Kalman Noncoherent Detection of CPFSK Signals

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Abstract — **The paper addresses the problem of detecting Continuous Phase Frequency Shift Keying, (CPFSK) signals embedded in Gaussian noise. The paper introduces a description of CPFSK signals using a state-space model. Based on this model, a noncoherent detection method for CPFSK signals that uses an Extended Kalman Filter was developed. The performances of the detection method were tested on Minimum Shift Keying, (MSK) and Gaussian Minimum Shift Keying, (GMSK) modulated signals embedded in Gaussian noise. The results leaded to the conclusion that for moderately low SNR's, the Extended Kalman Filtering (EKF) algorithm provides satisfactory, even good results in noncoherent demodulation of CPFSK signals.**

Keywords - Kalman filtering, state-space representation, continuous phase modulation, noncoherent detection

I. INTRODUCTION

Linear modulation schemes without memory exhibit phase discontinuity in the modulated waveform [1]. These phase transitions cause problems for band limited and power-efficient transmission especially in an interference limited environment. The sharp phase changes in the modulated signal cause relatively prominent side-lobe levels of the signal spectrum compared to the main lobe. In a wireless communication system these side-lobe levels should be as small as possible. The abrupt phase transitions generate frequency components that have significant amplitudes. Thus the resultant power in the side-lobes causes co-channel and inter-channel interference. In a practical situation, it may be necessary to use either a linear power amplifier or a non-linear amplifier using extensive distortion compensation or selective pre-distortion to suppress out-of-band frequency radiation. Continuous phase modulation schemes are preferred to counter these problems.

Continuous Phase Frequency Shift Keying (CPFSK) is a non-linear modulation scheme where frequency shifts are used to transmit information. CPFSK refers to a family of continuous phase modulation schemes where in the phase is constrained to be continuous during a symbol transition. The constraint of continuous phase affects the signal by same important ways: 1.) transient effects are lessened at the symbol transitions, thereby offering spectral bandwidth advantages [2]; 2.) memory, imposed upon the waveform by continuous phase transitions, improves performance by providing for the use of several symbols to make a decision rather then more common approach of making independent symbol-by-symbol decisions; 3.) has a constant envelope [3]. Constant envelope schemes have the advantage of simplicity in the transmitter and in the front end stages of the receiver in that these stages don't have to be linear. The use of class C amplifiers would be a definite advantage for small scale implementations which have limitations in power consumption and power dissipation. Because of the above mentioned advantages, CPFSK schemes are preferred modulation schemes in most of the wireless communication applications.

Minimum Shift Keying (MSK) is a special case in the family of constant envelope continuous phase modulation (CPM) signals [4] in which the carrier phase is modulated over the full 360 degree range. MSK is equivalent to CPFSK with a modulation index of 0.5. Being a frequency modulated signal, the MSK modulated signal has a constant envelope. Thus, the modulated signal is fairly insensitive to transceiver nonlinearity which gives rise to modulated signal amplitude and phase distortion. Other desirable properties of MSK are its relatively compact spectral main lobe and the ability to detect the signal using coherent or non-coherent means. Gaussian Minimum Shift Keying (GMSK) can be viewed as a derivative of MSK. In GMSK, the binary data is passed through a Gaussian filter before being modulated. A Gaussian pre-modulation low-pass filter is generally used with MSK at base-band for smoothing the sharp phase transitions and limiting the out of band power. The Gaussian filter response is appropriate for this purpose as it maintains the constant envelope property and preserves the pattern-averaged phase-transition trajectory [5]. GMSK modulation is used in many important wireless communications standards.

