A Study on Turbo Coding Systems with π/4 Shifted DQPSK Modulation

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***Abstract*— In this paper some methods of using turbo coding in conjunction with π/4 shifted Differential Quadrature Phase Shift Keying (π/4 shifted DQPSK) modulation are proposed. The models used in the simulation of the running turbo coded system and their BER performances are presented for each case. The turbo coding rate was R=1/3 and R=1/2. The MAP algorithm, 12 iterations were used. A Log Likelihood Ratio (LLR) stop criterion was selected. The interleaving length was N=1784 bits.**

The performances were obtained for AWGN and for flat Rayleigh fading channels.

***Keywords*: —turbo code, duo-binary turbo codes, π/4 shifted DQPSK modulation**

I. INTRODUCTION

The major advantage of the differential modulation [1] is its immunity to the missing of the high duration phase coherence. (the fading channels case). The quadrature modulation gives the best compromise between band efficiency and BER performance. Moreover, in π/4 shifted DQPSK modulation the maximum phase shift is restricted to ±135°, as compared to 180° for Quadrature Phase Shift Keying (QPSK).

This paper proposes to mix the modulation advantages with the turbo coding advantages [2]. So, an adaptation between them must be made, the differential quadrature modulation (which implies a demultiplexing 1→2) and turbo coding (which can be considerate like a demultiplexing of order R_c). Moreover, the π/4 shifted DQPSK modulation also presupposes, versus the Binary Phase Shift Keying (BPSK) or QPSK modulations, a change of the "amplitude" of the information carrier signal (that will be shown in the following).

A possible scheme for a π/4 shifted DQPSK modulation transmission system is shown in Fig.1. For the simulation, the energetic "balance sheets" are very important. These can be expressed as following. We suppose that the carrier signals (the versors) are:

$$\begin{aligned} \phi_1(t) &= \sqrt{\frac{2}{T}} \cdot \cos(\omega_c \cdot t) \\ \phi_2(t) &= \sqrt{\frac{2}{T}} \cdot \sin(\omega_c \cdot t), \end{aligned} \qquad (1)$$

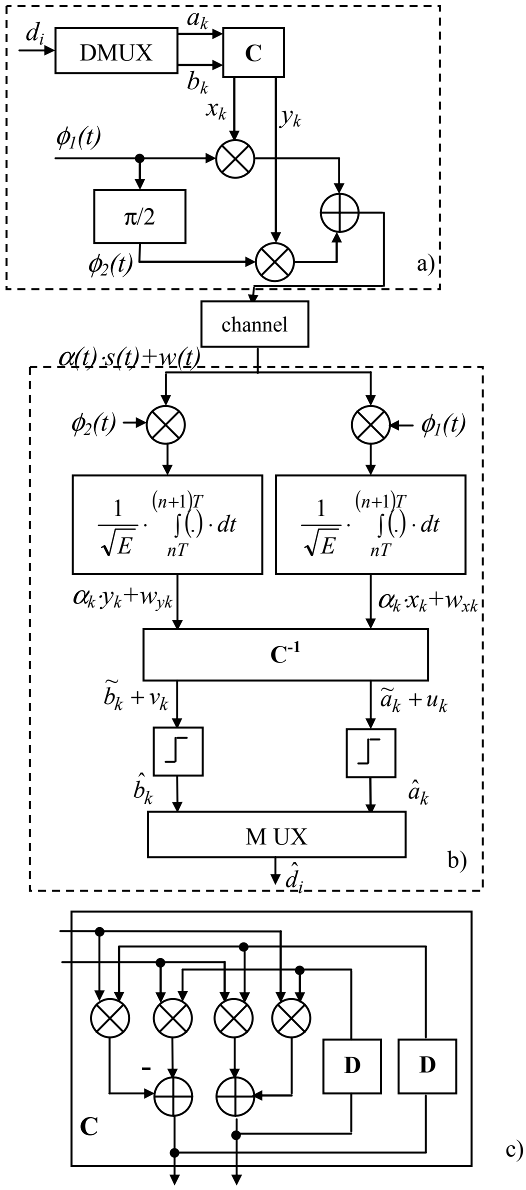

Figure 1. Transmission system with π/4 shifted DQPSK modulation : a) modulator; b) demodulator; c) The phase shifting computation block.

