Electrocardiogram Baseline Wander Removal Using Stationary Wavelet Approximations

[1]Beatrice ARVINTI, [1]Dumitru TOADER, [1]Marius COSTACHE, [2]Alexandru ISAR
[1]Electric Engineers Faculty, "Politehnica" University, Timisoara, Romania,
[2]Electronics and Telecommunications Faculty, "Politehnica" University, Timisoara, Romania
Contact e-mail: beatrice.arvinti@yahoo.com

Abstract- In this paper, a method to reduce the baseline wandering of an electrocardiogram signal is presented. The described method is based on stationary wavelet approximation of the whole signal. The main advantage of this method, compared with others, is the fact that it is a non-supervised method, allowing the process to be used in an automatic analysis of electrocardiograms. Moreover, the results are as accurate as those obtained with other methods of baseline wandering removal, methods considered as references in the scientific bibliography.

I. INTRODUCTION

An accurate ECG signal, unaffected by low-frequency and high-frequency interferences, is seldom encountered in practice. Usually an electrocardiogram is affected by noise and the artifactual data is due to the movement, perspiration or breathing of the patient, electrode contact, power-line interferences, etc. This noise influences the baseline of the ECG signal, introducing a wandering which can make the inspection difficult and even mask some significant features. Baseline wandering removal is an important preprocessing task, especially in an automatic system where one wants to avoid the failure of the processing task, such as wave detection. In order to reduce as much as possible the negative effects, methods to remove the baseline wandering have been elaborated and some of them are presented in this paper, each method showing advantages and disadvantages. For example, in [1] a method using cubic splines is presented. In order to obtain results using this method, one needs the coordinates of baseline points to estimate it, a fact which makes the procedure for baseline wandering removal more difficult. Other methods such as [2] use signal processing techniques, the results being more easily obtained, but with deficiencies. The solution proposed in [2] is based on the use of a FIR filter with linear phase having a reduced number of taps. The disadvantage is the fixed value of the cut-off frequency of the high-pass filter. Indeed, the maximal frequency of the base-line differs from an ECG to another. So, the filter should be adaptive.

In [3] is proposed a cascade adaptive filter for removing the baseline wander preserving the low frequency components of the ECG. It works in two stages. The first stage is an adaptive notch filter at zero frequency. The second stage is an adaptive impulse correlated filter that, using a QRS detector, estimates the ECG signal correlated with the QRS occurrence. In this way, all the signal components correlated with the QRS complex are preserved. This adaptive filter can be seen as a comb filter without the dc lobe. The solutions already presented are based on linear systems.

In [4] is presented a baseline wander reduction solution based on nonlinear systems, the morphological filters. The algorithm consists of only one stage of morphological processing (while similar morphological filters need two stages). The morphological operators are applied to approximate the baseline drift. Then it is subtracted from the input signal so as to leave a corrected-baseline signal. Compared with all existing morphological methods, there is a substantial improvement, especially in reducing distortion of the baseline waveform in any part of the signal. The experimental results prove that the proposed method is less sensitive to the size of the structuring element, if a reasonable size is considered.

Usually, the application of these methods is only suitable in particular cases, with known conditions. In this study, a more general method is developed, without need of interaction with the user and offering acceptable results. The proposed method belongs to the class of time-frequency methods. A type of solutions in this class uses wavelets. The method in [5] is based on the Discrete Wavelet Transform, DWT. The main reasons for using this transform are the properties of good representation of nonstationary signals such as ECG signals and the possibility of dividing the signal into different bands of frequency. This makes the detection and the reduction of ECG baseline wander in low frequency sub-signals possible. The mechanism of division of the signal into different sub-bands proper to the DWT is not very flexible. A more flexible division mechanism is associated with the wavelet packets transform, WPT. For this reason a baseline wander reduction mechanism based on WPT is proposed in [6]. In this reference is presented a wavelet based search algorithm using the energy of the signal in different scales in order to isolate baseline wander from ECG signal. The algorithm calculates the wavelet packet coefficients. In each scale the energy of the signal is calculated. Comparison is made and the branch of the wavelet binary tree corresponding to higher energy wavelet spaces is chosen. The DWT is obtained by the discretization of a time-frequency representation named Continuous Wavelet Transform, CWT. This time-frequency representation is generalized in [7] obtaining the Multi-adaptive Bionic Wavelet Transform, MABWT. It can be applied to ECG signals in order to remove low-pass noisy interference effects on the baseline of ECG.

