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# The Performances of Convolutional Codes used in Turbo Codes

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**Abstract – In this paper are presented and compared the BER performances obtained by the simulation of a transmission system, which utilizes the forward error correcting by codes concatenation and iterative decoding (turbo coding). There have been investigated all the systematic convolutional codes having the constraint length K less or equal to 6, under three diffrent concatenated forms: parallel PCCC (pure turbo code), serial SCCC and hybrid HCCC, at the following rates: 1/3, 1/4 and 1/3, respectively, all unpunctured.**

**Two interleaver types were used: pseudo-random and S-interleaver having the same length, 1784.**

**The AWGN channel and the BPSK modulation were employed.**

**The used iteration number was eight. For increasing the work speed an iterations stop criterion was used. When the resulting error number from the decoding of a data block is zero, the remaining iterations are not effectuated, passing to the next block.**

**For decoding, the MAP, MaxLogMAP and Log MAP algorithms were used. In all the cases, a tail off was employed for the first code, with the decreasing transmission rate price.**

**The transmitted data block numbers for a simulation were chosen in function of the signal to noise ratio, SNR, i.e. to keep a good precision for obtained curve.**

**Keywords: convolutional codes, turbo codes, interleaver, iteration.**

## I. INTRODUCTION

Two procedures which improve the performances of convolutional codes, CCs, (and block codes), from point of view of error rate (BER), are concatenation and iterative decoding.

Fig.1 illustrates the possible ways of convolutional codes concatenation. Concatenated convolutional schemes tend to fall into three categories: parallel concatenated convolutional codes, PCCCs, (as in Fig.1 a)), serial concatenated convolutional codes, SCCC, (as in Fig.1 b)) and hybrid concatenated convolutional codes, HCCC, (as in Fig.1 c)).

The PCCC (or Turbo code) was introduced by Berrou *et al.* [1], in 1993, and it was the beginning of turbo code era.

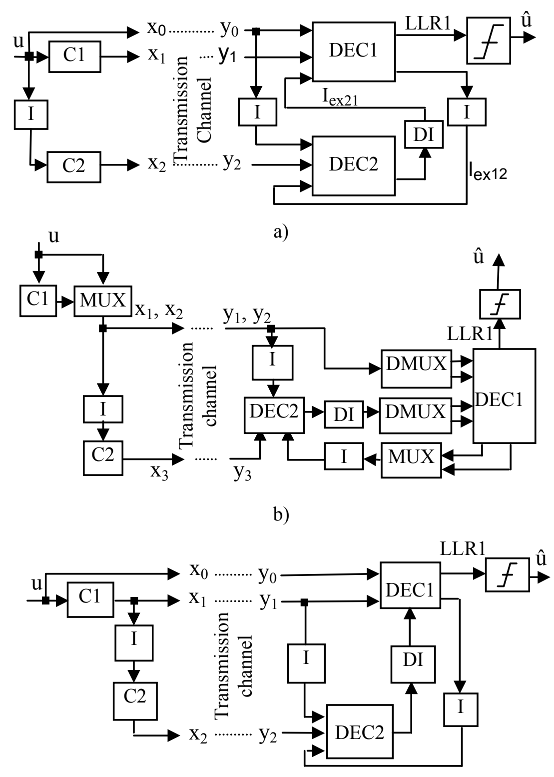


Fig.1 a) Parallel concatenated convolutional codes, b) Serial concatenated convolutional codes, c) Hybrid concatenated convolutional codes

Blocks I and DI realize interleaving and deinterleaving functions. We used in this paper two interleavers: pseudo-random [2] and S-interleaver [3]. Our simulations prove that the interleavers have an essential influence on performances of Turbo codes.

DEC1 and DEC2 are iterative decoder blocks [4], which implement algorithms like: the MAP algorithm, the first and the must important, proposed by Bahl and *al.* [5], the MAXLogMAP algorithm [6], and the Log MAP algorithm, proposed by Robertson and *al.* [7].

The constitutive codes can be convolutional codes or block codes. In this paper we studied the first exclusively. The general scheme of a recursive

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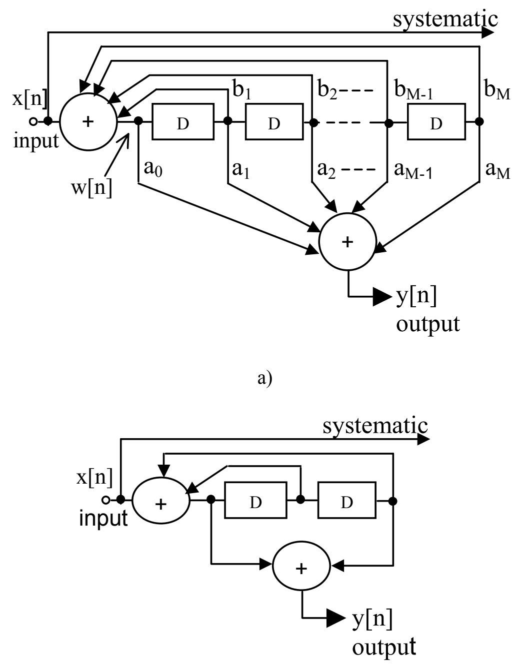
systematic convolutional code, RSC, is shown in Fig. 2 a) and an example of RSC is shown in Fig. 2 b).




a)

b)

