SOME PRACTICAL CONSIDERATIONS REGARDING THE IDENTIFICATION OF A NONLINEAR SYSTEM

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Key words: Identification, nonlinear system, Wiener kernels estimation, signal reconstruction.

The nonlinear system identification based on the Volterra model is applicable only for systems whose order is known and finite. A lot of practical applications need the identification of unwanted nonlinearities, of unknown order, for compensating them. The solution is an accurate nonlinear model. We propose in our article a method of constructing a Wiener model. Two applications are considered and the Wiener kernel technique is applied in both cases. Finally original contributions have been made regarding signal reconstruction using the Wiener model.

1. **INTRODUCTION**

In our previous work [1] we have studied two types of nonlinear filters the Volterra filter and the Wiener filter. An important part of modeling with Volterra and Wiener series is the accurate measurement of the coefficients model or kernels.

The Volterra filter is based on the Volterra series modeling technique and has been widely applied with considerable success. However, at present, no general methods exist to calculate the Volterra kernels for nonlinear systems, although they can be calculated for systems whose order is known and finite. This method is not generally applicable since the order of a system is not typically known apriori. In our article [1], we have mentioned practical methods for measuring the Volterra kernels. The method proposed by Lee and Schetzen based on the intercorelation function between the input and the output signal of the Volterra filter is discussed in detail. We have introduced a method for measuring the kernels of a second order filter based on its orthogonally property in the presence of a white Gaussian input signal.

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Rev. Roum. Sci. Techn.– Électrotechn. et Énerg., **49**, *4,* p. , Bucarest, 2004

However, a lot of practical applications need the identification of so call unwanted nonlinearities, of unknown order. The purpose is to compensate them.

The Wiener filter overcomes all these problems by forming an orthogonal set of functionals from the Volterra functionals [2]. The key of this approach is in viewing the scalar input signal as a vector process via time-delay embedding. The Wiener theory of nonlinear system identification uses a time-delay of the embedded input signal to estimate the system behavior [2].

In this article we apply the Wiener model for identifying a nonlinear system of unknown order. We have considered two applications: the first one is represented by a nonlinear system with memory. The system consists of a linear filter followed by a nonlinearity without memory. The second one is represented by a JFET amplifier working in the saturation domain. In both cases the accuracy of the measured kernels is demonstrated by the comparison made between the output signals, reconstructed according to the Wiener model and the actual output response of filters.

2. A BRIEF SUMMARY ON THE NONLINEAR WIENER MODEL

According to the Wiener model the input-output relation of the nonlinear filter is given by [3], [4]:

$$y[n] = \sum_{i=0}^{p} G_i\left[k_i ; x[n]\right] \tag{1}$$

where $G_i[\cdot]$ are orthogonal. That is, if the input, x, is the Gaussian white noise with a mean zero and a given power spectral density, then:

$$\overline{G_i[k_i ; x] G_j[k_j ; x]} = 0 \quad if \quad i \neq j \tag{2}$$

$G_i[\cdot]$ is the *i*-th order Wiener *G*-functional, where *G* denotes that the functionals have been orthogonalized with respect to a particular stationary, Gaussian, white input process.

Suppose that the power spectral density of the particular Gaussian white input is σ^2. Then the first Wiener functionals are given via Gram-Schmidt orthogonalization as [5]:

$$G_0\left[k_0 ; x(t)\right] = k_0 \tag{3}$$

$$G_1\left[k_1;x(t)\right]=\int_0^{\infty}k_1(\tau_1)x(t-\tau_1)d\tau_1 \tag{4}$$

$$G_2\left[k_2;x(t)\right]=\int\int_0^{\infty}k_2(\tau_1,\tau_2)d\tau_1 d\tau_2-\sigma^2\int_0^{\infty}k_2(\tau,\tau) \tag{5}$$

$$G_3\left[k_3;x(t)\right]=\int\int\int_0^{\infty}k_3(\tau_1,\tau_2\tau_3)d\tau_1 d\tau_2 d\tau_3-3\sigma^2\int\int_0^{\infty}k_3(\tau_1,\tau_2,\tau_2)d\tau_1 d\tau_2 \tag{6}$$

Where the k_i is named the *i*-th order Wiener kernel. The seconds terms in relations (5) and (6) are called the zero order derivative kernel, $k_{0(2)}$, of the second kernel respectively the first order derivative kernel, $k_{1(3)}$, of the third kernel.