Another class of filters used for the removal of the baseline wander of ECG signals is the Kalman filter (KF), as proposed in [8]. The method is based of an iterative approach, the hypothesis that an ECG signal can be characterized by an autoregressive model being assumed. The baseline wander is estimated using a polynomial approximation, independent of the characteristics of the signal. This model is then integrated with the KF and the state variables are calculated.

II. PROPOSED METHOD

Inspired from the analysis of temporal series, [9], the method proposed in this paper tries to estimate the "overall tendency" of the ECG. This signal represents the baseline of the ECG.

The multiresolution analysis (MRA) is a procedure of analysis of a signal $s(t)$ which takes into account its representation at multiple time resolutions. When the original signal $s(t)$ is involved, the maximal resolution is exploited. When a variant of the original signal (for example the signal $s(2t)$) is used then a poorer resolution is exploited. Combining few analyses realized at different resolutions, a MRA is obtained.

Generally the MRA is implemented on the basis of Mallat's algorithm (which corresponds to the computation of the DWT). The disadvantage of Mallat's algorithm is the decreasing of the length of the coefficient sequences with the increasing of the iteration index due to the decimators' utilization. Another way for the implementation of a MRA is the use of Shensa's algorithm (which correspond to the computation of the Stationary Wavelet Transform (SWT)), represented in figure 1, [10]. In the present case the utilization of decimators is avoided but, at each iteration K different low-pass (h_k) and high-pass (g_k) filters are used. Their impulse responses are constructed by interpolation, starting from h_1 and g_1:

$$f_{k+1}[n] = \begin{cases} f_k\left[\frac{n}{2}\right], & n \vdots 2 \\ 0, & \text{if not} \end{cases} \qquad (1)$$

where f can be g or h. Two types of coefficients are obtained at each decomposition level K, the approximation coefficients at the output of the filter with impulse response h_k (denoted by $a_k[n]$) and the detail coefficients at the output of the filter with the impulse response g_k (denoted by $d_k[n]$).

The aim of the method proposed in this paper is to estimate the baseline wander and to eliminate it by subtraction from the acquired ECG. The estimation is realized by low-pass filtering. To do this, the SWT of the acquired ECG is computed, using K decomposition levels. Next, all the detail coefficients are met to zero and a new sequence is obtained. Finally, the inverse SWT, ISWT, of the new sequence is computed and the baseline estimation is obtained.

The new sequence, already mentioned, is obtained by the low-pass filtering of the considered ECG with the filter with the impulse response $h_e = h_1 * h_2 * \ldots h_K$. Its frequency response is $H_e = H_1 \cdot H_2 \cdot \ldots H_K$. Taking into account relation (1) the expression of the frequency response of the equivalent low pass filter becomes:

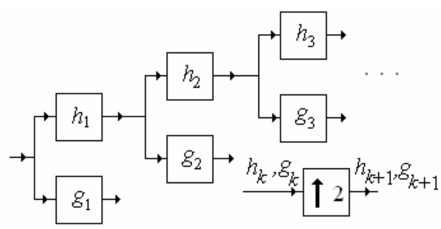


Fig. 1. The scheme of the system that computes the stationary wavelet transform. The systems with the impulse responses h_k are low-pass filters and the systems with impulse responses g_k are high-pass filters. These impulse responses are constructed iteratively by interpolation.

$$\begin{aligned} H_e(\Omega) &= H_1(\Omega) \cdot H_2(\Omega) \cdot \ldots H_K(\Omega) = \\ &= H_1(\Omega) \cdot H_1(2\Omega) \cdot \ldots H_1\left(2^{K-1}\Omega\right). \end{aligned} \qquad (2)$$

It depends on the mother wavelets used for the computation of the SWT (which gives the expression H_1) and on the resolution level K.