CPFSK signals can be detected by a variety of coherent and non-coherent detection techniques [6]. Coherent detection techniques are those requiring phase information of the carrier. These detection schemes require the local oscillator at the receiver to extract the phase information of the carrier signals used at the transmitter side, from the received signal, in order to generate coherent reference carrier signals. Non-coherent CPFSK detection differs from coherent detection it that it does not require the phase information of the carrier signals used on the transmitter side to generate the reference carriers at the local oscillator on the receiver side. Thus, in a non-coherent receiver, the reference carrier might have a different phase than those at the transmitter and it may still be possible to secure the correct detection. In this paper we propose a noncoherent procedure that uses Extended Kalman Filtering (EKF) to demodulate the MSK and GMSK signals.

This paper is structured as follows. Section II introduces the state-space model of CPFSK signal affected by additive Gaussian noise. In section III we describe the EKF algorithm used in non-coherent detection of CPFSK signals. Section IV provides simulation results. Finally, Section V gives the concluding remarks and sketches the prospective work to be done.

II. THE STATE-SPACE REPRESENTATION OF CONTINUOUS PHASE FREQUENCY SHIFT KEYING MODULATED SIGNAL MODEL

A continuous phase frequency shift keying modulated signal with constant amplitude embedded in additive noise $w[n]$ is expressed as:

$$y[n] = A\cos\varphi[n] + w[n] \tag{1}$$

where the positive real-valued A is the constant amplitude of the signal and $\varphi[n]$ is the signal's phase. The additive noise is white and Gaussian, having zero-mean and variance σ_w^2.

A. *The state space model and the transition equation of CPFSK modulated received signal*

We define the following 2×1 state vector $\mathbf{x}[n]$:

$$\mathbf{x}[n] = \begin{bmatrix}\varphi[n] & \Omega[n]\end{bmatrix}^T \tag{2}$$

with

$$\Omega[n] = \varphi[n] - \varphi[n-1] \tag{3}$$

where $\Omega[n]$ stands for the first order difference of the phase function $\varphi[n]$.

From (2) the signal phase can be written as:

$$\varphi[n] = \varphi[n-1] + \Omega[n-1] \tag{4}$$

Considering only the phase variations of a continuous phase frequency shift keying modulated signal, the state transition equation is written as:

$$\mathbf{x}[n] = \mathbf{F}\mathbf{x}[n-1] \tag{5}$$

where the 2×2 transition matrix $\mathbf{F}$ has the form:

$$\mathbf{F} = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \tag{6}$$

We will assume that the frequency of the signal follows a random-walk model:

$$\Omega[n+1] = \Omega[n] + v[n] \tag{7}$$

where $v[n]$ is a sequence of i.i.d. random scalars with the distribution $N\left(0,\sigma_v^2\right)$.

The last equation must be added to (5) in order to obtain the complete description of the state evolution for the CPFSK modulation received signal:

$$\mathbf{x}[n] = \mathbf{F}\mathbf{x}[n-1] + \mathbf{G}v[n] \tag{8}$$

where $\mathbf{G}$ is a 2×1 vector:

$$\mathbf{G} = \begin{bmatrix}0 & 1\end{bmatrix}^T \tag{9}$$

As reveals (8) the state transition equation of continuous phase frequency shift keying modulation received signal model is linear.

B. *The observation equation*

In order to estimate the parameters of continuous phase frequency shift keying modulated signal corrupted by noise, a nonlinear observation equation is used. In this sense, the measured signal $y[n]$ is expressed as:

$$y[n] = A\cos\left(\mathbf{l}^T\mathbf{x}[n]\right) + w[n] \tag{10}$$

where

$$\mathbf{l}^T = \begin{bmatrix}1 & 0\end{bmatrix} \tag{11}$$

From (10), we can conclude that the observation equation is nonlinear, which means that, in order to use EKF, we apply the first order linearization procedure to

$$\mathbf{h}\left(\mathbf{x}[n]\right) = \cos\left(\mathbf{l}^T\mathbf{x}[n]\right) \tag{12}$$

in (10) around the estimation of the state vector $\hat{\mathbf{x}}[n|n-1]$:

$$\mathbf{h}\left(\mathbf{x}[n]\right) = \mathbf{h}\left(\hat{\mathbf{x}}[n|n-1]\right) + \left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}[n|n-1]}\left(\mathbf{x}[n] - \hat{\mathbf{x}}[n|n-1]\right) \tag{13}$$

with:

$$\mathbf{H}[n] = \left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}[n|n-1]} = -\mathbf{l}\sin\left(\mathbf{l}^T\hat{\mathbf{x}}[n|n-1]\right) \tag{14}$$