The Wiener kernels are not in general the same as the Volterra kernels [6] of corresponding order. For example the zero-order Volterra kernel is simply the system output when the system input is zero, whereas the zero-order Wiener kernel is the mean output for the particular white Gaussian input used. However the corresponding first and second-order Volterra and Wiener kernels are identical when the system has no higher-order kernels because of the orthogonality of the second order Volterra model.

As with the Volterra model [7], discrete time versions of the relations above can be easily written by following the method used for the Volterra representation. A Wiener model that uses symmetric Wiener kernels can also be derived. For example, a third order discrete time Wiener model with symmetric kernels can be written as [1]:

$$y[n]=G_0\left[k_0;x[n]\right]+G_1\left[k_1;x[n]\right]+G_2\left[k_2;x[n]\right]+G_3\left[k_3;x[n]\right] \tag{7}$$

$$G_0\left[k_0;x[n]\right]=k_0 \tag{8}$$

$$G_1\left[k_1;x[n]\right]=\sum_{m_1=0}^{N-1}k_1[m_1]x[n-m_1] \tag{9}$$

$$G_2\left[k_2;x[n]\right]=\sum_{m_1,m_2=0}^{N-1}k_2[m_1,m_2]x[n-m_1]x[n-m_2]-\sigma^2\sum_{m_1=0}^{N-1}k_2[m_1,m_1] \tag{10}$$

$$G_3[k_3;x[n]]=\sum_{m_1=0}^{N-1}\sum_{m_2=0}^{N-1}\sum_{m_3=0}^{N-1}k_3[m_1,m_2,m_3]x[n-m_1]x[n-m_2]x[n-m_3]-$$
$$3\sigma^2\sum_{m_1=0}^{N-1}k_3[m_1,m_1,m_1]x[n-m_1]-$$
$$\sigma^2\sum_{m_1=0}^{N-1}\sum_{m_2=0}^{N-1}k_3[m_1,m_2,m_2]x[n-m_1]x[n-m_2] \quad (11)$$

The main advantage of the orthogonal property is that it allows simple Wiener kernel measurement by cross-correlation approach.

3. **EXPERIMENTS AND RESULTS**

We have considered two practical applications of the Wiener model. Both use the kernel measurement approach based on the intercorrelation function between the input and the output signal, when the input signal is a white Gaussian sequence.

The first one is the identification of a nonlinear system of unknown order. The system consists of a linear filter followed by a nonlinearity without memory as indicated in figure 1.