The appropriate selection of these two parameters assures the performance of the proposed compensation method. The optimal choice supposes the use of mother wavelets with good time-frequency localization, as shown in [11]. The value K must be selected in accordance with the sampling frequency used for the acquisition of the ECG, f_s. The ECG is a quasi-periodic signal. Let us denote its fundamental period with T. This value can be determined measuring the pulse of the patient. For a healthy patient the number of pulses per minute belongs to the interval [60-80]: $(T \in [0.75, 1]\text{s})$.

The equivalent low-pass filter with the frequency response $H_e(\Omega)$ should not affect the spectrum of the useful component of the ECG. Hence, its cut-off frequency must be smaller then $1/T$. In consequence the time resolution of the K^{th} decomposition level must be higher or at least equal to T.

The time resolution of the K^{th} decomposition level equals $2^k / f_s$. So, the value of K must be selected to satisfy the condition:

$$2^K / f_s \geq 1 \qquad (3)$$

For a value of the sampling frequency of 360 Hz the last condition becomes $K \geq 8$. It can be observed, analyzing equation (2), that the value of the cut-off frequency of the equivalent filter decreases with the increase of K. The bandwidth of the baseline signal is not a priori known. So, its maximal possible value must be considered. This is of K=8, for $f_s = 360$ Hz. The selection of K is very important. A value too small produces distortions of the useful components of the ECG. A value too high produces attenuations of the waveform of the baseline wander, making impossible its complete rejection. In figure 2 is presented the scheme of the proposed correction system.

One of the advantages of the SWT versus the DWT is the translation invariance. Indeed, the system in figure 1 is time-invariant, as a collection of linear time-invariant subsystems (any linear filter is time-invariant). This invariance is very important in the present method because the estimation is subtracted from the original ECG. A drawback of the method could be constituted by the increased number of computations the length of the signal obtained computing the SWT being much higher than the length of the original signal. Nevertheless, the SWT offers a more flexible analysis tool than the DWT.

For the mother wavelets Dau_2 (the first mother wavelets proposed by Ingrid Daubechies [12] which has 2 vanishing moments) the corresponding impulse response is:

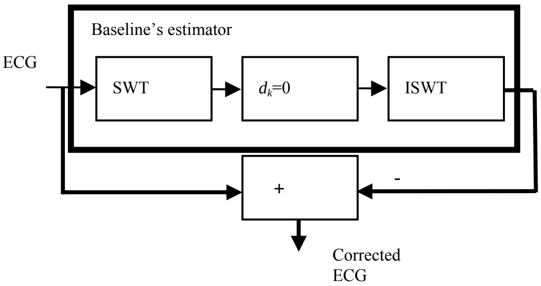


Fig. 2. The architecture of the proposed baseline's correction system.

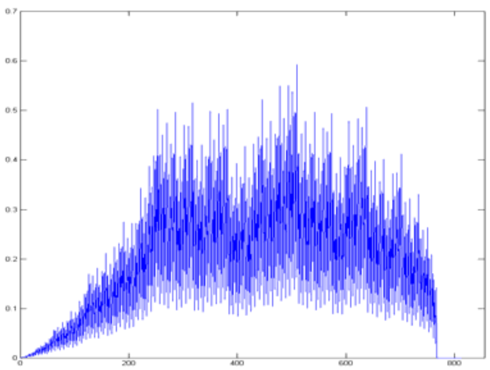


Fig. 3a) The impulse response of the equivalent low-pass filter.

