Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 53(67), Fascicola 1-2, 2008

A New Time-Frequency Adaptive Filter

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Abstract – This paper presents a generalization of the concept of adapted filter to a frequency modulated signal. This generalization is inspired by the time-frequency representations theory. A new time-frequency filter is proposed, the tracking filter. Its properties are shortly described.

Keywords: time-frequency, adaptive, filter, chirp

I. ADAPTED FILTERS

The aim of this paper is to respond affirmatively to the question " There exist ***adaptive*** *continuous-time filters ? ". To establish a possible definition of a continuous-time adaptive filter we present, for the beginning, some well-known results in the theory of adapted filters.*

Let $x(t)$ be the observed signal:

$$x(t) = s(t) + n(t)$$

where $s(t)$ is a finite energy signal and $n(t)$ is a stationary random signal with the power spectral density $\Phi_n(\omega)$, and $y(t)$ the response of a linear time-invariant system, with the frequency response $H(\omega)$ to the signal $x(t)$:

$$y(t) = u(t) + n_0(t)$$

The output SNR has the value:

$$SNR_0(t) = \frac{|u(t)|^2}{P_{n_0}}$$

where the power of the output noise can be computed using the relation:

$$P_{n_0} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(\omega)|^2 \Phi_n(\omega) d\omega$$

The expression of the signal $u(t)$ is:

$$u(t) = s(t) * h(t)$$

Its value at the moment T is:

$$u(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) S(\omega) \cdot e^{j\omega T} d\omega$$

and the value of the output SNR at the same moment is:

$$SNR_0(T) = \frac{\left(\frac{1}{2\pi}\right)^2 \left| \int_{-\infty}^{\infty} H(\omega) S(\omega) \cdot e^{j\omega T} d\omega \right|^2}{\left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} |H(\omega)|^2 \Phi_n(\omega) d\omega} \quad (1)$$

Using the Schwartz inequality we obtain:

$$SNR_0(T) \leq \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{|\Phi_n(\omega)|} d\omega$$

The sign equal, in the last relation is obtained for:

$$H(\omega) = \frac{K S^*(\omega) e^{-j\omega T}}{|\Phi_n(\omega)|} \quad (2)$$

where K is a constant.

This is the expression of the frequency response of the system that maximizes its output SNR at the moment T, when the signal $x(t)$ is processed. This system represents the adapted filter for the signal $x(t)$. If the input noise is white, with the power spectral density N_0, then the expression of the impulse response of the adapted filter becomes:

$$h(t) = \frac{K}{N_0} s(T - t) \quad (3)$$

The response $u(t)$ becomes:

$$u(t) = \frac{K}{N_0} R_s(T - t) \quad (4)$$

The output SNR takes the maximum value:

$$SNR_{0_{\max}}(T) = \frac{E}{N_0} \quad (5)$$

where E represents the energy of the signal $s(t)$.

When the input signal is a chirp:

$$s(t) = \begin{cases} \cos\left(\omega_{00} t + \frac{\Delta\omega}{2t_0} t^2\right), & |t| \leq \frac{t_0}{2} \\ 0, & |t| > \frac{t_0}{2} \end{cases} \quad (6)$$

with:

$$\alpha = \frac{\Delta\omega}{2\pi} t_0 > 25$$

then, [1]:

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$$|S(\omega)| \cong \begin{cases} \frac{t_0}{2\sqrt{\alpha}} \;, & \omega_{00} - \frac{\Delta\omega}{2} \le |\omega| \le \omega_{00} + \frac{\Delta\omega}{2} \\ 0 \;, & \text{in rest} \end{cases} \quad (7)$$

For:

$$K = \frac{2\sqrt{\alpha}}{t_0}$$

the magnitude of the frequency response of the adapted filter to a chirp signal becomes:

$$|H(\omega)| \cong \begin{cases} 1 \;, & \omega_{00} - \frac{\Delta\omega}{2} \le |\omega| \le \omega_{00} + \frac{\Delta\omega}{2} \\ 0 \;, & \text{in rest} \end{cases} \quad (8)$$

So, the adapted filter, to a chirp signal corrupted with white noise, is an ideal band-pass filter with the central frequency ω_{00} and the bandwidth $\Delta\omega$. **Unfortunately this system maximizes the output SNR only at the moment T.** In the following is presented a generalization of this concept.