Fig.2 Recursive systematic convolutional code: a) general scheme, b) example

The following ecuations result:

$$y[n]=\sum_{k=0}^{M} a_k w[n-k] \quad (1)$$

$$w[n]=\sum_{k=0}^{M} b_k w[n-k]+x[n] \quad (2)$$

$$Y(D)=\left(\sum_{k=0}^{M} a_k D^k\right)\cdot W(D) \quad (3)$$

$$W(D)\cdot\sum_{k=0}^{M} b_k D^k = X(D) \quad (4)$$

$$\frac{Y(D)}{X(D)}=\frac{\sum_{k=0}^{M} a_k D^k}{\sum_{k=0}^{M} b_k D^k} \quad (5)$$

The code generator matrix, G(D), is:

$$G(D)=\left[1,\frac{a(D)}{b(D)}\right] \quad (6)$$

The two polynomials attached, *a* and *b*, define in totality the convolutional code. The maximum degree of *a* and *b* polynomials give the coder's memory (equal with K-1) and it is a measure of the complexity and of the volume of the effectuated computation of each component decoder. It increases exponentially with K or M [6].

Practically, convolutional codes with K=3÷6 are used. Table 1 shows the generator polynomials with degree inferior or equal with 5 which can be selected like *a* or *b*. Because of the restriction that *a* and *b* to be prime the table shows also the possible divisors for each polynomial.

Table1

| Degree | Ireductibil | Primitive | Polynomials in octal and their divisors | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | * | | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | * | * | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | | | 5 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| | * | * | 7 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | | | 11 | 1 | 3 | 7 | 0 | 0 | 0 | 0 | 0 |
| | * | * | 13 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | * | * | 15 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 17 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | | | 21 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| | * | * | 23 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 25 | 1 | 0 | 7 | 0 | 0 | 0 | 0 | 0 |
| | | | 27 | 1 | 3 | 0 | 0 | 15 | 0 | 0 | 0 |
| | * | * | 31 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 33 | 1 | 3 | 7 | 0 | 0 | 0 | 0 | 0 |
| | | | 35 | 1 | 3 | 0 | 13 | 0 | 0 | 0 | 0 |
| | * | | 37 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | | | 41 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 37 |
| | | | 43 | 1 | 0 | 7 | 0 | 15 | 0 | 0 | 0 |
| | * | * | 45 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 47 | 1 | 3 | 0 | 13 | 0 | 0 | 0 | 0 |
| | * | * | 51 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 53 | 1 | 3 | 0 | 0 | 0 | 0 | 31 | 0 |
| | | | 55 | 1 | 3 | 7 | 0 | 0 | 0 | 0 | 0 |
| | * | * | 57 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 61 | 1 | 0 | 7 | 13 | 0 | 0 | 0 | 0 |
| | | | 63 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 65 | 1 | 3 | 0 | 0 | 0 | 23 | 0 | 0 |
| | * | * | 67 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 71 | 1 | 3 | 0 | 0 | 15 | 0 | 0 | 0 |
| | * | * | 73 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | * | * | 75 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | 77 | 1 | 3 | 7 | 0 | 0 | 0 | 0 | 0 |

## II. EXPERIMENTAL RESULTS

Table 2 and Table 3 present the simulation results obtained with parallel concatenated convolutional code, Fig.1 a), rate R=1/3, RSC, with the pseudo-random interleaver (table 2), and S-interleaver (table 3).

Table 2 (pseudo-random interleaver)