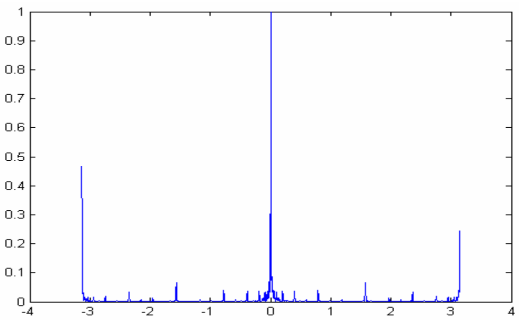


Fig. 3 b) The normalized magnitude of the frequency response of the equivalent low-pass filter.

$$h_1 = \begin{bmatrix} \frac{1+\sqrt{3}}{8} & \frac{3+\sqrt{3}}{8} & \frac{3-\sqrt{3}}{8} & \frac{1-\sqrt{3}}{8} \end{bmatrix} \qquad (4)$$

The impulse response of the equivalent filter is represented in figure 3 a). It contains 766 not null coefficients. Its waveform is not symmetric. This is not a linear phase filter. The magnitude of the frequency response of the same filter is ploted in figure 3 b).

It is a very narrow band low-pass filter, having a low cut-off frequency. Similar low-pass filters can be obtained using the other mother wavelets belonging to the Daubechies family, but these filters have poorer time-frequency localizations.

III. SIMULATION RESULTS

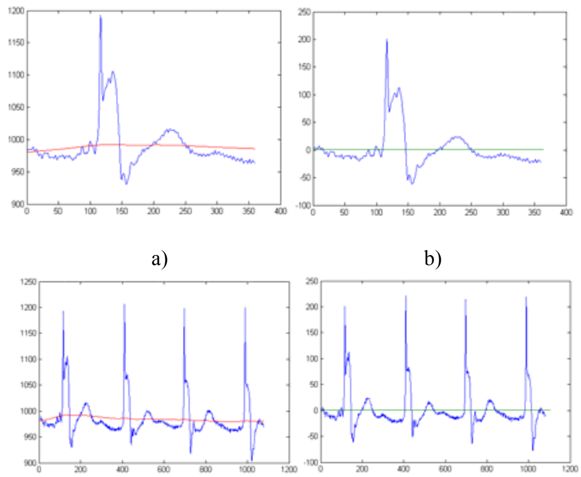
The proposed method was tested using the MIT-BIH arrhythmia database, which can be found at the address http://www.physionet.org/physiobank/database/mitdb/

The MIT-BIH Arrhythmia Database contains 48 half-hour excerpts of two-channel ambulatory ECG recordings, obtained from 47 subjects studied between 1975 and 1979. Twenty-three recordings were chosen at random from a set of 4000 24-hour ambulatory ECG recordings collected from a mixed population of in-patients (about 60%) and out-patients (about 40%); the remaining 25 recordings were selected from the same set to include less common but clinically significant arrhythmias that would not be well-represented in a small random sample.

The recordings were digitized at 360 samples per second per channel with 11-bit resolution over a 10 mV range. Two or more cardiologists independently annotated each record; disagreements were resolved to obtain the computer-readable reference annotations for each beat (approximately 110,000 annotations in all) included with the database.

This database was used in all the papers dedicated to the study of ECGs referenced here. So, direct comparisons can be done, using the results reported in these papers, excepting [8].

Considering the first minute of the ECG named 102 from the MIT-BIH database, represented in blue in figure 4 a), we have applied the proposed compensation method. The estimation of the baseline is represented in red on the same figure. It follows precisely the slow variations of the ECG. Making the difference between the original ECG and the estimation of the baseline, the result of the proposed compensation method, represented in figure 4 b), is obtained. Regarding the result, it can be observed that its baseline is very close to 0 at any moment. To better appreciate the performance of the proposed method, a closer look is presented in the following figure. It contains some zooms of different regions of figure 4. In figure 5 a) is represented the first beat of the considered ECG associated with its baseline (represented in red). The corresponding result of the proposed method is represented in figure 5 b). Its baseline is represented in green. It can be observed that it is zero everywhere.