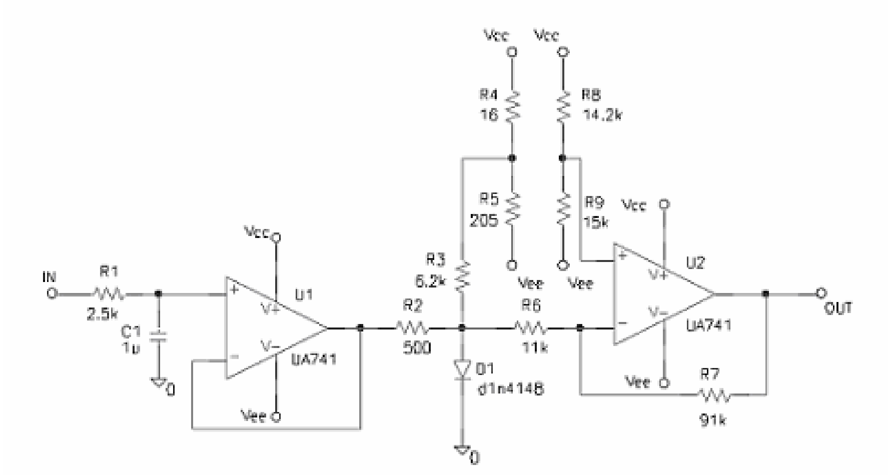

Fig.1 – Nonlinear system with memory

Simulations have been made using the PSPICE program. The kernels estimations have been made using MATLAB program. To measure the Wiener kernels $\overline{k_0, k_3}$ we have used the relations (15-22) proposed in our article[1]. The most significants results are presented in the figures below.

The first and the second order kernels are represented in figure 2 respectively in figure 3. They are the results of averaging 30 individual experiments.

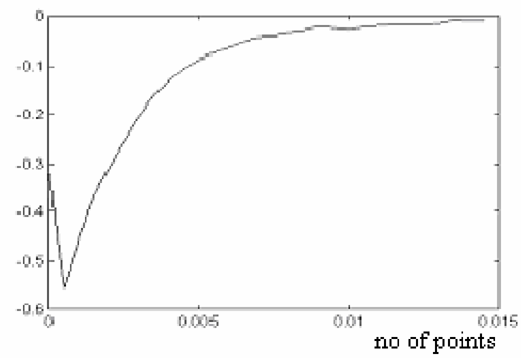

Fig.2 – The first order Wiener kernel

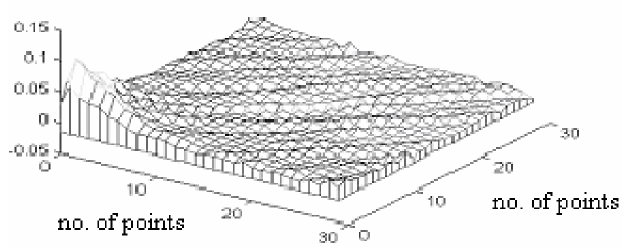

Fig.3 – The second order Wiener kernel

To prove the accuracy of our method we have calculated and compared the model response with the nonlinear system response. The input signal applied was a sine wave having the frequency equal to $100 Hz$.

The model response was calculated according to equation (7). The results are shown in figure 4.

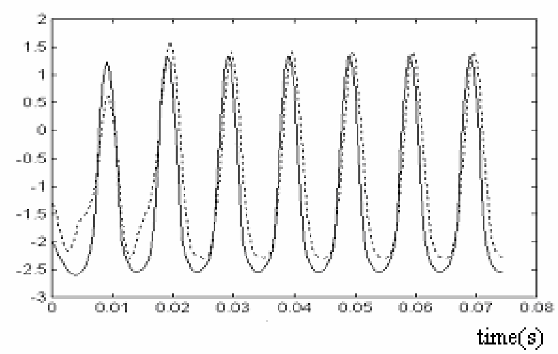

Fig.4 – The model response and the nonlinear system response to a sine wave

Figure 5 represents the Fourier Transform of the nonlinear system response, indicating the presence of components of different frequencies: $100Hz$, $200Hz$ and $300Hz$. A continues component is also present.

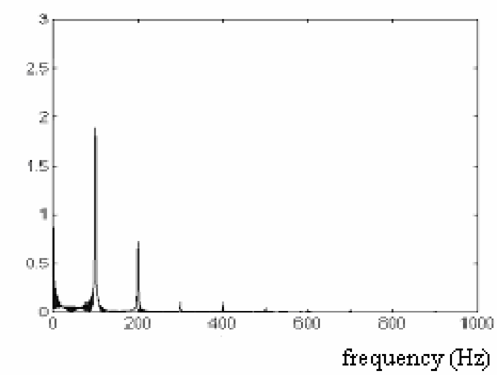

Fig. 5 – The Fourier Transform of the nonlinear system response

In figure 6 we have represented the Fourier Transform of the component $G_2[k_2;x]$ of the model response. It can be seen the presence of the continues term due to the zero order derivative kernel, $k_{0(2)}$, and the second order component having the frequency equal to $200Hz$.

