A New Method for the Simulation of the Nakagami Flat Fading (Radio) Transmission Channels

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***Abstract* – In this paper a new method for the simulation of Nakagami frequency-nonselective (flat) fading multichannel transmission channels are presented. The generating methods of the random sequences (numbers) with uniform distribution, Gauss and Nakagami, are used. The Bit Error Rate (BER) curves in conjunction with Binary Phase Shift Keying (BPSK) modulation for this type of fading are shown from simulations. Besides the fading phenomenon we considered an Additive White Gaussian Noise (AWGN) type noise. A comparison between the fading channel performances and no fading (it is considered only the AWGN noise) channel performances is made.**

***Keywords:* fading channel, Nakagami distribution.**

I. INTRODUCTION

The fading phenomenon occurs on radio transmission channels and it is due to the transmissions on multiple paths of the useful signal, [1]. If the time response duration of the fading channel is more inferior to the symbol duration then the fading is frequency-nonselective. The type of the fading differs in function by the application: the frequency band used, the distance and the space geometry between transmitting and receiving antennas, the modification speed of this geometry, the atmosferique conditions, [2]. Although the radio channel modeling is in progress of the investigation, a sufficient model, acceptable, for many real channels is to consider the input-output relation of the digital channel by the form:

$$y_k = \alpha_k * x_k + w_k, \qquad (1)$$

where x_k and y_k are the emitted and received values of the date signal, w_k is the random value which is responsamble by the time fluctuant character (the fading). The type of the random variable, α_k, gives the channel type (Rayleigh, Rice or Nakagami), [3]. Thus, for the channel simulation is necessary to generate the sequences α_k, x_k and y_k, respecting their each nature. In Fig. 1 the considered channel model and the BER computation are shown. The input sequence x_k is binary, random, in NRZ bipolar format.

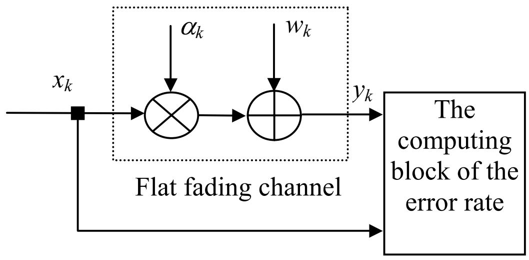

Fig. 1 Model for the simulation and for the study of the BER performances of the flat fading channels.

To generate the sequence x_k it is generated a sequence, u_k, by random numbers with uniform distribution in the interval *[0,1)* as the method presented in the following paragraph. The wanted sequences is obtained by following transformation:

$$x_k = 2 * [2 * u_k] - 1 \qquad (2)$$

where [ ] marks the truncated operation at the integer part. Because u_k has an uniform distribution in *[0,1)*, $[2 * u_k]$ can takes the *0* or *1* values with *1/2* probability, and x_k will take the coresponding values, *-1* or *+1*. The sequence α_k will have a Nakagami distribution and w_k is a sequence with Gauss distribution which simulate the AWGN noise.

II. THE GENERATION OF THE UNIFORM DISTRIBUTION RANDOM SEQUENCES

The generating of the uniform distribution numbers is made by recurrently methods. In this paragraph the residues method is presented, but in [4] we can find other methods. The residue method use the formula:

$$X_k = [a \cdot X_{k-1} + c] \text{ modulo } M. \qquad (3)$$

All the terms of the equation (3) are positive integer, M is a high value number (prime, eventually) to assure a high repetition period, a is the multiplication factor $(a<M)$, and c is an increment, *0* or *1* usually. The method generates integer numbers, distributed uniform in the interval *[0, M]*. To obtain uniform distribution numbers in the interval *[0, 1]*, X_k must be divided with M. The most used recurrences (for a processor of 32 bits) are:

$$X_k = 16807 \cdot X_{k-1} \text{ modulo } (2^{31}-1);$$
$$X_k = [69069 \cdot X_{k-1} + 1] \text{ modulo } 2^{32}. \quad (4)$$

The first variant has a repetition period of $(2^{31}-2)$, and the second has 2^{32}.

III. THE GENERATION OF THE RANDOM NUMBERS WITH KNOWN PARTICULARLY DISTRIBUTION

In this paragraph a generating method of the random numbers sequence, X_k, with a known particularity distribution, is described. It is considered that the analytic form of the repartition function, $F(x)$, of the random variable probability X is known. Because the function codomain $F(x)$ is the interval $[0 ,1]$ and because F is monotone on the entire real axe, we can construct the random variable X starting from a random variable U, which has the distribution constant on the interval $[0, 1]$, using the transformation:

$$X = F^{-1}(U). \quad (5)$$

Thus, generating a number sequence U_k with a constant distribution on the interval $[0, 1]$ (like in previous paragraph) and by applying the relation (5), we obtain the numbers sequence with the distribution $dF(x)/dx$.