the modulated signal is:

$$s(t)=\sqrt{E}\cdot\sqrt{\frac{2}{T}}\cdot\cos(\omega_c\cdot t+\theta_k)$$
$$=\mathrm{Re}\left\{\sqrt{E}\cdot\sqrt{\frac{2}{T}}\cdot e^{j\omega_c\cdot t}\cdot e^{j\theta_k}\right\}\quad kT\le t<kT+T \tag{2}$$

where E and T - are the energy and the duration of the symbol, ω_c – the carrier pulsation and θ_k - the phase of the k symbol. Let S_k be the complex amplitude:

$$S_k=\sqrt{E}\cdot e^{j\theta_k}=\sqrt{E}\cdot(x_k+j\cdot y_k) \tag{3}$$

It results that:

$$S_k=S_{k-1}\cdot e^{j\varphi_k} \tag{4}$$

where φ_k represents the phase shift of the k symbol. This shift depends on the input dibit:

$$(a_k b_k)=(d_i d_{i+1}),\qquad k=2i \tag{5}$$

A possible correspondence is done in Table 1. The possible phase shifts, taking into account that $S_{k-1}=1=e^{j0}$, are shown in Fig.2. If it is considered a bipolar representation for a_k and b_k bits then, according to Table 1, it can be written:

$$\cos(\varphi_k)=\frac{\sqrt{2}}{2}\cdot a_k\qquad \sin(\varphi_k)=\frac{\sqrt{2}}{2}\cdot b_k \tag{6}$$

TABLE I. π/4 SHIFTED DQPSK MODULATION MAPPING

a_k b_k	φ_k	$\cos(\varphi_k)$	$\sin(\varphi_k)$	Frame
0 1	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$+\frac{\sqrt{2}}{2}$	*II*
1 1	$\frac{\pi}{4}$	$+\frac{\sqrt{2}}{2}$	$+\frac{\sqrt{2}}{2}$	*I*
1 0	$-\frac{\pi}{4}$	$+\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	*IV*
0 0	$-\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	*III*

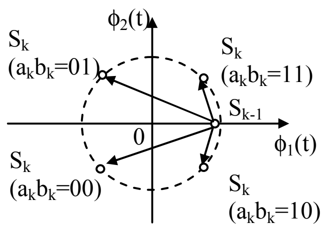

Figure 2. The constellation of the π/4 shifted DQPSK modulation.

So, the relation (4) becomes:

$$\begin{bmatrix}x_k\\y_k\end{bmatrix}=\frac{\sqrt{2}}{2}\cdot\begin{bmatrix}a_k & -b_k\\ b_k & a_k\end{bmatrix}\cdot\begin{bmatrix}x_{k-1}\\y_{k-1}\end{bmatrix}=\frac{\sqrt{2}}{2}\cdot\begin{bmatrix}x_{k-1} & -y_{k-1}\\ y_{k-1} & x_{k-1}\end{bmatrix}\cdot\begin{bmatrix}a_k\\b_k\end{bmatrix}=\frac{\sqrt{2}}{2}\cdot\begin{bmatrix}a_k\cdot x_{k-1}-b_k\cdot y_{k-1}\\ a_k\cdot y_{k-1}+b_k\cdot x_{k-1}\end{bmatrix} \tag{7}$$

Due of the fact that ($\sqrt{E}\cdot x_k$, $\sqrt{E}\cdot y_k$) are the coordinates of a single point from the circle with $\sqrt{E}$ ray, the matrix *[x_{k-1} −y_{k-1}; y_{k-1} x_{k-1}]* is reversible and:

$$\begin{bmatrix}a_k\\b_k\end{bmatrix}=\sqrt{2}\cdot\begin{bmatrix}x_{k-1} & y_{k-1}\\ -y_{k-1} & x_{k-1}\end{bmatrix}\cdot\begin{bmatrix}x_k\\y_k\end{bmatrix}=\sqrt{2}\cdot\begin{bmatrix}x_k\cdot x_{k-1}+y_k\cdot y_{k-1}\\ -x_k\cdot y_{k-1}+y_k\cdot x_{k-1}\end{bmatrix} \tag{8}$$

The equations (7) and (8) aid the modulator and demodulator implementations.