As is obvious, the replacement of $\mathbf{h}\left(\mathbf{x}[n]\right)$ by its first order approximation has dramatic effects on the stability and the convergence of the EKF algorithm, which implies the appearance, especially at very low S/N ratios, of "lack of convergence" cases [7].

III. THE EKF ALGORITHM

The extended Kalman filter is an approximate solution that allows us to extend the Kalman filter idea to non-linear state-space models [8]. As far as the observation model is nonlinear, in order to apply the Kalman filtering procedure as it was shown, a first order linearization around estimated $\hat{\mathbf{x}}[n|n-1]$ is needed at each step of the standard Kalman algorithm. The procedure is well known as Extended Kalman Filter algorithm and it uses state-space equations (8) and (1) as well as the linearization of the observation function around the current vector estimate (10). The EKF generally has better robustness because it uses linear approximation over smaller ranges of state space.

Assume that the initial state $\mathbf{x}[1]$, the observation noise $\mathbf{w}[n]$ and the state noise $v[n]$ are jointly Gaussian and mutually independent. Let $\hat{\mathbf{x}}[n|n-1]$ and $\mathbf{R}[n|n-1]$ be the conditional mean and the conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{y}[1],\ldots,\mathbf{y}[n-1]$ and let $\hat{\mathbf{x}}[n|n]$ and $\mathbf{R}[n|n]$ be the conditional mean and conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{y}[1],\ldots,\mathbf{y}[n]$. We have:

Measurement update equations

$$\mathbf{H}[n] = -\mathbf{l}\sin\left(\mathbf{l}^T\hat{\mathbf{x}}[n|n-1]\right) \quad (15)$$

$$\mathbf{K}[n] = \mathbf{R}[n|n-1]\mathbf{H}^T[n]\left(\mathbf{h}[n]\mathbf{R}[n|n-1]\mathbf{H}^T[n]+\sigma_w^2\right)^{-1} \quad (16)$$

$$\hat{\mathbf{x}}[n|n] = \hat{\mathbf{x}}[n|n-1]+\mathbf{K}[n]\left(y[n]-\cos\left(\mathbf{l}^T\hat{\mathbf{x}}[n|n-1]\right)\right) \quad (17)$$

$$\mathbf{R}[n|n] = \mathbf{R}[n|n-1]-\mathbf{K}[n]\mathbf{H}[n]\mathbf{R}[n|n-1] \quad (18)$$

Time update equations

$$\hat{\mathbf{x}}[n+1|n] = \mathbf{F}\hat{\mathbf{x}}[n|n] \quad (19)$$

$$\mathbf{R}[n+1|n] = \mathbf{F}\mathbf{R}[n|n]\mathbf{F}^T+\mathbf{G}\mathbf{G}^T\sigma_v^2 \quad (20)$$

where $\mathbf{K}[n]$ is the Kalman gain matrix at moment n.

As initial conditions the following equations are used:

$$\hat{\mathbf{x}}[1|0] = [\Omega_0 \quad 0]^T \quad (21)$$

$$\mathbf{R}[1|0] = \frac{\pi^2}{9}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (22)$$

IV. SIMULATION RESULTS

This section gives some preliminary results of the simulations obtained for the noncoherent detection by the Extended Kalman Filter algorithm of continuous phase frequency shift keying modulated signal in additive Gaussian noise.