A. The Nakagami distribution

The Nakagami probability density has the relation:

$$p_N(x) = \frac{x^{m-1}}{\Gamma(m)} \cdot \left(\frac{m}{\bar{x}}\right)^m \cdot \exp\left(-\frac{m}{\bar{x}} \cdot x\right), \quad x \geq 0, \quad (6)$$

where $\bar{x}$ is the mean value of the variable x, and m is a positive parameter. $\Gamma(m)$ is the gamma function with the expression:

$$\Gamma(m) = \int_0^\infty t^{m-1} \cdot e^{-t} \cdot dt \quad (7)$$

We make the notation:

$$a = m / \bar{x}. \quad (8)$$

The relation (6) become:

$$p_N(x) = \frac{x^{m-1}}{\Gamma(m)} \cdot a^m \cdot e^{-a \cdot x}, \quad x \geq 0. \quad (9)$$

The repartition function of the probability of the Nakagami random variable is obtained by the integration of the relation (9):

$$F_N(x) = \int_0^x p_N(t) \cdot dt = \frac{a^m}{\Gamma(m)} \cdot \int_0^x t^{m-1} \cdot e^{-a \cdot t} \cdot dt =$$

$$= \frac{1}{\Gamma(m)} \cdot \int_0^{a \cdot x} u^{m-1} \cdot e^{-u} \cdot du = \frac{\Gamma_{ax}(m)}{\Gamma(m)}. \quad (10)$$

If m is natural number then the integral is:

$$I_{m-1}(x) = \int_0^x \frac{t^{m-1}}{(m-1)!} \cdot a^m \cdot e^{-a \cdot t} \cdot dt, \quad (11)$$

and it can be written as:

$$I_{m-1}(x) = \frac{t^{m-1}}{(m-1)!} \cdot a^{m-1} \cdot e^{-a \cdot t} \Big|_x^0 + \int_0^x \frac{t^{m-2}}{(m-2)!} a^{m-1} e^{-a \cdot t} dt$$
$$= -\frac{(a \cdot x)^{m-1}}{(m-1)!} \cdot e^{-a \cdot x} + I_{m-2}(x). \quad (12)$$

Using the obtained recurrence we find the relation:

$$F_N(x) = I_{m-1}(x) = -e^{-a \cdot x} \cdot \sum_{k=1}^{m-1} \frac{(a \cdot x)^k}{k!} + I_0(x), \quad (13)$$

with:

$$I_0(x) = \int_0^x a \cdot e^{-a \cdot t} \cdot dt = 1 - e^{-a \cdot x}. \quad (14)$$

Thus:

$$F_N(x) = 1 - e^{-a \cdot x} \cdot \sum_{k=0}^{m-1} \frac{(a \cdot x)^k}{k!}, \quad x \geq 0, \quad m = \text{integer}. \quad (15)$$

Observation: 1) If $m=1$ then $F_N(x) = 1 - e^{-a \cdot x}$ and if $a \cdot x = r^2 / 2\sigma^2$ it is obtained Raylegh distribution.
2) The relation (13) can be written as:

$$F_N(x) = 1 - Q(x) \cdot e^{-a \cdot x}, \quad (16)$$

where $Q(x)$ is un $m-1$ degree polynomial on x.

B. The graphic of the function $F_N(x)$

Obviously, from relation (10), we obtain that $F_N(0) = 0$ and $F_N(\infty) = 1$. Moreover, $dF_N(x)/dx = p_N(x)$ is positive on entire $(0, \infty)$ defintion domaine. $F_N(x)$ represents an inflection point, x_F, gave by the second derivation:

$$\frac{d^2 F_N(x)}{dx^2} = \frac{dp_N(x)}{dx} =$$
$$= \frac{a^m}{\Gamma(m)} \cdot \left[(m-1) \cdot x^{m-2} - a \cdot x^{m-1}\right] \cdot e^{-a \cdot x}, \quad x \geq 0, m \neq 1, \quad (17)$$

with:

$$x_F = \frac{m-1}{a}. \quad (18)$$

The functions graphic $p_N(x)$ and $F_N(x)$, for $m=2, 3, 4$ *and* 5 is shown in Fig. 2.

