow I, \quad with \ I = \{1,2, ... N\}, \tag{1}$$

where N represents the length of the interleaved sequence. Receiver has a complementary deinterleaver, which implements the reverse function:

$$\pi^{-1} : I \rightarrow I, \quad with \ \pi^{-1}(\pi(i)) = i, \ \forall i \in I. \tag{2}$$

The interleaving distance between the positions i and j is:

$$d(i, j) = |i - j| + |\pi(i) - \pi(j)|, \ \forall i,j \in I, \ i \neq j. \tag{3}$$

Then, the minimum interleaving distance is given as, [1]:

$$d_{min} = \min_{\substack{i,j \in I \\ i \neq j}} d(i,j). \tag{4}$$

The paper is organized as follows. Section II presents some interleaver: random interleaver, S-interleaver, block interleaver, pseudo-random interleaver, and Takeshita-Costello interleaver. A new type, the block random interleaver is proposed. Section III presents the performances of these interleavers. The last section is dedicated to some concluding remarks.

II. Interleaver Design

A. Random interleaver (IN_rS1)

The random interleaver, noted IN_rS1 (N_r takes values from Table 1), has a simple design, which provides a good original sequence spreading, but it generally has $d_{min} = 2$, i.e. the smallest possible value. The construction of a random interleaver is as following. Knowing the interleaving length N_r, the ensemble $A = \{1,2 ...N_r\}$ is built. We choose in a random way a number $n_1 \in A$. Then, the allocation $\pi(1) = n_1$ is made and this value (n_1) is eliminated from A. The process is repeated while A has no elements. So, the following random interleaver function:

$$\pi_r(i) = rand(i), \ \forall i \in I = \{1,2 ...N_r\}, \tag{5}$$

is used.

A main disadvantage for the random interleaver is the irreproducibility of the generation process of mapping π_r: it should be memorized for reproduction, after its construction.

B. The S-interleaver (IN_sSY)

The S-interleaver, noted IN_sSY (N_s and Y take values from Table 1) is a random type interleaver. However, unlike the pure random interleaver, by construction a minimum interleaving distance equal with S is forced. The interleaving mapping construction algorithm is as follows. We select a possible future position for the current bit. This position is compared to the last S positions already selected.

TABLE I. The Interleavers Used

N≅	400	900	1800	3600
IXRY	*I19R21*	*I29R31*	*I41R45*	*I59R61*
IN_rS1	*I392S1*	*I896S1*	*I1784S1*	*I3568S1*
IN_sSY	*I392S5* *I392S10* *I392S13*	*I896S5* *I896S10* *I896S20*	*I1784S10* *I1784S20* *I1784S29*	*I3568S10* *I3568S20* *I3568S40*
IXP8	*I392P8*	*I896P8*	*I1784P8*	*I3568P8*
IXTC	*I512TC*	*I1024TC*	*I2048TC*	*I4096TC*
IXBRLY	*I49BRL8*	*I81BRL11*	*I121BRL15*	*I225BRL16*
IXBLRY	*I7BLR56*	*I9BLR99*	*I11BLR165*	*I15BLR240*

If the condition:

$$|\pi_s(i) - \pi_s(j)| > S \text{ for } i \text{ and } j \text{ with } |i - j| < S, \quad (6)$$

is satisfied we go further. If the condition is not true, we select another position for the current bit, which will also be tested. The process will be repeated up to the moment when all the positions for the N_s bits will have been found. The simulations demonstrated that, if $S<\sqrt{N_s/2}$, then the process will converge. The design of this interleaver is difficult because the difficulty of the accomplishment of the condition increases with the rise in the number of bits already tested.

C. The block interleaver (IXRY)

The block (rectangular) interleaver, noted *IXRY* (*X* and *Y* are given by Table 1) presents the simplest structure, [2]. To obtain a block interleaver function it is necessary to factorize its length.

$$N_b = X \times Y, \quad (7)$$

X and *Y* are closed natural numbers, [2]. The block interleaver function is:

$$\pi_b(i+j*X+1) = i*Y+j+1,$$
$$\forall i\in I=\{0,1\ldots X\text{-}1\} \text{ and } \forall j\in J=\{0,1\ldots Y\text{-}1\}. \quad (8)$$

Any two bits, initially situated at a distance less than $d_{min}=min(I,J)$, will be situated, after interleaving, at a distance superior to d_{min}.