In order to emulate the proposed detection method, a transmission system that uses MSK or GMSK modulation was simulated using the program MATLAB. The message consists of a unipolar random sequence generated by MATLAB function *randsrc*. The CPFSK modulated signal propagates through a communication channel AWGN affected by additive, white and Gaussian noise, having zero-mean and variance σ_w^2. The extended Kalman filter algorithm presented in the previous Section implements a noncoherent detection of the received signal. Simulations were performed for different values of the sampling rate (nSamp) and of the signal to noise ratio (SNR). The sampling rate indicates the number of samples by message bit that EKF uses for detection. It was found that the EKF algorithm performs satisfactorily even at low levels of SNR.

Figure 1 presents a part of a binary message sequence sent by MSK modulation together with the detected signal given by the EKF filter output, the instantaneous frequency $\Omega[n]$. The sampling rate chosen for this example was $\text{nsamp} = 16 \text{ samples/bit}$ and the signal to noise ratio $\text{SNR} = 8\,\text{dB}$. In Fig. 1 can be observed that for the conditions mentioned above, the estimated signal concords to the message sent even for fast bit changes in the sent message. Performance evaluation of a data transmission system has among the most important parameters the bit error rate (BER). In order to test the capabilities of the noncoherent detection of CPFSK signals by EKF detection, an algorithm was developed that search the optimal value of the decision threshold in order to restore the bits received.

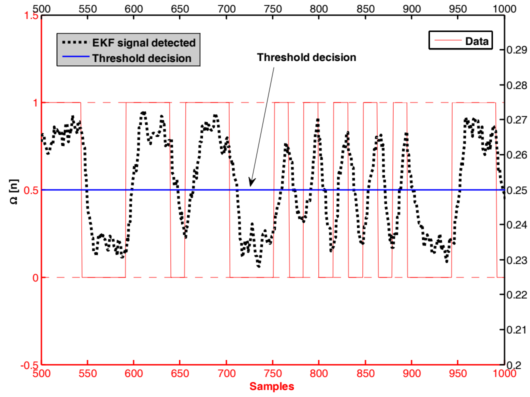

Figure 1. MSK with SNR=8dB signal detected by EKF.

The same procedure is used both for MSK and GMSK demodulation. To make a comparative analysis of MSK and GMSK modulations, similar sets of measurements was taken in consideration. The performances of the extended Kalman filter for the two types of modulation will be described below.

Figure 2 illustrates the dependence of the MSK system's BER performance with the sampling rate. Simulations were performed for three different values of the SNR (SNR=4, 6 and 8) and for three different sampling rates (nSamp=4, 8 and 16). It can be observed from the figure that performance increases with the sampling rate.

Figure 3 presents the dependence of performance of GMSK detection by extended Kalman filtering with the sampling rate of the algorithm. Simulations were carried out identical conditions to those used in the case of MSK detection.

In this case simulations were performed for three different values of the sampling rate (nSamp = 4, 8, and 16) and for two different values of the bandwidth-time product (BT), (BT=0.3 and BT=0.5). It can be observed that for BT=0.5 the system's performance is better than for BT=0.3.

Figure 4 shows in the case of MSK signal detection by EKF

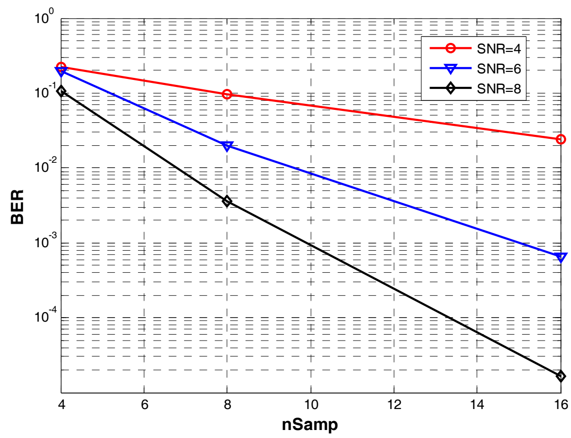

Figure 2. The dependence of MSK demodulation performance to sampling rate of EKF algorithm for three different values of SNR.