| | 1 | 3 | 5 | 7 | 11 | 13 | 15 | 17 | 21 | 23 | 25 | 27 | 31 | 33 | 35 | 37 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2954035 | 3021300 | 1935725 | 2864349 | 1188340 | 1177130 | 1093049 | 3157698 | 1073326 | 1821748 | 710482 | 1242932 | 585061 | 560538 | 704876 |
| 3 | 3612356 | 0 | 0 | 1358637 | 0 | 527925 | 498583 | 0 | 0 | 243060 | 156083 | 0 | 318487 | 0 | 0 | 392376 |
| 5 | 4516335 | 0 | 0 | 100348 | 0 | 61305 | 62825 | 0 | 0 | 5311 | 1471412 | 0 | 5259 | 0 | 0 | 20042 |
| 7 | 1262331 | 919632 | 5710 | 0 | 0 | 5163 | 6663 | 378923 | 1419 | 1445 | 0 | 8216 | 1916 | 0 | 5840 | 289992 |
| 11 | 4684497 | 0 | 0 | 0 | 0 | 37148 | 37295 | 0 | 0 | 47924 | 0 | 0 | 37591 | 0 | 0 | 437 |
| 13 | 4684497 | 33594 | 6755 | 7043 | 1720 | 0 | 312 | 292 | 220 | 3144 | 243 | 8839 | 265 | 1192 | 0 | 155 |
| 15 | 108671 | 34188 | 9633 | 6457 | 1294 | 235 | 0 | 403 | 204 | 579 | 313 | 0 | 2910 | 1077 | 7302 | 229 |
| 17 | 445882 | 0 | 0 | 230633 | 0 | 9302 | 9040 | 0 | 0 | 8749 | 1741 | 0 | 10009 | 0 | 0 | 249017 |
| 21 | 4886023 | 0 | 0 | 13579 | 0 | 11594 | 11975 | 0 | 0 | 176919 | 159130 | 0 | 173697 | 0 | 0 | 8037 |
| 23 | 178171 | 20128 | 1102 | 521 | 2061 | 1629 | 230 | 691 | 6165 | 0 | 578 | 476 | 1032 | 1283 | 3005 | 586 |
| 25 | 1420029 | 5267 | 966445 | 0 | 0 | 751 | 1159 | 926 | 86082 | 34388 | 0 | 3327 | 1676 | 0 | 1904 | 1999 |
| 27 | 21498 | 0 | 0 | 6201 | 0 | 12060 | 0 | 0 | 0 | 345 | 1105 | 0 | 373 | 0 | 0 | 546 |
| 31 | 429482 | 6548 | 1855 | 600 | 2888 | 418 | 1823 | 344 | 10420 | 3162 | 207 | 828 | 0 | 776 | 534 | 1443 |
| 33 | 10949 | 0 | 0 | 0 | 0 | 2602 | 2471 | 0 | 0 | 417 | 0 | 0 | 450 | 0 | 0 | 1374 |
| 35 | 10609 | 0 | 0 | 6446 | 0 | 0 | 9675 | 0 | 0 | 241 | 558 | 0 | 368 | 0 | 0 | 270 |
| 37 | 135016 | 123269 | 1379 | 114297 | 1445 | 236 | 449 | 104996 | 956 | 757 | 10480 | 1418 | 737 | 901 | 731 | 0 |
| 41 | 4107943 | 0 | 0 | 2430 | 0 | 1991 | 1651 | 0 | 0 | 2242 | 2031 | 0 | 1643 | 0 | 0 | 0 |
| 43 | 424030 | 8621 | 7204 | 0 | 0 | 456 | 0 | 2488 | 6217 | 551 | 0 | 0 | 2140 | 0 | 6606 | 3117 |
| 45 | 299338 | 13935 | 5521 | 1638 | 3316 | 901 | 555 | 6442 | 1873 | 12670 | 4607 | 854 | 3971 | 5999 | 4711 | 13557 |
| 47 | 38916 | 0 | 0 | 10547 | 0 | 0 | 3771 | 0 | 0 | 1679 | 9612 | 0 | 13089 | 0 | 0 | 6179 |
| 51 | 205797 | 7216 | 6101 | 1502 | 3468 | 678 | 630 | 2024 | 1006 | 3910 | 5735 | 3119 | 17711 | 5515 | 3467 | 15763 |
| 53 | 9846 | 0 | 0 | 599 | 0 | 2125 | 6769 | 0 | 0 | 962 | 681 | 0 | 0 | 0 | 0 | 1890 |
| 55 | 16898 | 0 | 0 | 0 | 0 | 3796 | 3708 | 0 | 0 | 1940 | 0 | 0 | 13015 | 0 | 0 | 16663 |
| 57 | 5160 | 1513 | 2592 | 2879 | 913 | 748 | 4673 | 879 | 4271 | 1351 | 1768 | 8903 | 30764 | 20257 | 3587 | 2084 |
| 61 | 510340 | 12737 | 2421 | 0 | 0 | 0 | 607 | 721 | 6592 | 4919 | 0 | 18487 | 744 | 0 | 0 | 5890 |
| 63 | 125731 | 0 | 0 | 38990 | 0 | 29225 | 38412 | 0 | 0 | 39576 | 43539 | 0 | 36760 | 0 | 0 | 38388 |
| 65 | 28245 | 0 | 0 | 698 | 0 | 2550 | 3709 | 0 | 0 | 0 | 2436 | 0 | 2649 | 0 | 0 | 1497 |
| 67 | 2075 | 8583 | 2910 | 2383 | 8190 | 355 | 2117 | 607 | 22152 | 26281 | 20200 | 1009 | 2018 | 1191 | 3883 | 11581 |
| 71 | 21641 | 0 | 0 | 7941 | 0 | 18221 | 0 | 0 | 0 | 4869 | 8983 | 0 | 945 | 0 | 0 | 2437 |
| 73 | 2587 | 6523 | 3030 | 4425 | 12835 | 5061 | 471 | 313 | 114070 | 997 | 16231 | 1540 | 12697 | 977 | 424 | 3386 |
| 75 | 9336 | 4653 | 11238 | 1249 | 968 | 12662 | 961 | 2685 | 6851 | 104145 | 2325 | 7577 | 781 | 16290 | 1835 | 450 |
| 77 | 94043 | 0 | 0 | 0 | 0 | 1404 | 2169 | 0 | 0 | 1751 | 0 | 0 | 2123 | 0 | 0 | 73523 |