D. The pseudo-random interleaver (IXP8)

The pseudo-random interleaver, noted *IXP8* (*X is* given by Table 1) recommended by the "CCSDS Recommendation for Telemetry Channel Coding", [3]. For this interleaver type there is a controlled spreading.

It is a high performance interleaver, which combines the advantages of the random and block interleavers, i.e. it presents a good spreading at enough large minimum distance.

The recommended permutation for each block of length N_p is given by a particular reordering of integers: *1, 2, ...* N_p. It is generated by the following algorithm.

• $N_p = k_1 \cdot k_2$ where k_1 is a fixed parameter, and k_2 varies according to the interleaver length;

• next do the following operations for $s=1$ to $s= N_p$ (the current position before interleaving) to obtain the permutation numbers $\pi_p(s)$:

• $m = (s-1) \bmod 2$
• $i = [(s\text{-}1)/(2k_2)]$
• $j = [(s\text{-}1)/2] - i\cdot k_2$
• $t = (19\cdot i+1) \bmod (k_1/2)$
• $q = t \bmod 8+ 1$
• $c = (p_q\cdot j+ 21\cdot m) \bmod k_2$
• $\pi_p(s)=2\cdot (t + (N_p+ 1)/2+ 1)\cdot s - m$

where $[x]$= the largest integer less than or equal to x, "mod"= the operation modulo and $q=1\div 8$, $p_1=31$, $p_2=37$, $p_3=43$, $p_4=47$, $p_5=53$, $p_6=59$, $p_7=61$, $p_8=67$, [4].

E. The Takeshita-Costello interleaver (IXTC)

The Takeshita-Costello interleaver, noted *IXTC* (*X is* given by Table 1) is presented in [5], and it supposes the interleaver length, N_{TC}, to be equal with 2^j (j is a natural number). The construction of the interleaver function is as follows. The vector $c_{i=1\div N}$, is built with the relation:

$$c(i)=k\cdot i\cdot(i+1)/2. \quad (9)$$

where k is, usually, equal with one. The interleaver function is obtained using the relation:

$$\pi(c(i)) = c(i+1),\ \forall i\in I. \quad (10)$$

F. The block-random interleavers

This new interleaver type aims to continue the qualities of the block interleavers (high d_{min}) and of the random interleavers (a good spreading). This kind of interleaver represents an alternative to the S-type interleaver, which is difficult to be designed. Two variants of the block-random interleaver were studied, the Block Random in Line Interleaver (BRL), noted *IXBRLY* (*X* and *Y* are given by Table 1) and the Block with Random Lines Interleaver (BLR), noted *IXBLRY* (*X* and *Y* are given by Table 1). In the following, their design is presented. We will suppose that the length of those interleavers is given by the relation:

$$N_{br}=X\times Y.$$

i) The BRL interleaver (IXBRLY).

–we build the following matrix:

$$c(i,j)=1+i+j\cdot X,\ i\in I=\{0,1\ldots X\text{-}1\},\ j\in J=\{0,1\ldots Y\text{-}1\}. \quad (11)$$

–the lines of this matrix are permuted, $c(i,J)$, using the relation (5), obtaining the matrix b:

$$b(i,Y)= \pi_r(c(i,Y))\ \forall i\in I. \quad (12)$$

–the block permutation is finished, reading in order the columns of the matrix b, first the even and after that the odd ones:

$$\pi_{BRL}(j+k\cdot Y+1)=b(i,j),\ i=(2\cdot k)\in I,\ j\in J.$$
$$\pi_{BRL}(j+k\cdot Y+Y_2+1)=b(i,j),\ i=(2\cdot k-1)\in I,\ j\in J. \quad (13)$$

where $Y_2 = floor[(Y-1)/2]$.

ii) The BLR interleaver (IXBRLY).

