Permutations in Communications

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There are few applications of the permutations in the field of communications, for example: the symetric data encription algorithms use permutations, an interleaver is a permutation device, the allocation of sub-carriers' frequencies in the OFDMA system (described in the 802.16.e standard) used for the management of the access of the last generation of WiMAX mobile devices is made in fact by a permutation. A permutation system is very attractive from an electronic point of view because it has a very simple implementation requiring only few wires.

There are two goals for a permutation system: the decorrelation of a sequence and the spreading of the neighbors from the initial sequence.

Why decorrelation? The most eloquent answer is the shuffle of playing cards, required for destroying the "playing cards groups" formed in a previous game. In communications the correlation of groups of bits is unwanted when we desire to obtain from each bit its particular information independent of the particular information of its neighboring bits. If the bits from a sequence are correlated then all of them will give the same information like the first bit of the considered sequence, so the second bit, the third one and so on will not give any additional information. We suppose that the permutations used in the symmetric data encryption algorithms were conceived to maximize the decorrelation of the symbols composing the current block.

Why spreading? In the case of the mobile WiMAX system the Doppler Effect can be observed. Due to the mobility of the devices the value of the frequency of a sub-carrier can vary between the input and the output of a communications channel producing interference with the sub-carriers having neighbor frequencies (if for example these frequencies were already allocated to a fix user). By spreading the distance between the initial neighbor frequencies is increased and the interference can be reduced or avoided. So, we suppose that the permutations included in the OFDMA system (FUSC, PUSC and so on) are conceived to maximize the spreading of sub-carriers' frequencies.

In the case of an interleaver both effects are required. It is possible to maximize simultaneously the decorrelation and the spreading? This question does not receive an answer yet.

In this paper is proposed a qualitative and quantitative method for the analysis of the spreading realized by a given permutation system, based on the computation of the Distances Spectrum (DS). It is used to analyze the quality of the PUSC permutation system.

In [1], the function DS, $s: J \rightarrow \mathbf{N}$ was defined as:

$$s(k) = no[d(i,j) = k], \quad k \in J \tag{1}$$

where $J = \{2, 3, \dots 2N-2\}$, and *no*[*condition*] represents the number of cases in which the *condition* is true, $\mathbf{N}$ denotes the natural numbers set and $d(i,j)$ is the distance between the positions i and j defined as:

$$d(i,j) = j - i + |\pi(i) - \pi(j)|, \quad \forall i, j \in I, j > i, \tag{2}$$

where π is the permutation map:

$$\pi : I \rightarrow I, \quad I = \{1, 2, \dots N\}, \tag{3}$$

In the last years a significant effort was made on the study and on the design of the turbo codes (TCs) owing to their huge performance. Implicitly, the best interleaving methods were searched and investigated. One of the best interleavers is the S-interleaver, [2]. To appreciate it in a quantitative way we define the spreading degree as:

$$G = supp\{s(k)\} \;/\; \max_{k \in J} s(k), \tag{5}$$

i.e. the ratio between the support of the function s (the number of distances d in which $s(d) > 0$) and the maximum value of DS.

We will compare in the following (from the DS point of view) a practical S-interleaver with the length N=140 bits with the interleaver defined by the frequencies' permutation of a segment in the DL-PUSC case, [3]. We consider only the ordering relation between the 140 frequencies, and then we obtain a sequence of positions from 1 to 140, which is permuted. Following now the place conquered by the positions in the "arrangement" given by the standard 802.16e, we obtain the result of the permutation with the DS presented in Fig. 2 b. The two diagrams from Fig. 2 show a similitude between the frequencies' permutation and the permutation realized by the S-interleaver. In Table 2 is indicated for both permutation systems (in the columns 2 and 3) the minimum and the maximum interleaving distances (d_{min}, D_{max}), the maximum DS value (s_{max}), the support of DS (*supp(s)*), and the spreading degree (G). The parameters from Table 2 prove the similitude between the two permutation systems. In conclusion the DL-PUSC permutation of the sub-carriers' frequencies is very good. It minimizes the sub-carriers interference, spreading their frequencies at an important distance. This conclusion is in accord with the conclusion of a study having the same subject, but using a different approach based on link level simulations, [4]. On the other hand, the fact that there are permutations with a higher spreading degree, lead us to the idea that this permutation can be improved. An analysis more detailed can leads to results more precise.