| | 41 | 43 | 45 | 47 | 51 | 53 | 55 | 57 | 61 | 63 | 65 | 67 | 71 | 73 | 75 | 77 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2558092 | 1291785 | 1193479 | 523168 | 1207492 | 480732 | 644618 | 277521 | 1286689 | 589732 | 508488 | 246833 | 586229 | 251741 | 312032 | 409436 |
| 3 | 0 | 145046 | 50884 | 0 | 41916 | 0 | 0 | 31688 | 119502 | 0 | 0 | 137957 | 0 | 139590 | 32654 | 0 |
| 5 | 0 | 6347 | 50865 | 0 | 52682 | 0 | 0 | 20946 | 5037 | 0 | 0 | 1560 | 0 | 1861 | 23048 | 0 |
| 7 | 348 | 0 | 262 | 13007 | 216 | 329 | 0 | 737 | 0 | 257 | 363 | 18076 | 11557 | 14284 | 351 | 0 |
| 11 | 0 | 0 | 43879 | 0 | 44683 | 0 | 0 | 401 | 0 | 0 | 0 | 416 | 0 | 445 | 662 | 0 |
| 13 | 7029 | 416 | 199 | 0 | 250 | 1679 | 1008 | 195 | 0 | 392 | 242 | 153 | 215 | 1689 | 265 | 424 |
| 15 | 1396 | 0 | 198 | 122 | 151 | 482 | 1639 | 499 | 370 | 282 | 1975 | 2524 | 0 | 466 | 235 | 359 |
| 17 | 0 | 8663 | 1597 | 0 | 5878 | 0 | 0 | 8822 | 9446 | 0 | 0 | 8446 | 0 | 8800 | 9174 | 0 |
| 21 | 0 | 155602 | 9816 | 0 | 9225 | 0 | 0 | 1372 | 159927 | 0 | 0 | 14240 | 0 | 12664 | 3965 | 0 |
| 23 | 1917 | 254 | 616 | 140 | 17637 | 261 | 604 | 368 | 3818 | 1717 | 0 | 1465 | 13100 | 471 | 14498 | 2881 |
| 25 | 1267 | 0 | 865 | 15090 | 359 | 2107 | 0 | 2429 | 0 | 4654 | 2191 | 17575 | 15184 | 9580 | 1931 | 0 |
| 27 | 0 | 0 | 586 | 0 | 1273 | 0 | 0 | 4623 | 23549 | 0 | 0 | 297 | 0 | 585 | 13015 | 0 |
| 31 | 6477 | 3924 | 21692 | 5640 | 332 | 0 | 490 | 27704 | 250 | 1406 | 185 | 379 | 253 | 3520 | 365 | 14592 |
| 33 | 0 | 0 | 13682 | 0 | 16959 | 0 | 0 | 6718 | 0 | 0 | 0 | 1994 | 0 | 2053 | 2180 | 0 |
| 35 | 0 | 12497 | 820 | 0 | 265 | 0 | 0 | 15250 | 0 | 0 | 0 | 547 | 0 | 251 | 3434 | 0 |
| 37 | 0 | 617 | 6125 | 912 | 13870 | 3399 | 21406 | 885 | 723 | 229 | 5265 | 1037 | 837 | 972 | 909 | 90250 |
| 41 | 0 | 52386 | 51113 | 0 | 54221 | 0 | 0 | 3930 | 44754 | 0 | 0 | 74738 | 0 | 11696 | 4972 | 0 |
| 43 | 12883 | 0 | 7156 | 5074 | 4128 | 6676 | 0 | 2928 | 0 | 8412 | 18832 | 18168 | 0 | 14370 | 14586 | 0 |
| 45 | 7550 | 7623 | 0 | 4591 | 23549 | 16302 | 4950 | 16167 | 4498 | 37402 | 2142 | 19979 | 28922 | 17354 | 20076 | 24917 |
| 47 | 0 | 29280 | 13511 | 0 | 24500 | 0 | 0 | 13531 | 0 | 0 | 0 | 13803 | 0 | 16293 | 113221 | 0 |
| 51 | 8407 | 4568 | 39701 | 11210 | 0 | 525 | 12873 | 24937 | 3149 | 14515 | 129624 | 24140 | 3039 | 15473 | 27847 | 69808 |
| 53 | 0 | 11421 | 16531 | 0 | 4511 | 0 | 0 | 23816 | 149866 | 0 | 0 | 13064 | 0 | 9364 | 7155 | 0 |
| 55 | 0 | 0 | 8543 | 0 | 6700 | 0 | 0 | 21371 | 0 | 0 | 0 | 11158 | 0 | 14395 | 11214 | 0 |
| 57 | 17131 | 27777 | 26594 | 1752 | 14839 | 10324 | 8576 | 0 | 23236 | 15364 | 10750 | 15603 | 6537 | 6747 | 23012 | 6439 |
| 61 | 10064 | 0 | 5965 | 0 | 2505 | 23247 | 0 | 35109 | 0 | 3933 | 3802 | 9639 | 3637 | 89620 | 14001 | 0 |
| 63 | 0 | 7429 | 35644 | 0 | 29862 | 0 | 0 | 19276 | 19300 | 0 | 0 | 6196 | 0 | 14240 | 18842 | 0 |
| 65 | 0 | 113080 | 5937 | 0 | 20680 | 0 | 0 | 14321 | 7637 | 0 | 0 | 19730 | 0 | 14989 | 4491 | 0 |
| 67 | 12965 | 25181 | 13423 | 15669 | 12916 | 7387 | 7400 | 13738 | 3109 | 13093 | 9074 | 0 | 6612 | 2022 | 20297 | 30407 |
| 71 | 0 | 0 | 15047 | 0 | 13372 | 0 | 0 | 14385 | 5929 | 0 | 0 | 14642 | 0 | 12603 | 7323 | 0 |
| 73 | 12240 | 1979 | 40387 | 16590 | 24624 | 14869 | 5216 | 19117 | 35211 | 7683 | 12892 | 3100 | 4207 | 0 | 6536 | 93671 |
| 75 | 15321 | 20015 | 23252 | 40156 | 9457 | 106484 | 6020 | 112287 | 78940 | 18715 | 2690 | 26072 | 2081 | 9212 | 0 | 18490 |
| 77 | 0 | 0 | 33231 | 0 | 26997 | 0 | 0 | 1144 | 0 | 0 | 0 | 1291 | 0 | 1185 | 1035 | 0 |

We used an AWGN noise and a BPSK modulation. All the simulations were made for signal/noise ratio equal with 1 dB and for a number of 500 errors, at least.

An iterations stop criterion was used for each decoder. When the resulting errors number for a data block is zero, the remaining iterations are not effectuated, passing to the next block.

The tables contain bit error rate ($BER\cdot 10^8$) obtained for each polynomial pair indicated in octal, at the beginning of each row (the denominator, b(D), from relation 6), or column (the numerator, a(D), from relation 6).