Because the coordinates x_k and y_k of the signal point can take values from the sets: *{-1, +1}* with probability *25%*, *{0}* with probability *25%*, $\{-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\}$ with probability *50%*, "the power" of the sequences x_k and y_k (if $E=1$) is:

$$\overline{x_k^2}=(\pm)^2\cdot\frac{1}{4}+(0)^2\cdot\frac{1}{4}+\left(\pm\frac{\sqrt{2}}{2}\right)^2\cdot\frac{1}{2}=\frac{1}{2} \tag{9}$$

And the signal/noise ratio (SNR) at the output from each integration block is:

$$SNR=\frac{\overline{\alpha_k^2}\cdot\overline{x_k^2}}{\overline{w_{xk}^2}}=\frac{\frac{1}{2}}{\frac{N_0}{2\cdot E}}=\frac{1}{2}\cdot\frac{2\cdot E}{N_0}=\frac{2\cdot E_b}{N_0} \tag{10}$$

In conclusion, the flat fading "digital channel" model with π/4 shifted DQPSK modulation is presented in Fig.3. If α=1 we will obtain an AWGN channel.

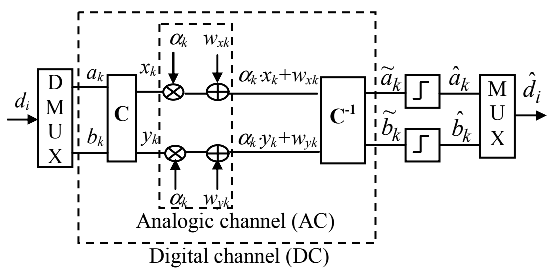

Figure 3. The flat fading digital channel and π/4 shifted DQPSK modulation model.

II. The Methods for the Simulation of the Turbo Coded System

A. π/4 shifted DQPSK & Turbo Code Classic (dq1TC)

Due to the fact that the turbo decoder operates with soft sequences ($\tilde{a}_k$ and $\tilde{b}_k$), the hard decision blocks are missing from the turbo-coded scheme. A first solution will be the application of the turbo code on the points d_i (turbo coder, TC) and, respectively $\hat{d}_i$ (turbo decoder, TD). This variant is named classical because the turbo code has one input. The scheme from Fig.4 is obtained.

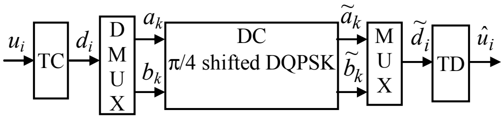

Figure 4. The π/4 shifted DQPSK & Turbo Code classic system model.

The major disadvantage of the scheme from Fig.4, from a practical point of view, is the necessity of the multiplexing operation on the soft sequences.

B. π/4 shifted DQPSK & 2 Turbo Code Classics (dq2TC)

To eliminate the disadvantage of the "*A*" method, we propose three solutions. The first one would be to apply two turbo codes (classical) to each of the branches a_k and b_k. As the result we obtain the scheme from Fig.5.

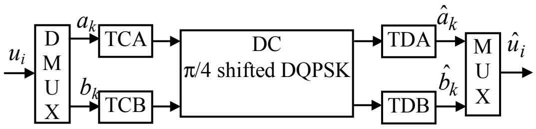

Figure 5. The π/4 shifted DQPSK & 2 Turbo Code classics system model.

This method implies the doubling of the coding equipment, but conserves the BER performances.

C. π/4 shifted DQPSK & Turbo Code Duo-binar (dqTCDB)

The second solution proposed is the using of the duo-binary turbo code (Fig. 6). Because the duo-binary turbo code accepts two inputs, others operations for adaptation are not necessary. Moreover, the duo-binary turbo code offers some advantages [3]: better convergence, larger minimum distances, less sensitivity to puncturing patterns, reduced latency, robustness of the decoder.