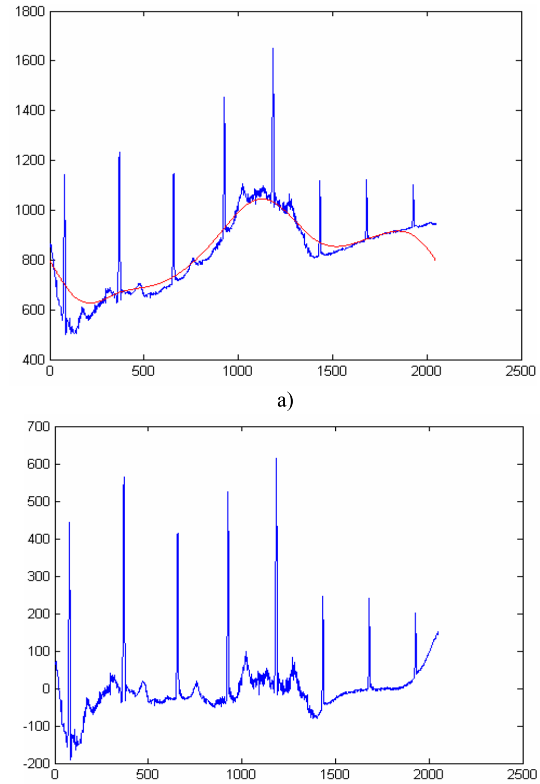
a) b)

c) d)

Fig 5. First beat. a) Before treatment (the baseline is represented in red), b) After treatment (the new baseline is represented in green). First three beats. c) Before treatment (the baseline is represented in red), d) After treatment (the new baseline is represented in green).

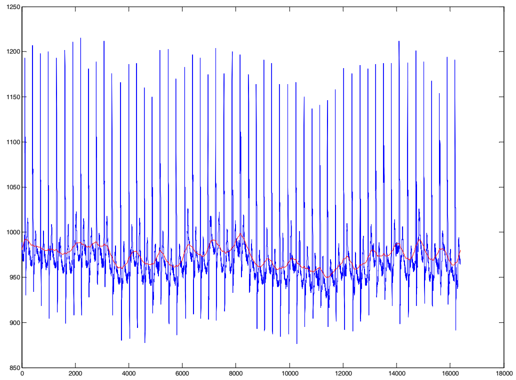
The first three beats of the considered ECG are represented in figure 5 c) and the corresponding result in figure 5 d). Once again, the new baseline equals zero everywhere. Analyzing the image in figure 5, it can be observed that the considered ECG is a little bit perturbed by noise. So, another goal of the ECG's pretreatment is its denoising.

Another simulation result is presented in figure 6. This time, a signal with a high baseline wander was selected, in conformity with [1]. In this reference is proposed the signal 103 from the MIT-BIH arrhythmia database, and more precisely its waveform starting at the moment 18'20''. The waveform of the selected ECG is presented in figure 6 a) and the result obtained applying the proposed method in figure 6 b). Indeed the wander of the baseline of this waveform is high (see fig. 6 a)) and the result of the method proposed in this paper is not perfect (see fig. 6 b)). The limitation appears because the cut-off frequency of the low-pass filter used for the estimation of the baseline is too low and this system is not able to follow the rapid variations of the baseline. Still, the probability of putting a correct diagnosis can be considerably increased because of the smaller drift of the baseline, which enables the ECG to be easier to read. Comparing figure 6 with figure 8 in [1] (the two figures refer to the same ECG) it can be observed that the error introduced by the proposed method is comparable with the error introduced by the method based on the interpolation of the baseline with the aid of cubic splines and is higher than the error introduced by the method based on adaptive filter. This comparison is not very easy to be done due to the lack of resolution of figures in [1]. The method based on adaptive filter's theory makes a better correction of the baseline but it is possible that this method distorts stronger the useful component of the considered electrocardiogram. It must be also mentioned that the method proposed in [1, 3], based on the adaptive filter's theory requires a higher computational effort, because it applies two times the LMS algorithm. The proposed method can be compared also with the method in [2], where for the compensation of the baseline wander is proposed a low-pass filter. The equivalent filter, characterized in figure 3, has a higher transition from the pass band to the stop band than the filter proposed in [2] assuring a higher precision for the compensation method proposed here, but it has also a higher number of coefficients and it is not of linear phase requiring a higher computational effort for its implementation. The method proposed in this paper belongs to the class of nonlinear signal processing methods (because all the detail coefficients are met to zero) like the method proposed in [4]. Comparing the results presented in figure 4 with those reported in [4] it can be observed that the method proposed here is comparable with the method based on mathematical morphology proposed in [4]. The advantage of the method proposed in this paper versus the method proposed in [4] is the reduced computational effort required. In [5] is proposed another non-linear baseline drift compensation method which uses the DWT. First the DWT is applied to the selected ECG. The single iteration Mallat's algorithm is implemented.