Table 3 (S- interleaver)

| | 1 | 3 | 5 | 7 | 11 | 13 | 15 | 17 | 21 | 23 | 25 | 27 | 31 | 33 | 35 | 37 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2782307 | 2943825 | 1786715 | 3066143 | 1207492 | 1339953 | 1123232 | 2853139 | 1250243 | 1811238 | 611821 | 1159192 | 575409 | 691787 | 657002 |
| 3 | 3713565 | 0 | 0 | 1513452 | 0 | 480592 | 586229 | 0 | 0 | 188714 | 158108 | 0 | 235426 | 0 | 0 | 515695 |
| 5 | 4532351 | 0 | 0 | 38628 | 0 | 54530 | 45274 | 0 | 0 | 2482 | 1401345 | 0 | 3080 | 0 | 0 | 11095 |
| 7 | 1040109 | 1022982 | 3436 | 0 | 0 | 2994 | 2956 | 305910 | 832 | 817 | 0 | 6151 | 850 | 0 | 4025 | 241462 |
| 11 | 3979820 | 0 | 0 | 0 | 0 | 36253 | 41956 | 0 | 0 | 41470 | 0 | 0 | 35644 | 0 | 0 | 477 |
| 13 | 3979820 | 24035 | 5447 | 4684 | 3999 | 0 | 143 | 242 | 167 | 763 | 185 | 1311 | 479 | 182 | 0 | 100 |
| 15 | 118335 | 36556 | 7559 | 4703 | 1166 | 267 | 0 | 185 | 266 | 738 | 145 | 0 | 707 | 401 | 1421 | 228 |
| 17 | 394618 | 0 | 0 | 162101 | 0 | 332 | 365 | 0 | 0 | 302 | 3075 | 0 | 307 | 0 | 0 | 193675 |
| 21 | 4155989 | 0 | 0 | 594 | 0 | 781 | 830 | 0 | 0 | 42699 | 46773 | 0 | 48925 | 0 | 0 | 10501 |
| 23 | 186127 | 5812 | 539 | 658 | 1333 | 752 | 144 | 333 | 24686 | 0 | 313 | 743 | 6509 | 1428 | 2622 | 794 |
| 25 | 1198150 | 2598 | 1069026 | 0 | 0 | 539 | 162 | 640 | 2893 | 4516 | 0 | 731 | 2887 | 0 | 908 | 758 |
| 27 | 7770 | 0 | 0 | 2034 | 0 | 1874 | 0 | 0 | 0 | 258 | 732 | 0 | 506 | 0 | 0 | 432 |
| 31 | 180019 | 6177 | 1006 | 568 | 2092 | 220 | 939 | 501 | 5303 | 5826 | 437 | 1721 | 0 | 945 | 439 | 1285 |
| 33 | 5135 | 0 | 0 | 0 | 0 | 1273 | 1560 | 0 | 0 | 302 | 0 | 0 | 252 | 0 | 0 | 1161 |
| 35 | 9702 | 0 | 0 | 2503 | 0 | 0 | 3214 | 0 | 0 | 460 | 487 | 0 | 439 | 0 | 0 | 324 |
| 37 | 102566 | 114577 | 322 | 76369 | 275 | 385 | 317 | 81253 | 543 | 397 | 4446 | 565 | 421 | 570 | 404 | 0 |
| 41 | 4212043 | 0 | 0 | 1934 | 0 | 1139 | 824 | 0 | 0 | 639 | 1355 | 0 | 903 | 0 | 0 | 0 |
| 43 | 186387 | 5332 | 2908 | 0 | 0 | 1305 | 0 | 2480 | 9810 | 550 | 0 | 0 | 1105 | 0 | 15923 | 6397 |
| 45 | 359028 | 10573 | 6894 | 2182 | 7325 | 413 | 737 | 2858 | 1180 | 2040 | 3133 | 2808 | 4826 | 15890 | 3769 | 14232 |
| 47 | 9265 | 0 | 0 | 13877 | 0 | 0 | 2067 | 0 | 0 | 992 | 9972 | 0 | 7480 | 0 | 0 | 9044 |
| 51 | 228619 | 14099 | 4668 | 2885 | 2697 | 1039 | 1665 | 9634 | 1735 | 8191 | 3568 | 3668 | 8333 | 13754 | 1056 | 13976 |
| 53 | 12047 | 0 | 0 | 1625 | 0 | 8100 | 2027 | 0 | 0 | 777 | 1633 | 0 | 0 | 0 | 0 | 2386 |
| 55 | 29230 | 0 | 0 | 0 | 0 | 4696 | 6396 | 0 | 0 | 3125 | 0 | 0 | 1994 | 0 | 0 | 6292 |
| 57 | 8908 | 4020 | 1333 | 2419 | 612 | 1843 | 19325 | 949 | 4723 | 739 | 16047 | 1396 | 71654 | 16720 | 4934 | 4592 |
| 61 | 227980 | 4904 | 2968 | 0 | 0 | 0 | 846 | 703 | 6697 | 2943 | 0 | 10200 | 291 | 0 | 0 | 3813 |
| 63 | 10090 | 0 | 0 | 263 | 0 | 417 | 384 | 0 | 0 | 307 | 167 | 0 | 190 | 0 | 0 | 387 |
| 65 | 8205 | 0 | 0 | 817 | 0 | 2784 | 2383 | 0 | 0 | 0 | 1867 | 0 | 2581 | 0 | 0 | 1138 |
| 67 | 3536 | 4241 | 1694 | 2789 | 12315 | 190 | 2025 | 287 | 28519 | 130035 | 26804 | 1066 | 648 | 1183 | 1525 | 2540 |
| 71 | 6549 | 0 | 0 | 2588 | 0 | 2093 | 0 | 0 | 0 | 2634 | 2889 | 0 | 1310 | 0 | 0 | 24800 |
| 73 | 3793 | 12398 | 1442 | 43816 | 15414 | 2714 | 672 | 788 | 33805 | 1276 | 113225 | 12719 | 19350 | 2428 | 714 | 3802 |
| 75 | 12376 | 2481 | 3854 | 2155 | 1844 | 10375 | 687 | 377 | 11232 | 19005 | 8164 | 5724 | 329 | 15031 | 934 | 2206 |
| 77 | 70523 | 0 | 0 | 0 | 0 | 2842 | 2491 | 0 | 0 | 5198 | 0 | 0 | 3437 | 0 | 0 | 48073 |