The generating algorithm, similar with BRL has following modifications. At the second step, the permutations are realized between lines (not in each line):

$$d(i,J)= c(\pi_r(i), J))\ \forall i\in I=\{0,1\ldots X\text{-}1\}. \quad (14)$$

and at the third step the columns are read in order:

$$\pi_{BLR}(j+i\cdot Y+1)=d(i,j),\ i\in I,\ j\in J \tag{15}$$

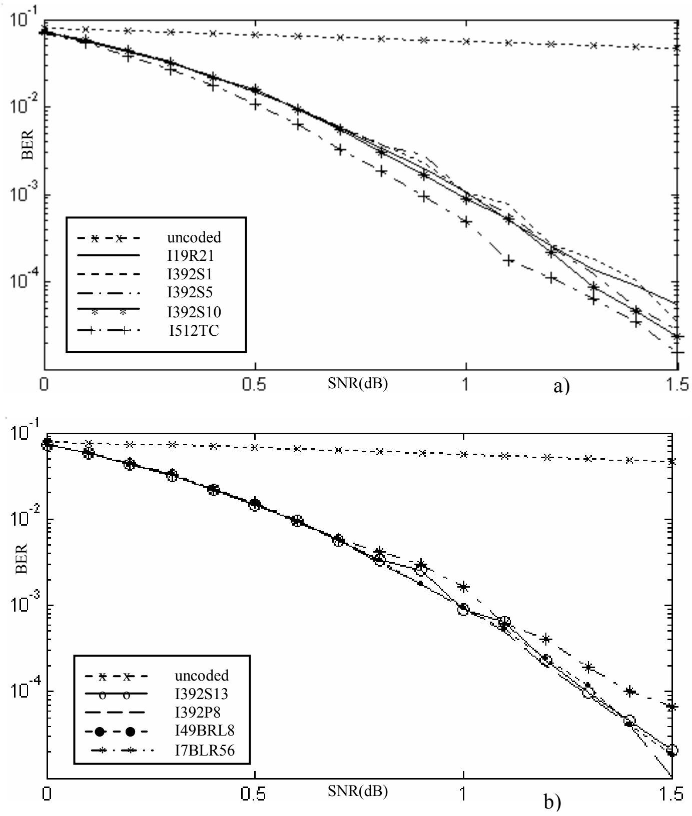
The minimum interleaver distance is $d_{min}\geq Y/2$ for the BRL interleaver, and $d_{min}\geq X$ for the BLR interleaver. For a good spreading, the dimension X must be chosen the greatest possible. For the BLR interleaver, the greatest possible dimension Y must be selected. A good compromise is obtained choosing $X\cong Y^2$ for the BRL interleaver and $Y\cong X^2$ for BLR.

III. The Interleavers BER and FER Peformance

On the diagrams from Fig.1, some BER performance curves obtained using different interleavers are presented: random interleaver, S-interleaver, block interleaver, pseudo-random interleaver, Takeshita-Costello interleaver, BLR and BRL interleavers.

These interleavers were introduced into an unpuncturated turbo code at the rate of 1/3, in which two identical recursive systematic convolutional (RSC) 15/13 codes, having the constraint length, K=4, are connected in parallel, [6].

For the interleaving length these values are around: 400, 900, 1800 and 3600. The exact values take into account the limitation of the design of each interleaver type. These interleavers are presented in Table I.