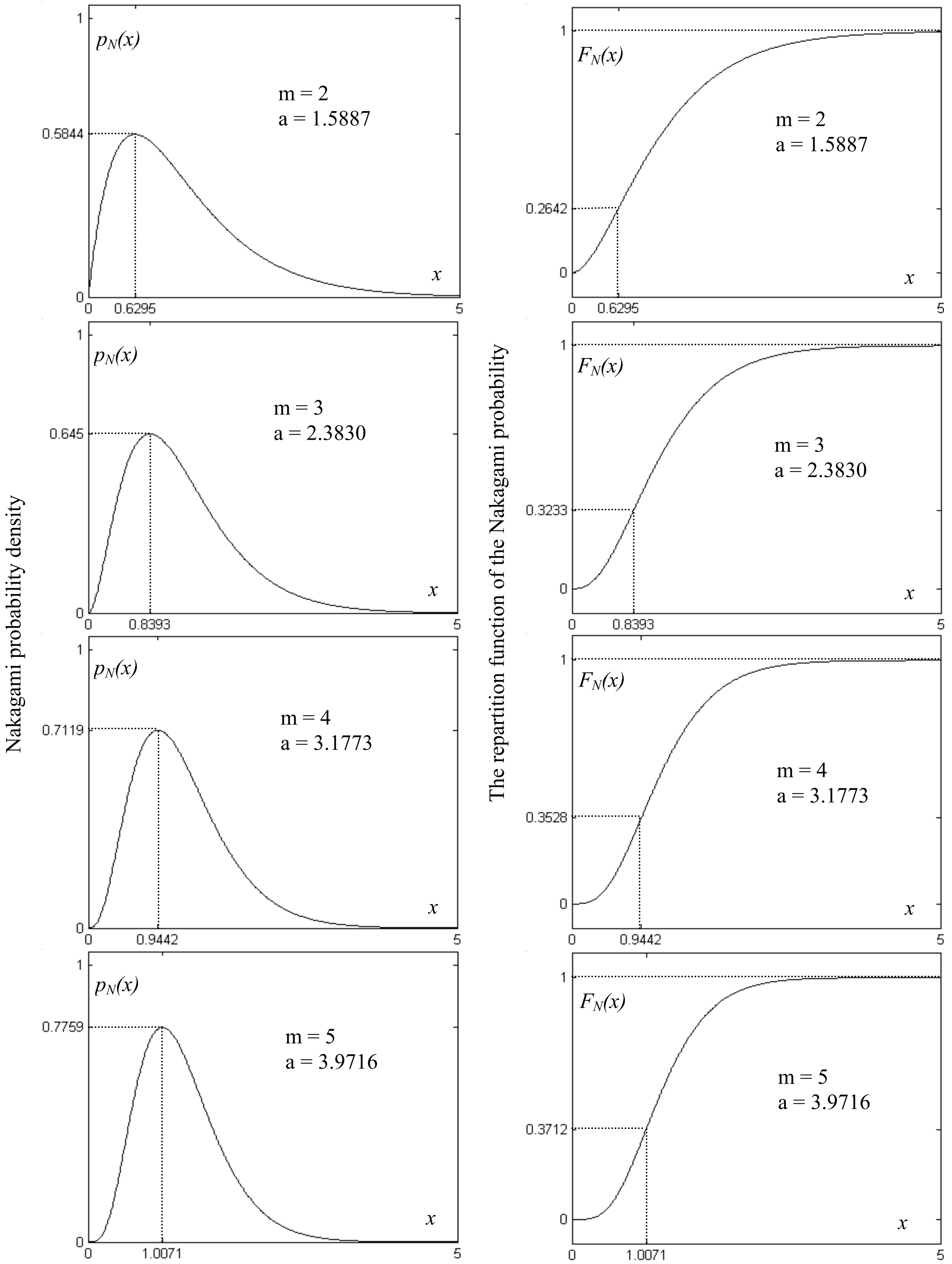

Fig. 2 The probability density and the repartition function of the probability for a random variable distributed Nakagami, with the mean $1.2589=10^{0.1}$.

C. The random numbers generation with Nakagami distribution

Because, for m≠1, the function $F_N(x)$ from relation (10) or relation (13), is not analytical inversable, a relation with the form $x = F_N^{-1}(u)$ can be find only through aproximations. An algorithm to obtain this aproximation, with $u \in [0, 1]$, is described on the following:

Step 1. It is generate $u \in [0, 1]$, a realisation of the uniform distribution random variable;

Step 2. It is computed a, x_F, $p_N(x_F)$ and $F_N(x_F)$, knowing m and $\bar{x}$;