| | 41 | 43 | 45 | 47 | 51 | 53 | 55 | 57 | 61 | 63 | 65 | 67 | 71 | 73 | 75 | 77 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2993273 | 1207492 | 1223435 | 556141 | 1188807 | 549547 | 717488 | 262103 | 1134529 | 583417 | 583417 | 260213 | 494650 | 250240 | 333025 | 367943 |
| 3 | 0 | 131135 | 31141 | 0 | 28824 | 0 | 0 | 39378 | 117391 | 0 | 0 | 104040 | 0 | 102492 | 33592 | 0 |
| 5 | 0 | 3172 | 41831 | 0 | 40679 | 0 | 0 | 9642 | 3278 | 0 | 0 | 358 | 0 | 727 | 8419 | 0 |
| 7 | 276 | 0 | 127 | 9802 | 74 | 197 | 0 | 373 | 0 | 258 | 125 | 8786 | 8039 | 11305 | 363 | 0 |
| 11 | 0 | 0 | 41854 | 0 | 49803 | 0 | 0 | 508 | 0 | 0 | 0 | 485 | 0 | 519 | 569 | 0 |
| 13 | 1649 | 83 | 80 | 0 | 61 | 386 | 473 | 127 | 0 | 147 | 935 | 196 | 121 | 396 | 193 | 287 |
| 15 | 1299 | 0 | 286 | 94 | 73 | 1313 | 499 | 208 | 186 | 128 | 518 | 350 | 0 | 342 | 122 | 575 |
| 17 | 0 | 333 | 532 | 0 | 533 | 0 | 0 | 335 | 335 | 0 | 0 | 488 | 0 | 782 | 402 | 0 |
| 21 | 0 | 45040 | 1133 | 0 | 588 | 0 | 0 | 1887 | 35638 | 0 | 0 | 1074 | 0 | 2472 | 2761 | 0 |
| 23 | 11315 | 233 | 1940 | 136 | 25743 | 437 | 1122 | 391 | 3591 | 2305 | 0 | 2710 | 4540 | 330 | 15135 | 2778 |
| 25 | 1331 | 0 | 538 | 13856 | 204 | 1010 | 0 | 1071 | 0 | 15702 | 878 | 13606 | 19005 | 9179 | 708 | 0 |
| 27 | 0 | 0 | 202 | 0 | 752 | 0 | 0 | 982 | 16096 | 0 | 0 | 520 | 0 | 249 | 152923 | 0 |
| 31 | 13931 | 2866 | 13021 | 16983 | 425 | 0 | 600 | 69932 | 269 | 1091 | 217 | 286 | 167 | 13009 | 207 | 4345 |
| 33 | 0 | 0 | 7457 | 0 | 14444 | 0 | 0 | 17526 | 0 | 0 | 0 | 919 | 0 | 1416 | 1690 | 0 |
| 35 | 0 | 23136 | 258 | 0 | 336 | 0 | 0 | 7477 | 0 | 0 | 0 | 218 | 0 | 335 | 2291 | 0 |
| 37 | 0 | 731 | 5433 | 390 | 22609 | 3707 | 14937 | 376 | 331 | 384 | 4299 | 1593 | 534 | 923 | 798 | 58997 |
| 41 | 0 | 36471 | 47788 | 0 | 40759 | 0 | 0 | 30444 | 46317 | 0 | 0 | 18534 | 0 | 401227 | 6071 | 0 |
| 43 | 9174 | 0 | 2550 | 2094 | 1518 | 4200 | 0 | 4059 | 0 | 5968 | 20458 | 20065 | 0 | 444 | 15899 | 0 |
| 45 | 10600 | 14247 | 0 | 23325 | 26952 | 22341 | 4008 | 22238 | 5201 | 31271 | 3309 | 16184 | 8513 | 98094 | 24113 | 10032 |
| 47 | 0 | 2094 | 5453 | 0 | 16476 | 0 | 0 | 3385 | 0 | 0 | 0 | 4066 | 0 | 30534 | 26549 | 0 |
| 51 | 20778 | 1979 | 25027 | 13750 | 0 | 2564 | 4858 | 21800 | 8835 | 52297 | 15758 | 99546 | 3659 | 14286 | 16219 | 68429 |
| 53 | 0 | 28429 | 44149 | 0 | 3700 | 0 | 0 | 4375 | 206608 | 0 | 0 | 14589 | 0 | 15480 | 13301 | 0 |
| 55 | 0 | 0 | 16959 | 0 | 14568 | 0 | 0 | 5918 | 0 | 0 | 0 | 15839 | 0 | 923983 | 6148 | 0 |
| 57 | 22967 | 36629 | 28408 | 616 | 8852 | 1526 | 3734 | 0 | 6688 | 29861 | 39126 | 15876 | 27662 | 17594 | 32804 | 18244 |
| 61 | 13068 | 0 | 3898 | 0 | 2653 | 23247 | 0 | 32727 | 0 | 2570 | 3929 | 3518 | 686 | 64571 | 4821 | 0 |
| 63 | 0 | 19832 | 53766 | 0 | 30534 | 0 | 0 | 8299 | 3021 | 0 | 0 | 7494 | 0 | 13151 | 30265 | 0 |
| 65 | 0 | 65910 | 14435 | 0 | 26699 | 0 | 0 | 8136 | 10792 | 0 | 0 | 33623 | 0 | 39829 | 3974 | 0 |
| 67 | 61580 | 63321 | 111671 | 5047 | 66312 | 7881 | 24005 | 29670 | 21168 | 10619 | 13226 | 0 | 18438 | 1958 | 19668 | 16879 |
| 71 | 0 | 0 | 27579 | 0 | 12537 | 0 | 0 | 192134 | 7742 | 0 | 0 | 22430 | 0 | 20300 | 5443 | 0 |
| 73 | 27606 | 3694 | 31173 | 17325 | 108604 | 14879 | 4281 | 17458 | 122750 | 99527 | 2136 | 3568 | 14395 | 0 | 61241 | 25116 |
| 75 | 24360 | 10319 | 17105 | 15974 | 18931 | 12671 | 2340 | 119031 | 27355 | 24284 | 3416 | 22923 | 1731 | 27310 | 0 | 48372 |
| 77 | 0 | 0 | 27526 | 0 | 27316 | 0 | 0 | 64 | 0 | 0 | 0 | 1055 | 0 | 1720 | 582 | 0 |