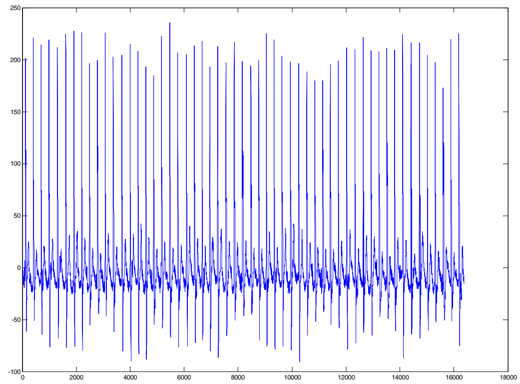


b)

Fig 6. Simulation results obtained for the ECG 103 with the start moment at 18'20''. a) Original waveform (in blue) and the estimation of the baseline (in red) and b) Result of the compensation method.



a)



b)

Fig. 4. Simulation results. The original signal (represented in blue in a) is the recording 102 from the MIT-BIH arrhythmia data base. The estimation of the base line obtained using the proposed method is represented in red on the same figure. The result of the proposed correction method is represented in blue in b).

Two sequences, a_1 and d_1 are obtained. The drift of the baseline affects only the low frequency coefficients a_1. This sequence is filtered with the aid of an averaging filter, AF. First the sequence a_1 is segmented in beats. Next, the P-Q segment in each beat is identified. Next the level of the baseline of each beat is estimated as the average of the samples of the corresponding P-Q segment. Next the baseline of each beat is corrected by computing the difference between the corresponding values of the ECG and the estimated level of the baseline, obtaining a new approximation sequence, a_{1n}. Applying the inverse DWT, IDWT to the couple (a_{1n}, d_1) the result of the baseline correction method proposed in [5] is obtained. Comparing the results presented in figure 4 with the results reported in [5] it can be observed that the two methods have similar performance for moderate distorted ECGs. The advantage of the method proposed in this paper is its higher simplicity. The localization of the P-Q segments required by the method proposed in [5] could be problematic, especially for ECGs with small signal to noise ratio, SNR. So, we can affirm that the method proposed here is more robust than the method proposed in [5]. Analyzing the wavelet decomposition in figure 1, it can be observed that only the output of the low-pass filters is reiterated. In the case of the WPT the output of the high-pass filters can be reiterated also. This way a tree is obtained. The selection of its nodes procedure is named the search of the best basis. In [6] a new best basis search procedure whose purpose is to reduce the wander of the baseline is proposed. Comparing the results presented in figure 4 with the results reported in [6] it can be observed that the two methods have similar performance for moderate distorted ECGs. The advantage of the method proposed in this paper is the reduced computational effort.

Finally, in [7] is proposed the MABWT. Comparing the results presented in figure 4 with the results reported in [7] for the baseline wander reduction, it can be observed that the two methods have similar performance for moderate distorted ECGs. The advantage of the method proposed in this paper is the reduced computational effort.

Regarding the method using an adaptative Kalman filter [8], the computational effort and implicitely the processing time required induces a major drawback. Real time processing is difficult under such conditions. As can be seen from the results shown in figure 4 of [8], dealing with KF, under special conditions, such as high frequency changes (as we have used also for our proposed method) the KF approach of the baseline wandering removal induces clear distortions in the S-T segment, due to the convergence factor and adaptability of the KF. The distortions are minimal at the method proposed in this paper.