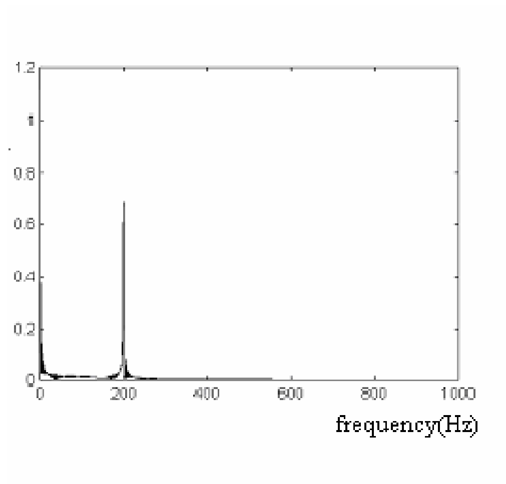

Fig.6 – The Fourier Transform of the model response due to functional G_2

Figure 7 represents the Fourier Transform of the component $G_3[k_3;x]$ of the model response and provide the presence of a first order component due to $k_{1(3)}$ ($100Hz$) and the presence of a third order component ($300Hz$).

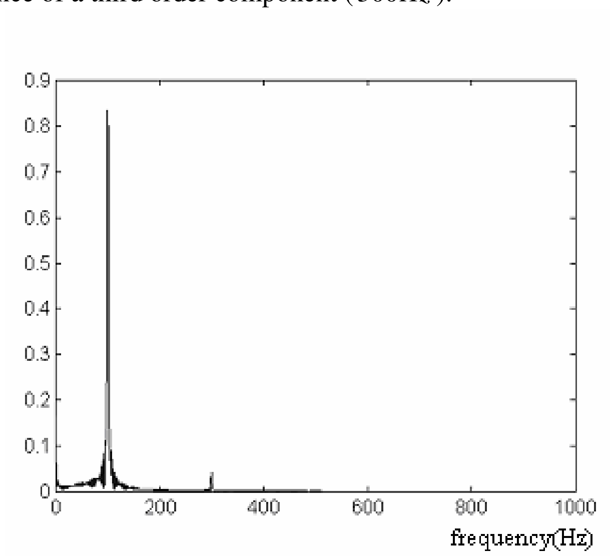

Fig.7 – The Fourier Transform of the model response due to functional G_3

Finally we have represented the Fourier Transform of the model response according to the proposed formula (8)-(11). It may be compared with the FT of the nonlinear system represented in figure 5 to confirm the accuracy of the model.

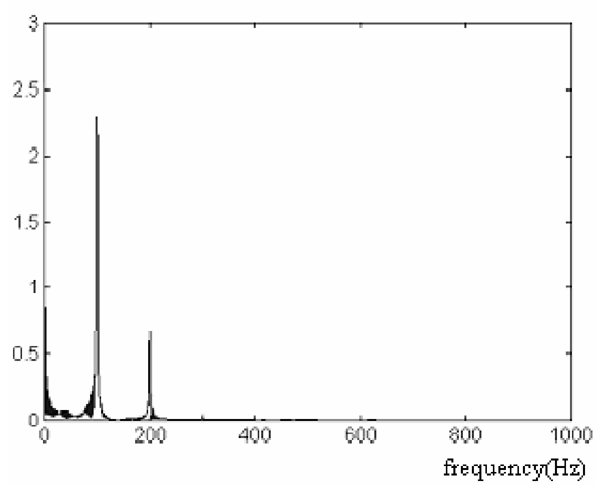

Fig. 8 – The Fourier Transform of the model response

In the second application we have modeled a JFET amplifier working in the saturation region.