We try to build a filter for the maximization of the SNR at its output, at every moment, when it has at its input a chirp corrupted by noise.

This time frequency filter is inspired by the response of a linear time-invariant system to a quasi-stationary signal.

II. A NEW TIME-FREQUENCY FILTER

Let $s(t)$ be a quasi-stationary signal:

$$s(t) = \cos(\omega_i(t)t) \quad (9)$$

The response of a linear time invariant band-pass filter, with the central frequency ω_0 and the frequency response $H(\omega)$ to this signal is:

$$u(t) = |H(\omega_i(t))| \cos(\omega_i(t)t + \arg\{H(\omega_i(t))\}) \quad (10)$$

Our new time frequency filter will be obtained making the central frequency of the band-pass filter already mentioned equal with the instantaneous frequency of the input signal $s(t)$. There are practical solutions to implement the condition already mentioned. For example, a phase look loop (PLL) system can be used to detect the instantaneous frequency of the signal $s(t)$. With the aid of a tuned band-pass filter (a band pass filter which central frequency can be set using a command voltage – for example a switched capacitor band-pass filter) the central frequency can be made equal with the instantaneous frequency of the input signal. To do this the tuned band-pass filter command signal must be the PLL system output signal. Such a filter, which central frequency tracks the instantaneous frequency of its input signal is called tracking filter.

The response of the new time-frequency filter, to the signal $s(t)$, is:

$$y(t) = |H(\omega,t)| \cos(\omega_i(t)t + \arg\{H(\omega,t)\}) \quad (11)$$

This time-frequency filter has a lot of useful properties. These properties depend on the expression of the frequency response $H(\omega)$ used.

III. A TRAKING FILTER

If the initial system is a second order band-pass filter, then the expression of the time-frequency representation of the impulse response of the new filter, introduced in this paper, becomes:

$$H(\omega,t) = \frac{2\xi A j \omega_i(t)\omega}{\omega_i^2(t) - \omega^2 + 2j\xi\omega\omega_i(t)} \quad (11)$$

This is not a linear or bilinear time-frequency representation. It is a time-frequency representation, of a new kind, first introduced in this paper.

This is the description of a frequency tracking filter. Indeed, at every moment, the central frequency of the band-pass filter is equal with the instantaneous frequency of the signal $s(t)$.

This system can be called continuous in time adaptive filter. It is a generalization of the concept of adapted filter to a chirp signal.

An example is presented in figure 1, for the case $\omega_i(t) = t \cdot \sigma(t)$.

The proposed time-frequency filter is concentrated around the instantaneous frequency of the signal *s*(*t*).

Indeed, the curve of the instantaneous frequency, in the time-frequency plane has the expression:

$$\omega = \omega_i(t) \quad (12)$$

If this expression is substituted in (11) it can be written:

$$|H(t,\omega)| = A \quad (13)$$

Hence, at every moment, the response of the tracking filter has the same value, A (where A represents the gain of the filter and has not any dimension). This is the maximum value of the time-frequency representation. So, the proposed time-frequency representation is concentrated around the instantaneous frequency of the signal $s(t)$, like the majority of time-frequency representations [2].

There are two important types of sections trough the surface of the proposed time-frequency representation:

- the intersection between the surface $|H(\omega,t)|$ and the plane $\{(\omega,t_p) \mid \omega \in R,\ p \in Z,\ p\text{-fixed}\}$ is called instantaneous characteristic;
- the intersection between the surface $|H(\omega,t)|$ and the vertical surface with the trace on the time-frequency plane:

  $$\omega = \omega_i(t)$$

  is called global characteristic.

The global and local characteristics corresponding to the example in figure 1 are presented in figures 2 and 3.