IV. Conclusions

The aim of this paper is a new method for the correction of the baseline wander of ECGs. The estimation is done with the aid of the SWT, computed using the mother wavelets Dau-2 for 8 decomposition level. The sequence $a_8[n]$ represents the SWT of the estimated baseline. Next, all the detail sequences $d_1[n], d_2[n],...,d_8[n]$ are met to zero and a new sequence corresponding to the SWT result is obtained. Next the ISWT is computed obtaining the estimation of the baseline. Finally the difference between the acquired ECG and the estimation of the baseline is computed obtaining the corrected ECG. We have proved the equivalence of the proposed estimation method with a low-pass filtering of the ECG using a special filter (the equivalent filter characterized in figure 3). An original method for the designing of such a special filter was also derived in this paper. It is based on the product of a prototype frequency response with some scaled versions of the same function, (8):

$$H_e(\Omega) = \prod_{k=0}^{7} H_1\left(2^k \Omega\right). \qquad (8)$$

The function $H_1(\Omega)$ is periodic of period 2π, the function $H_1(2\Omega)$ is periodic of period π and so on. If the cut-off frequency of the filter with the frequency response $H_1(\Omega)$ is Ω_c then the cut-off frequency of the filter with the frequency response $H_1(2\Omega)$ is $\Omega_c/2$ and so on. The cut-off frequency of the filter with the frequency response $H_e(\Omega)$ is smaller then the cut-off frequency of the filter with frequency response $H_1\left(2^7\Omega\right)$ which is equal with $\Omega_c/2^7$. So, with the aid of this designing method, very selective filters (see for example figure 3b)) can be obtained. The SWT has 2 parameters, the corresponding mother wavelets and the number of decomposition levels. We recommend the utilization of one of the best time-frequency localized mother wavelets, namely Dau-2. The most difficult task in the implementation of the proposed correction method is the selection of the number of decomposition levels for the SWT. This is equivalent with the selection of the cut-off frequency of the equivalent low-pass filter. This frequency must be small enough for the protection against distortions of the useful components of the ECG but it must be high enough to permit the correct estimation of the high frequency components of the baseline drift. The method proposed in this paper works well for ECGs moderately distorted by the drifts of their baseline (see for example figure 4). Unfortunately, for ECGs excessively distorted (like for example that represented in figure 6 a)) the proposed method is accompanied by limitations. This occurs because the proposed estimation method is not able to follow the rapid variations of the baseline in this case. The method proposed in [1, 3], based on a cascade of two adaptive filters, corrects better the baseline in this case, but the distortions of the useful component of the ECG produced by the proposed correction method seem to be smaller.

The approach proposed in this paper is qualitative, the analyses of the performances of the different baseline wander removal methods being judged as relevant, because they are applied to the same database and direct comparisons can be seen and made. The aim of a future study will be derived through the use of quantitative approaches, defined in terms such as SNR, which could be applied to simulated data.

We have compared the performance of the proposed method with the performance of several other baseline correction methods [1-8]. A very useful tool to make objective comparisons between different ECGs signal processing methods is the MIT-BIH arrhythmia database. It was used in all the papers dealing with ECGs referenced here, single exception being paper [8].

So, selecting the same ECG record and applying the proposed signal processing method, the result obtained can be directly compared with the result in the reference, making the knowledge of the corresponding algorithm unnecessary. The proposed method has similar performance with the following methods: the method based on the use o a FIR low-pass filter [2], the method based on mathematical morphology [4], the method based on the utilization of the WP [6] and the method based on the MABWT [7]. The proposed method is more robust than the method based on the utilization of the DWT in association with the estimation of the baseline of each beat [5]. The results shown in paper [8] are directly compared with those shown in paper [1] and the conclusion of the KF approach consists in a greater accuracy over the spline technique. Compared to the method proposed in this paper, KF need more computational effort and induce some distortions.

Concerning the volume of computation required, the method proposed in this paper is one of the best, being faster then the methods proposed in [1, 3, 4, 5, 6, 7, 8]. The slowest are the methods proposed in [1, 3, 8] followed by the method proposed in [7], by the method proposed in [4] and by the method proposed in [6].

The main advantage of this method, compared with the others already mentioned, is the fact that this is a non-supervised method, allowing the process to be used in an automatic analysis of electrocardiograms.

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