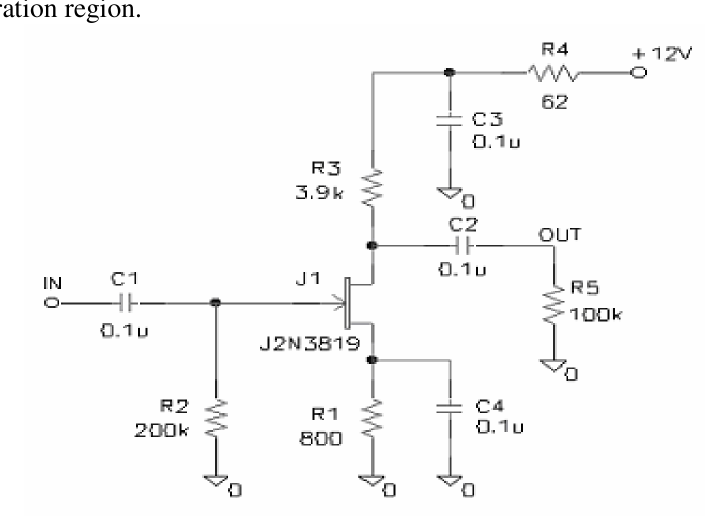

Fig. 9 – Nonlinear system working in the saturation region

To prove the accuracy of the method and the correctness of the model response we have compared the nonlinear response and the model response to a sine wave. The result is indicated in figure 10.

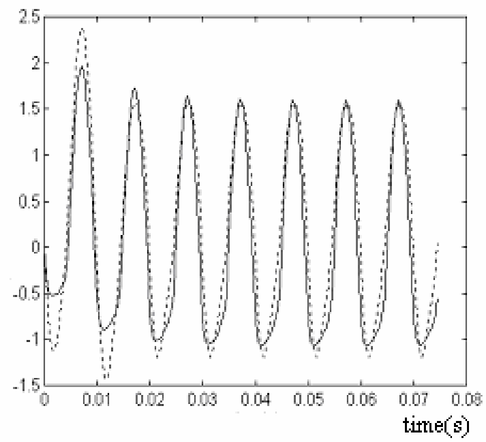

Fig. 10 – The model response and the nonlinear system response

Comparison was made also in the frequency domain and the results are indicated in figures 11 and 12.