Step 3. It is computed x_1 with the relation:

$$(x_1 - x_F) \cdot p_N(x_F) = u - F_N(x_F), \quad (19)$$

and it results :

$$x_1 = x_F + \frac{u - F_N(x_F)}{p_N(x_F)}; \quad (20)$$

Step 4. It is computed $F_N(x_1)$ which is compared with u. If:

$$|F_N(x_1) - u| < \varepsilon_o, \quad (21)$$

where ε_o is the desired precision on aproximation, then the algorithm is stoped. If the relation (21) it is not satisfied, then steps *3* and *4* are repeated, replacing x_1 with x_2 and x_F with x_1. This generating process is shown in Fig. 3. We considered the values: $m=2$, $\bar{x}=10^{0.1}=1.2589$, $u=0.8$, $c=10^{-3}$, obtained succesive $x_F=0.629463$, $x_1=1.546177$, $x_2=1.834194$, $x_3=1.883486$, $F_N(x_3)=0.799686$, $p_N(x_3)=0.238518$.

Observation: The algorithm presented before, in the case where m is not natural, impose the computing of two integrals, with the values $\Gamma_{ax}(m)$ and $\Gamma(m)$. These also can results through a numerical approximation (Riemann integrals). If m is natural, then relation (13) gives an analytic relation, computable, of the $F_N(x_k)$ values.

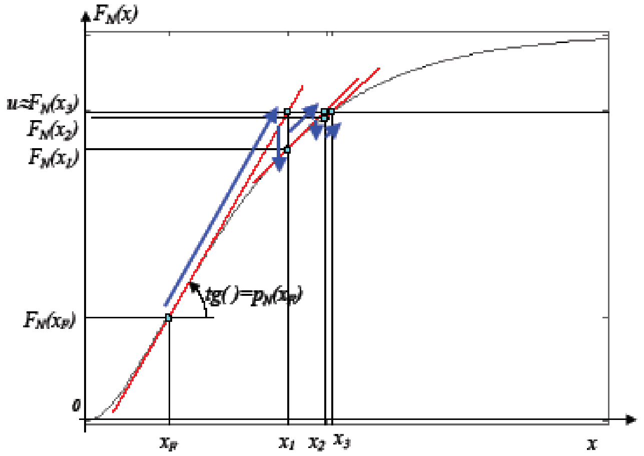

Fig. 3 The generating algorithm ilustration of the Nakagami distribution random variable

IV. EXPERIMENTAL RESULTS

The Nakagami flat fading channels, with the over unitary values of the parameter m, have the performances comprised between the Rayleigh flat fading performances (which correspond to the Nakagami flat fading channel with $m=1$) and no-fading channels (which correspond to the Nakagami flat fading channel with $m=\infty$) performances. The BER (SNR) curves resulted from the simulation of the Nakagami flat fading channel, for different values of the m parameter, are presented on the Fig. 4. A BPSK modulation and AWGN noise was used. Thus, w_k, from relation (1) is a normally distributed random variable with mean zero and variance equal to:

$$\overline{w_k^2} = \frac{1}{2 \cdot 10^{SNR/10}}, \quad (22)$$

where $SNR=E_b/N_0$ (dB), and:

$$\alpha_k = \sqrt{\frac{\gamma}{SNR}}, \quad (23)$$

where γ is a Nakagami distributed random variable. The Rayleigh type fading is a particular case of the Nakagami type, for $m=1$. When the m parameter grows the corresponding BER curves are more abrupt, at the limit, when $m \to \infty$ we obtain no-fading (AWGN) channel.

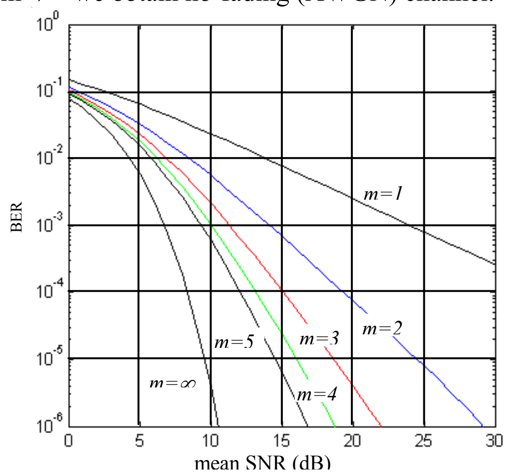

Fig. 4 The BER performances obtained through the simulation of the BPSK modulation transmission system on the Nakagami flat fading channels, with $m=2, 3, 4$ and 5.

V. CONCLUSIONS

In this paper, a model for the simulation of the Nakagami flat fading channel and the construction way of the random variables, which are implied in this model, are presented. We proposed a new method to generate random sequences with Nakagami distribution. The curves BER obtained with this model are identical with the curves that exist on literature, [3].

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