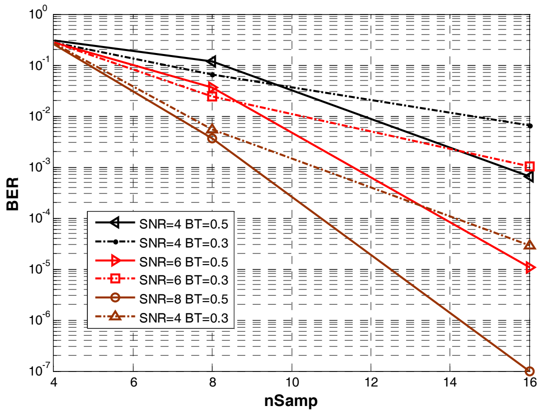

Figure 3. The dependence of GMSK demodulation performance of the sampling rate of the EKF algorithm for three different values of SNR and for two different BT values.

the dependence of BER performances on the signal to noise ratio (E_b/N_o) for three different values of the sampling rate, namely: nSamp = 4, 8 and 16. The results were compared with the theoretical ideal curve. As expected, with the increase of the sampling rate the EKF demodulator performances improve. It was found that for nSamp = 4, if $E_b/N_o < 4\text{dB}$, the extended Kalman filter's stability is no longer assured and therefore the algorithm becomes divergent. Consequently, the noncoherent demodulator by EKF algorithm can not be used in such conditions.

Figure 5 compares the BER performances of EKF algorithm for the two types of CPM modulation (MSK and GMSK). We used the same sampling rate, nSamp=8, in both cases. As seen from the figure, the MSK signals demodulation gives better results than the GMSK signal demodulation. It is noted that the curves obtained by EKF for MSK modulation is

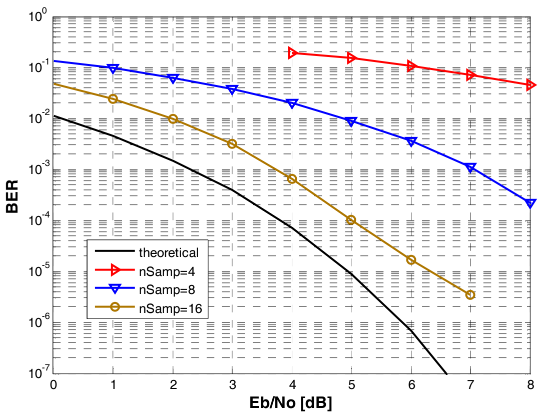

Figure 4. The dependence of MSK demodulation performance to SNR for three different values of sampling rate of EKF algorithm.

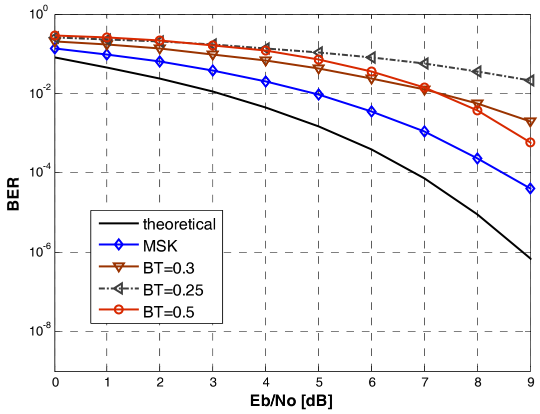

Figure 5. The comparison of performances of MSK and GMSK signal detection by EKF for the sampling rate nsamp =8.

closest to the ideal curve.

V. Conclusions

In this paper we introduce a description of CPFSK signals by a state-space model that permits the use of extended Kalman filter in order to demodulate this kind of signals. We have studied the performances of the method for two types of signals from this class: MSK and GMSK modulations. We have compared their BER performances with the theoretical curve. The results obtained from the simulations, leaded to the fact that for moderately low SNR's, the EKF algorithm provides satisfactory even good results in noncoherent demodulation.

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