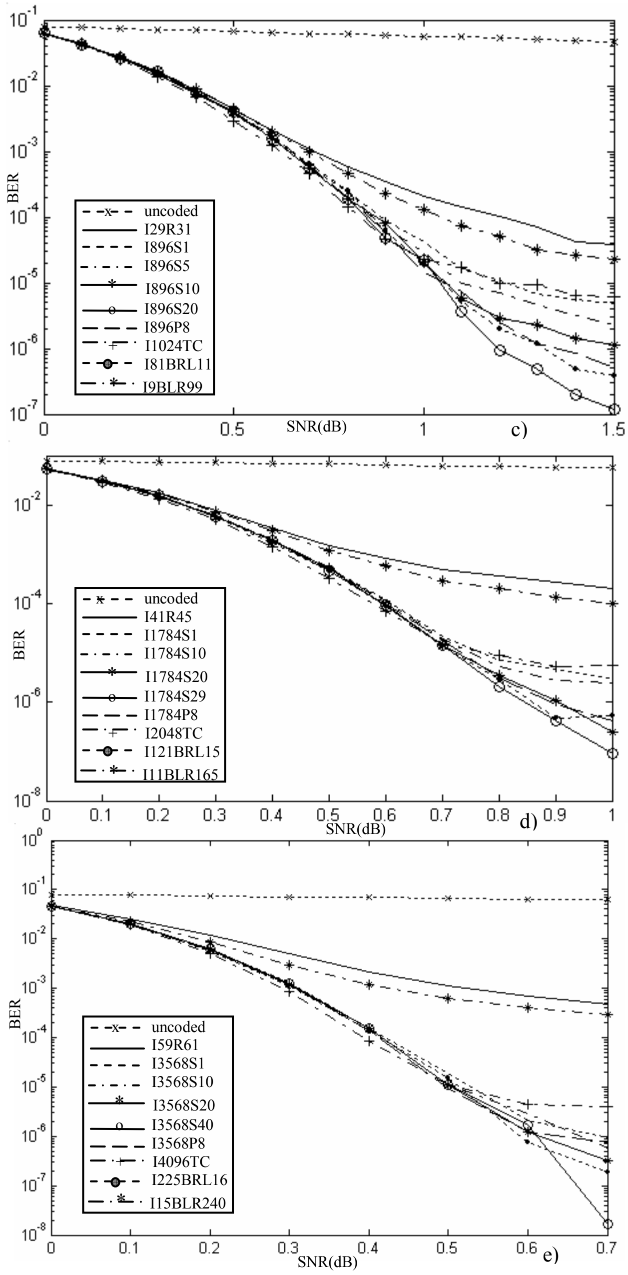


Figure 1. BER performence for the 8-state, rate 1/3, (1, 15/13), turbo code for interleavers with N ≅ 400 (a), b)) , N ≅ 900 (c)), N ≅ 1800 (d)) , N ≅ 3600 (e)).

Analyzing the Figure 1 it can be observed that the performances of the S-interleavers increase with the minimum interleaving distance, equal with *S*.

Hence, at a given spreading degree (because all are random interleavers they have the same spreading degree) the minimum interleaving distance represents the performances measure.

On the diagrams in Figure 2 the corresponding FER performance curves obtained using the same interleavers are presented: random interleaver (S=1), S-interleaver, block interleaver (R), pseudo-random interleaver (P), Takeshita-Costello interleaver (TC), BLR and BRL interleavers.