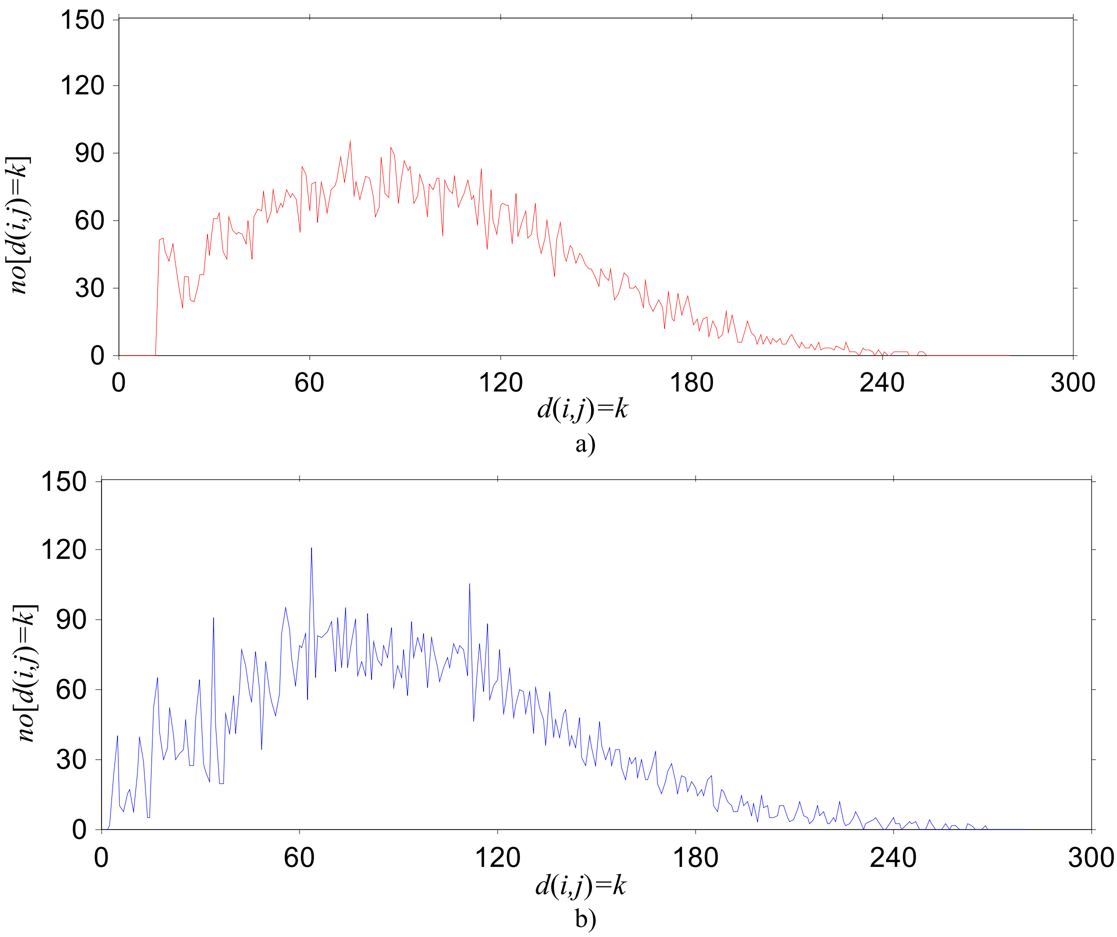

Fig. 2 The DS for: a) the S-interleaver with S=13 and N=140; b) the permutation system defined by the positions' sequence of the 140 sub-carriers' frequencies of a BS segment.

TABLE II
THE DSS PARAMETERS

Parameter	S-interleaver	Frequencies' positions permutation system DL-PUSC
d_{min}	13	3
D_{max}	253	268
s_{max}	95	121
supp	233	249
G	2.4526	2.0579

References

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