The zeros in the tables contain mark that the respective codes have common divisors (see Table 1) and can not be used together. With little exceptions, the results obtained with S-interleaver are superior to the results corresponding the pseudo-random interleaver. We also remark the superior results obtained with both interleavers in the case of the use at denominator, b(D), of the primitive polynomials, comparing to non-primitive polynomials, at the same constraint length. Despite of the fact that global performances increase proportionally with K, it must be remarked the performances obtained using, for the Feed Back loop, the polynomials $b_1=7=111$, $b_2=13=1011$ and $b_3=15=1101$ as in combination with the polynomials with the same degree (ex. 15/13 or 13/15) as in combination with the polynomials with superior degree (ex. 51/7, 51/13 or 51/15).

Good performances are obtained using the Feed Back with $b_4=23=10011$, indicated in [2] (25/23, 33/23, 37/23). The last two combinations (33/23 and 37/23), correspond to the situation when the pseudo-random interleaver is superior to S-interleaver.

In the following are presented some practical results.

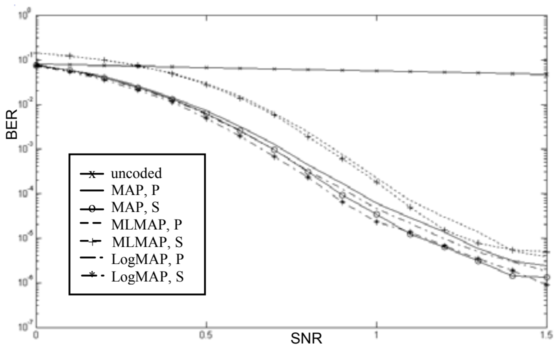



Fig.3 Simulation of rate 1/3 PCCC, 5/7 code, pseudo-random interleaver and S-interleaver.

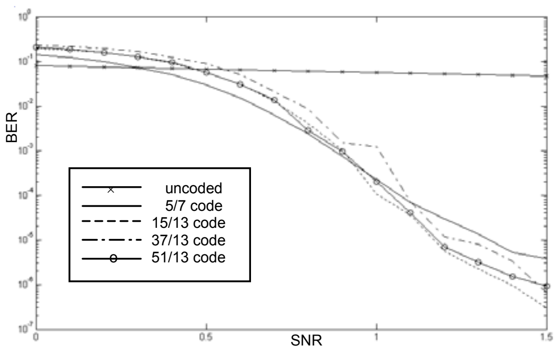



Fig.4 Simulation of rate 1/3 PCCC, P-interleaver, MaxLogMAP, for 5/7, 15/13, 37/13 and 51/13 codes.

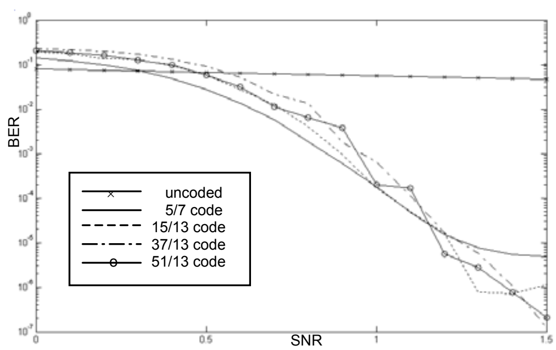



Fig.5 Simulation of rate 1/3 PCCC, S-interleaver, MaxLogMAP, for 5/7, 15/13, 37/13 and 51/13 codes.