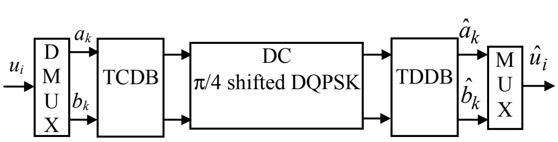

Figure 6. The π/4 shifted DQPSK & Turbo Code Duo-binary system model.

D. π/4 shifted DQPSK & Turbo Code Classic Punctured (dqTCPU)

The last solution proposed invokes the puncturing operation. The classical punctured turbo code eliminates the necessity of multiplexing, because it is made in the implicit way.

Remark:

In the last two variants a rate 1/2 turbo code is used whereas in the first two variants (dq1TC and dq2TC) a rate 1/3 turbo code is used.

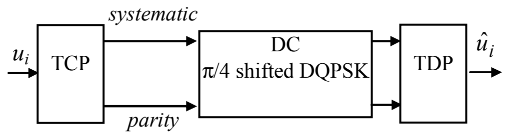

Figure 7. The π/4 shifted DQPSK & Turbo Code Classic Punctured system model.

III. Experimental Results

The BER performances of the transmission systems which use BPSK, respectively π/4 shifted DQPSK modulations, both uncoded, are comparatively presented in the diagram from Fig.8.

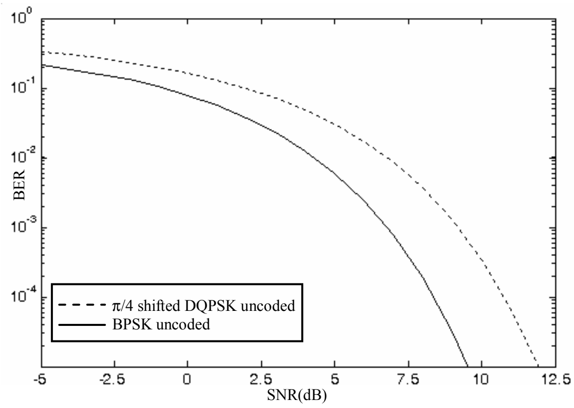

Figure 8. The BER performances of the BPSK and π/4 shifted DQPSK modulations

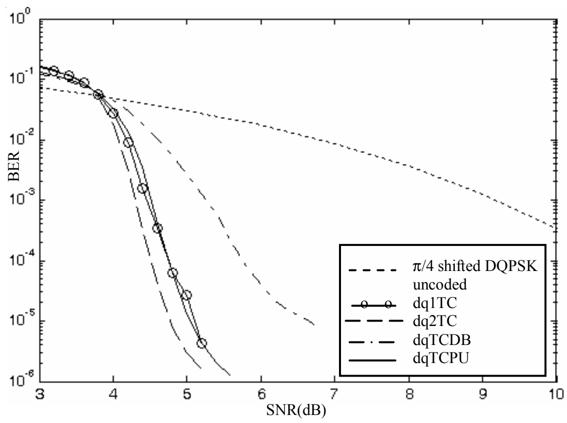

Figure 9. The BER performances of the methods presented above.

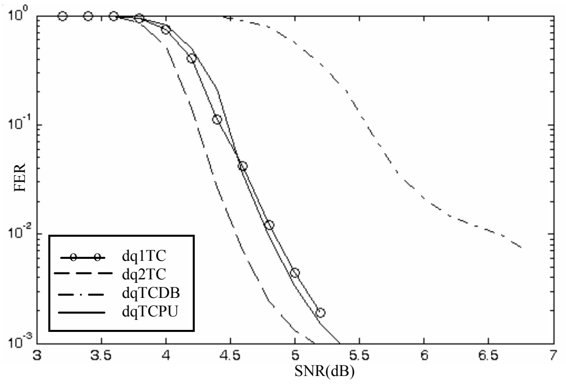

Figure 10. The FER performances of the methods presented above.