The tracking band of the band-pass tracking filter cannot be infinitely large due to technological reasons. So, it is rational to consider this band as an interval:

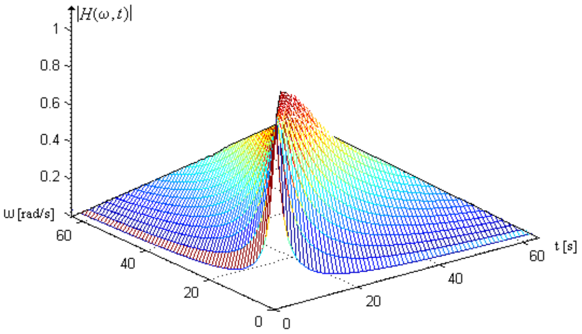

Figure 1. An example of magnitude of the time frequency response of a tracking filter.

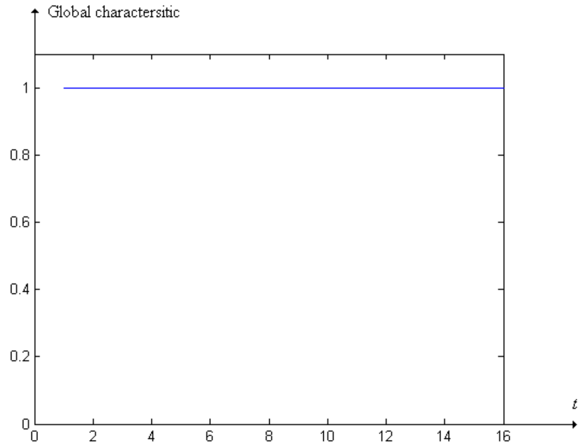

Figure 2. The global characteristic corresponding to the example in figure1.

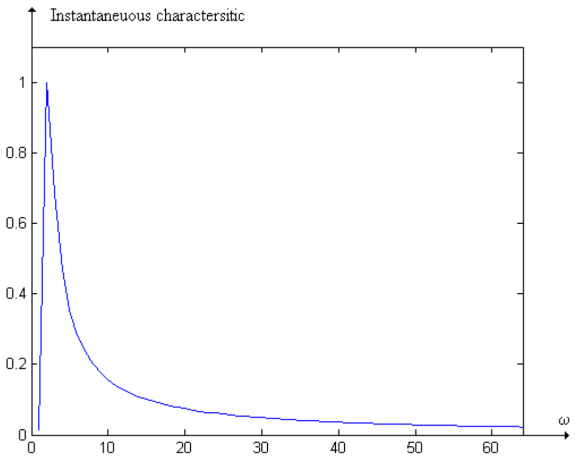

Figure 3. The instantaneous characteristic corresponding to the example in figure 1.

$$B = \left[\omega_{00} - \frac{\Delta\omega}{2},\ \omega_{00} + \frac{\Delta\omega}{2} \right]$$

This is why we can formulate the following property of the tracking filter:

P1. The magnitude of the global characteristic of the tracking filter is a good approximation of the magnitude of the frequency characteristic of an ideal band-pass filter in the bandwidth B.

The proof of this property is very simple and is based on the result in relation (11) (see also figure 2). So the tracking filter is an ideal band-pass filter. Hence this is an adapted filter for a chirp signal.

All the instantaneous characteristics of the tracking filter are of band-pass type. The filter can be designed such that every band-pass bandwidth of an instantaneous characteristic to be inferior to $\Delta\omega$, with a large quantity (see figure 2). All the instantaneous characteristics of the tracking filter have the same quality factor. This is the reason why, if the value of ξ in relation (11) is small enough, the tracking filter realizes, at every moment of time, an output SNR superior to the maximum output SNR of the adapted filter. So, we can assert that:

P2. The frequency tracking filter maximizes the *SNR* at its output at any moment.

Hence the frequency tracking filter represents a generalization of the adapted filter to a chirp signal.

Another aspect of this generalization is the fact that the signal $s(t)$ can be a chirp or any other frequency modulated signal.

Due to its time-frequency interpretation, the tracking filter can be called time-frequency filter.

From here becomes the title of this paper.

In [3] and [4] are presented some other properties of the instantaneous and global characteristics. We will present one of these properties here, without proof.