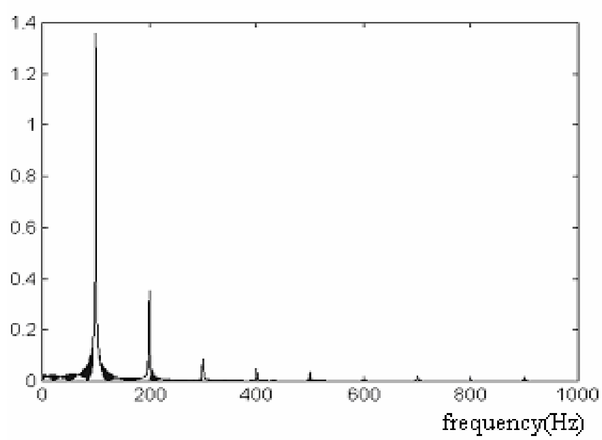

Fig. 11 – The Fourier Transform of the nonlinear system response

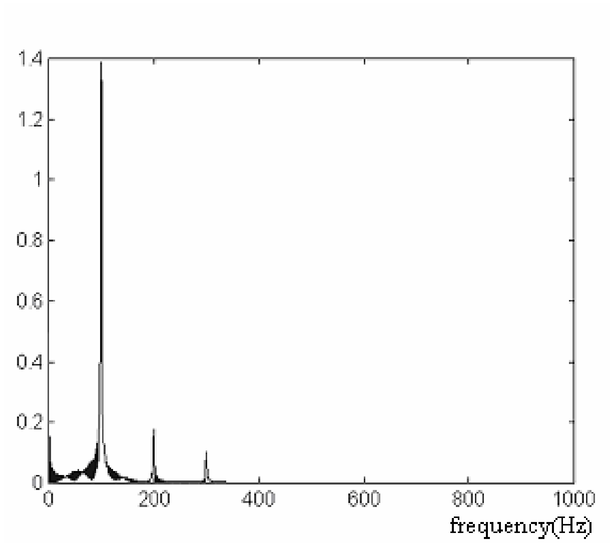

Fig. 12 – The Fourier Transform of the model response

Both applications considered have proved that the technique developed in this paper for Wiener kernels estimation lead us to accurate solutions. The model performances were appreciated by calculating the response to a sine input signal. Finally the model response was compared with the nonlinear system response. Comparisons were made in the time domain and in the frequency domain.

An advantage of the identification procedure is the absence of the convergence problems that may be encountered with adaptive methods. The major disadvantage of the considered method is that it requires a high quality Gaussian generator. In order to ensure the orthogonality of the higher order Wiener functionals and the accuracy of the cross-correlation method, the Gaussian generator must have higher order properties.

In some situations, when the compensation of unwanted nonlinearities is required, one may need to derive the Volterra kernels from a set of Wiener kernels. A set of Volterra model kernels up to *N*-th order may be obtained from a set of Wiener kernels up to *N*- th order. For the considered two applications the relations are:

$$h_0 = k_0 + k_{0(2)} \tag{12}$$

$$h_1[n] = k_1[n] + k_{1(3)}[n] \tag{13}$$

$$h_2[n_1, n_2] = k_2[n_1, n_2] \tag{14}$$

$$h_3[n_1, n_2, n_3] = k_3[n_1, n_2, n_3]] \tag{15}$$

The relationship between the Volterra and Wiener models clearly illustrates that the Voltera kernels would generally change with a change in the model order. For example, as the model order is increased, the Volterra kernels will change because higher order Wiener kernels would be used to calculate them.

4. **CONCLUSIONS**

Based on the contributions we have made to knowledge of modeling and identification techniques for nonlinear systems using Volterra and Wiener series [1], the aim of our article was to show how to implement some of these techniques in practical applications. The previous work in this area especially considered the identification of nonlinear system of known and finite order. Both applications we have presented in our article refer practical situations: the identification of a nonlinear system with memory of unknown order and the identification of a nonlinear system, which operates in the saturation region. In both cases we have constructed a Wiener model. To evaluate the technique accuracy, the model response was calculated and compared with the nonlinear system response. Comparisons were made in the time domain and in the frequency domain.

The main advantage of the Wiener model is based on the orthogonallity property which allows simple Wiener kernel measurement by cross- correlation approach. Also, there are other benefits related to the orthogonality of the model. For example, the orthogonality property leads to improved convergence compared to the Volterra model. Another advantage relates to the completeness of Wiener kernels.

Higher order models entail a high computational burden. So, often we are forced to use a Volterra or Wiener representation of lower nonlinearity order than the system to be modeled. Under the appropriate continuity conditions, either a finite order Volterra or a Wiener representation will give an arbitrary degree of accuracy. However, there is a significant difference between using the Volterra and Wiener models, that we want to emphasize.

The value of the Volterra kernels will depend on the order of the Volterra representation being used. If the order of the Volterra model is changed, the Volterra kernels will change and they must be recalculated. If the order of the Volterra representation reaches the actual system order, any further increase in the Volterra representation order would not cause changes in Volterra kernels.

The completeness property of the orthogonal Wiener model gives the following advantage. For a Wiener model of order N , the Wiener kernels are always optimum in least mean squared error sense. If the Wiener model order is increased, the lower order Wiener kernels remain the same and do not have to be re-estimated. Only the additional, higher order Wiener kernels need be measured.

Received on (*month day, year*)

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