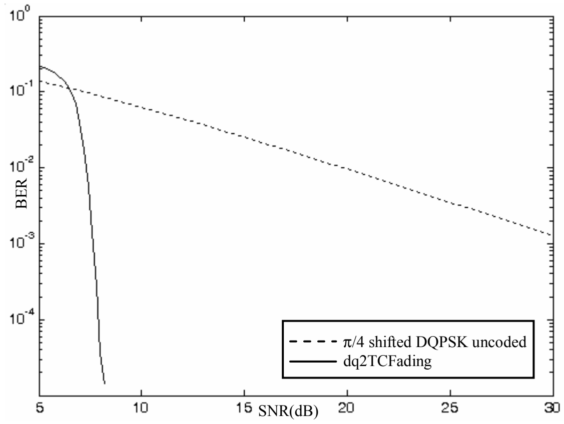

Figure 11. The BER performances for dq2TC system, Rayleigh fading channel.

The π/4 shifted DQPSK modulation is with 2,5dB inferior to that of the BPSK modulation. However, the advantage of the π/4 shifted DQPSK modulation is a higher immunity to the loss of coherence. This fact is not directly reflected from the curves in Fig.8, where a perfect synchronization was considerate.

In the simulations of the 4 variants presented in the anterior paragraph, the parameters from table 2 were used. The BER and FER performance of the 4 simulated systems are shown in Fig.9 and Fig. 10. Obviously, the scheme with 2 turbo codes (dq2TC) gives the best performances. It presents, an approximately *7,2 dB* coding gain, versus the un-coded system, for a Bit Error Rate *(BER)*=10^{-5}. Its coding gain versus dq1TC and dqTCPU variants is of approximately *0,35dB* coding gain.

The similar performances of the dq1TC and dqTCPU variants, even though they have different coding rate (1/3, respectively 1/2), denote the special importance of the multiplexing operation. The fact that this multiplexing operation is made implicit on dqTCPU system constitutes a gain, which is reflected in the BER performance.

The weak performance of duo-binary turbo code (dqTCDB) is a consequence of the inadequate interleaving and of the unclosed trellis. In future the optimum interleaving solutions for correcting this gap must be found.

As for the behaviour of the turbo codes in Rayleigh flat fading channel, the BER performance of the dq2TC system is presented in Fig.11.

TABLE II. THE TURBO CODES PARAMETERS

Var	Coding Rate	Generator Matrix	Trellis closing I Code	Trellis closing II Code	Puncturing Matrix
dq1TC	$\frac{1}{3}$	$[1,\ 5/7]$	*Yes*	*No*	-
dq2TC	$\frac{1}{3}$	$[1,\ 5/7]$	*Yes*	*No*	-
dqTCDB	$\frac{1}{2}$	$\begin{bmatrix}1,0,5/7\\0,1,3/7\end{bmatrix}$	*No*	*No*	-
dqTCPU	$\frac{1}{2}$	$[1,\ 5/7]$	*Yes*	*No*	$P=\begin{bmatrix}1 & 1\\1 & 0\\0 & 1\end{bmatrix}$

It is with approximately *3,5 dB* inferior to the AWGN channel, but the coding gain is of tens of dB, which justifies the price paid by the turbo decoding complexity.

IV. CONCLUSIONS

In this paper a study of using turbo coding in conjunction with π/4 shifted DQPSK modulation is presented. The AWGN and Rayleigh flat fading channels were used. Four turbo coding schemes were proposed. The Recursive Systematic Convolutional Codes (RSC) with constraint length K=3, 1/3 and 1/2 rates, turbo codes were used. In one variant a punctured turbo code is built, and in another variant a duo-binary turbo code is built. An S-interleaver with the length N=1784 bits was employed.

The most performing variant, which results from the BER curves, correspond to the most complex variant. The coding gain price, 0,35 DB at BER=10^{-5}, is the use of two 1/3 rate turbo codes. An attractive alternative, from a complexity-performance compromise point of view, is the punctured turbo code. Implicitly, it eliminates the multiplexing necessity that is a factor, which has an influence in the BER performances. The weak performance obtained with the duo-binary turbo code (dqTCDB) implies the finding of a better interleaver.

Another observation is that the performance of the turbo code in conjunction with π/4 shifted DQPSK modulation is not proportional with the turbo code in conjunction with the BPSK modulation (the difference between the gains coding of the turbo coded systems is with *1dB* higher than that of the uncoded systems).

The turbo coding utility in the fading channel case is totally justified by the big coding gain (tens of dB).

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