P3. With the aid of a band-pass tracking filter the ridge of a continuous wavelet transform of the input signal can be computed.

Analogous properties of the argument of the time-frequency representation, introduced in this paper can be also formulated.

IV. EXAMPLES

Two examples of tracking filters, operating at low frequencies, were already reported by the authors, one corresponding to a continuous-time system, [3], [4] and the other to a mixed system [5].

In the first example, presented in figure 4, a switched capacitor filter, and in the second example a digital "comb" filter were used. This second system is a digital band-pass filter. Its frequency response doesn't follow the relation (11), but the class of tracking filters is more general. In both examples a P.L.L, [6], was used to track the instantaneous frequency of the input signal and to generate the sampling frequency. Both systems have very good performances especially for the enhancement of the *SNR* of frequency modulated signals. So, using the continuous-time tracking filter, already mentioned, to process a sinusoidal signal perturbed with white noise, we have obtained an improvement of the signal to noise ratio of 35,2 dB.

Using the digital comb filter already mentioned, improvements of the *SNR* superior to 30 dB were also obtained.

The design of tracking filters, operating at low frequencies, (the design of the switched capacitors filter, or of the comb-filter and of the PLL) is very simple. The most difficult problem is the design of the

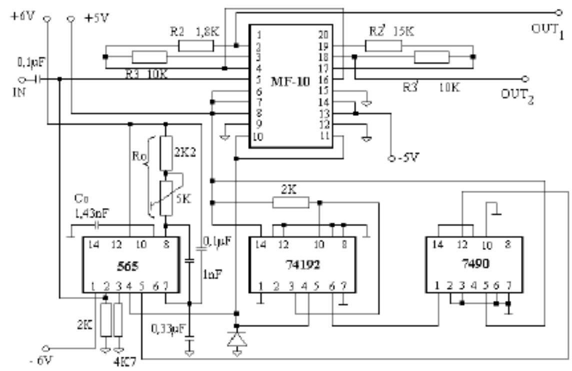

Figure 4. A tracking filter realized using the switching capacitors filter MF-10, the PLL 565 and a frequency multiplier realized with two counters. The instantaneous frequency of the input signal is detected with the PLL system. A multiple of this frequency is used to generate the clock for the switching capacitors filter.

tracking velocity. The value of this parameter can be selected by the design of the PLL system, [6].

V.CONCLUSION

We have presented a generalization of the concept of adapted filter to a frequency modulated signal), the tracking filter. Our approach is independent of the implementation of the electronic circuit. This is very important because a simple INTERNET research with the keyword tracking filter gives a lot of different results highlighting a large vaiety of implementations. We have introduced a new time-frequency representation. Its properties seem to be very interesting. A similitude between this time frequency representation and the continuous wavelet transform was reported in [4]. The study of the properties of this new time-frequency representation will represent the subject of a future paper.

The propositions presented in this paper were experimentally verified, by Matlab simulations (see figures 1-3) and by direct measurements, realized on the circuit presented in figure 4.

The tracking filter can be considered like an adaptive system because it maximizes the output *SNR* (hence it minimizes the mean approximation error of the signal $s(t)$ with the signal $u(t)$). So, continuous in time (and mixed) **adaptive systems** exist. This conclusion is in accord with the conclusion presented in [7].

There are a lot of interesting applications of the class of systems presented in this paper.

The tracking filters presented in this paper can be used with good results in industrial applications. These systems can be utilized for industrial measurements. For example the instantaneous pulsation of an electrical machine can be estimated using a tracking filter. So, all the parameters of an electrical machine that depend on this parameter can also be estimated.

Starting from the band-pass tracking filter already presented it is very easy to built a tracking notch filter. For this purpose is sufficient to substitute the prototype band-pass filter used in our derivation with a notch prototype. Such a system can be utilized for the rejection of industrial perturbations (for examples the perturbations due to the power supplies), or in spectral analysis [8].

These systems can be used for the estimation of some continuous wavelet transforms [9]. Finally, using a bank of such tracking filters the WiMAX signals can be analyzed, [10].

ACKNOWLEDGEMENT

This research was realised in the framework of the CNCSIS Grant number 351.

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