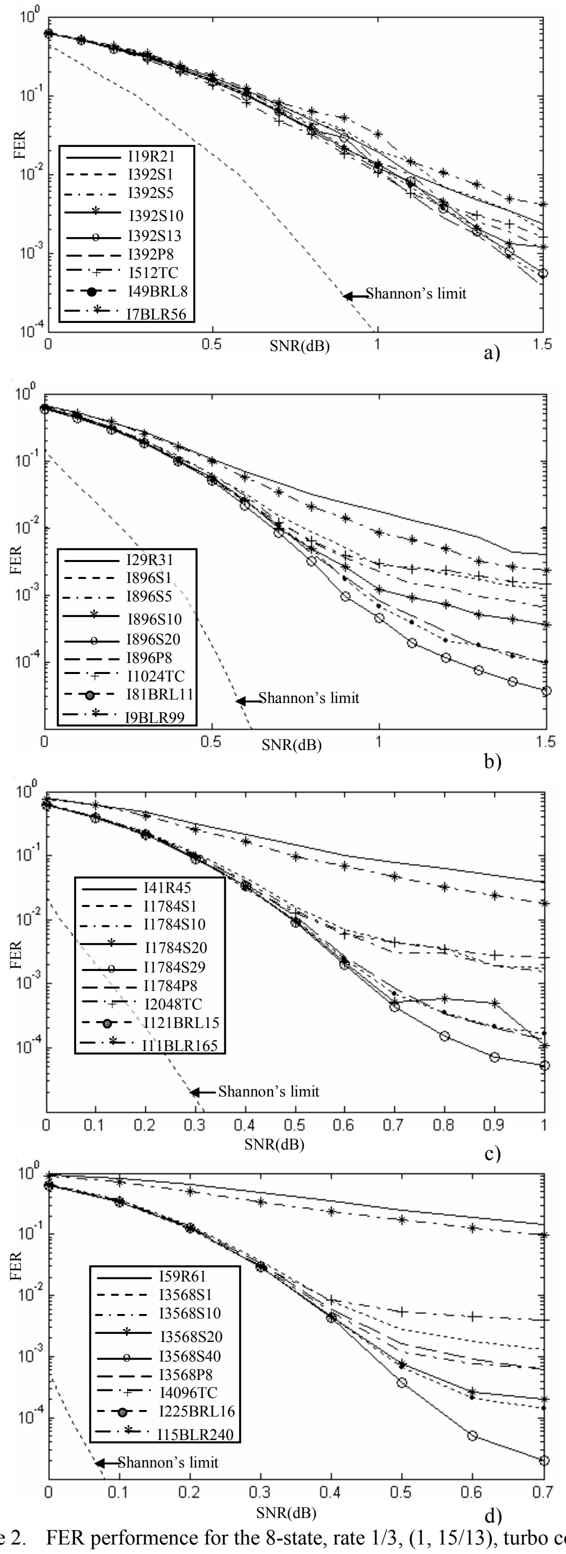


Figure 2. FER performence for the 8-state, rate 1/3, (1, 15/13), turbo code for interleavers with N ≅ 400 (a)) , N ≅ 900 (b)), N ≅ 1800 (c)) , N ≅ 3600 (d).

Analyzing the curves presented in figures 1 and 2, it can be observed that when the interleaving length is already increased, the spreading degree becomes the most important factor for the interleaver BER and FER performances. So, at small interleaving length, the interleavers have a „block character". In this case the interleavers R, TC, BLR have similar performances to the others'. The best interleaver is in this case the TC interleaver. At a length equal with 3600 these interleavers (R, TC, BLR) have the poorer performances

The P interleaver (recommended by CCSDS, [3]) has good performances at all the interleaving lengths. But with the increasing of the interleaving length, it also has a decreasing performance as compared to the one of the random interleavers.

Analyzing the performances of the proposed block-random interleavers we can make the following observations. The BLR interleaver has similar performances to block interleavers. The performances of the BRL interleaver are similar to the performances of the S-interleavers, having a comparable minimum distance. The similarities can be explained taking into account the design procedure. In the design of the BLR interleaver the length of the random permutations was equal to the half of the length of the random permutations realized in the design of the BRL interleaver. Moreover, in the case of the BLR interleaver, all the columns are permutated in the same manner. For the design of the BRL interleaver the permutations are realized independently from line to line.

Despite the fact that at a high interleaving length, *N>1000*, the s-interleaver with a high *S* is very good (in this paper only one code, 15/13 was used, but this conclusion is valid for every code, [7]) its design is difficult. This is the reason why the proposed BRL interleaver is attractive; it has similar performances and a simpler design.

IV. Conclusions

A new type of interleaver is proposed and their BER, FER are compared with others interleavers. The performances recommend the proposed interleaver as an alternative to the S-interleaver, which is the best at lengths superior to 1000 bits. The performances of the proposed BRL interleaver are very close to the performances of the S interleaver with maximum S, but the design of the new interleaver is simpler.

References

[1] L.Hanzo, T.H.Liew, B.L.Yeap, "Turbo Coding, Turbo Equalisation and Space-Time Coding for Transmission over Fading Channels", John Wiley & Sons Ltd, England, 2002.

[2] S. A. Bărbulescu, S. S. Pietrobon, "Turbo Codes: a tutorial on a new class of powerful error correcting coding schemes. Part I: Code Structures an Interleaver Design", 26 October, 1998.

[3] Consultative Committee for Space Data Systems "Telemetry Channel Coding", Blue Book, CCSDS 101.0-B-6, Oct 2002.

[4] M. Jézéquel, C. Berrou, C. Douillard, „Turbo codes (convolutifs)", seminar, Timisoara, Romania, 15-18 March, 2004, http://hermes.etc.utt.ro/cercetare/carti.html.

[5] O.Y. Takeshita, D.J. Costello - "New Classes of Algebraic Interleavers for Turbo-Codes", ISIT 1998, August 16-21, USA.

[6] C. Berrou, A. Glavieux, P. Thitimajshima - "Near Shannon limit error-correcting coding and decoding: Turbo-codes", Proc.ICC'93, Geneva, Switzerland, May 1993, pp. 1064 – 1070.

[7] H. Balta, M. Kovaci - "The BER Performances of Convolutional Codes used in Turbo Codes", Scientific Bulletin of the "Politehnica" University of Timişoara, România, Oct 2004, pp. 38 –43.