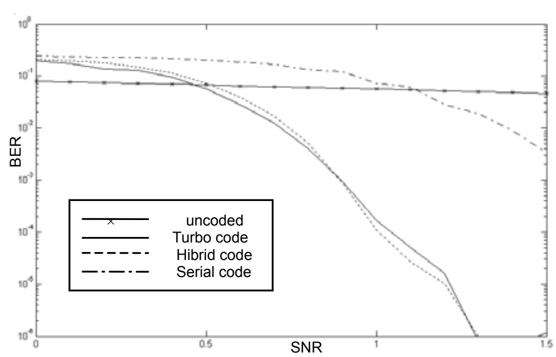



Fig.6 Simulation of rate 1/3 PCCC, 1/3 HCCC, 1/4 SCCC, S-interleaver, MaxLogMAP algorithm, 15/13 code.

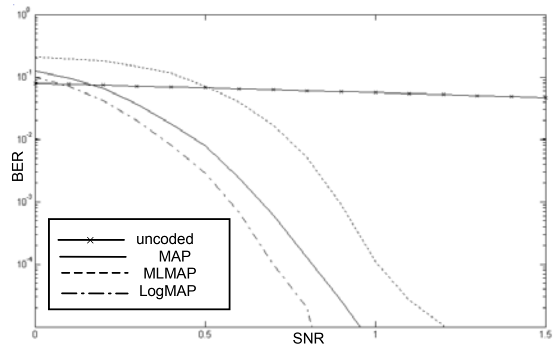



Fig.7 Simulation of rate 1/3 HCCC, 15/13 code, S-interleaver

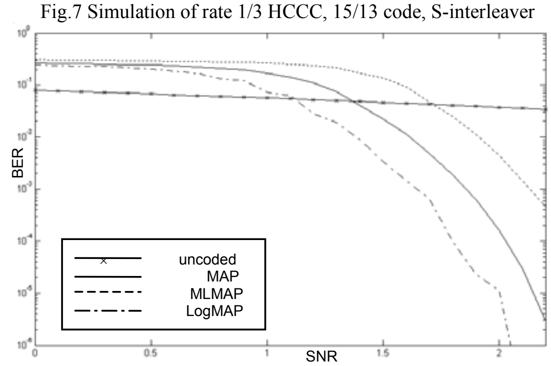



Fig.8 Simulation of rate 1/3 SCCC, 15/13 code, S-interleaver.

The results obtained with different decoding algorithms (MAP, MaxLogMap and LogMAP), with two types of interleavers: pseudo-random and S-type interleaver, with S=29, are comparted in Fig.3. In all the cases we used the 5/7 code (the most performant from all codes which have K=3) with data blocks size equal with N=1784 bits. We can remark the superiority of S-interleaver versus the pseudo-random interleaver. For example, in the case of MAP algorithm, the S-interleaver brings a SNR improvement of 0,1dB, at BER=$10^{-5}$, versus the pseudo-random interleaver. Surprisingly, in the algorithms performances hierarchy the first place is took by the LogMAP algorithm, especially under 1dB. The performances of MAP and Log MAP algorithms are with approximation equals over 1dB. As we expected, MaxLogMAP algorithm is with 0,2 dB inferior than the two enounced above.

Fig. 4 and Fig. 5 compare the most performant codes from each constraint length: for K=3 is the 5/7 code, for K=4 is the 15/13 code, for K=5 is the 37/13 code and for K=6 is the 51/13 code, in the case of using MaxLogMAP algorithm (the fastest in simulations) for pseudo-random interleaver and S-interleaver. Thought we used $10^8$ transmitted bits (or more) sometimes it was insufficient to obtain smooth curves. Besides of this observation, we notice that for SNRs inferior to 1 dB, the 5/7 code is the best. When SNR increases more than 1 dB, the codes with K>3 are more performants. This fact leads to the idea that the codes hierarchy can changes at higher SNR. Notice that turbo codes which have Feed Back loop realized on the basis of polynomial 13, for 1dB, are the best. Probably, for SNR higher than 1dB, maybe the xx/13

codes can lose their supremacy. This verification constitute the objective of a future study which we propose us.
The diagrams, for the last three figures, show the BER performances obtained with different concatenation modes (parallel, hybrid and serial) and with different algorithms (MAP, MaxLogMAP and LogMAP). In all the cases we used the 15/13 code and the S-interleaver.
Obvious, the serial concatenation is less performant than parallel and hybrid concatenations. Because of multiplexing and restriction using of interleavers with the same length, N=1784, in the case of SCCC code it results a number of N=1784/2=892 information bits per block, than N=1784 for PCCC and HCCC codes. Moreover, because the SCCC transmission rate is of 1/4 versus 1/3 in the case of PCCC and HCCC codes, for the equivalence of the ratio between the transmitted energy in information bit ($E_0$) and the noise power spectral energy ($N_0/2$), for the 1/4 transmission rate case, the channel noise power is higher than the corresponding power for the 1/3 transmision rate case.
We remark the falling of the SCCC codes, for SNR of 2 dB, falling what it's not find in the case of the PCCC and HCCC codes.
Finally, we also notice the good behavior of the LogMAP algorithm in the case of SCCC and HCCC, fact which "invite" us to make an investigation more detailed of this algorithm for the future.

## III. CONCLUSIONS

In this paper we presented and compared the BER performances obtained by the simulation of a transmission system, which utilizes the forward error correcting by codes concatenation and iterative decoding (turbo coding). We used two interleaver types, pseudo-random and S-interleaver with the length equal with N=1784. The BPSK modulation and the AWGN channel were employed. The MAP, MaxLogMAP and Log MAP algorithms were used for decoding.
There have been investigated all the systematic convolutional codes having the constraint length K less or equal to 6, under three different concatenated forms: parallel PCCC (pure turbo code), serial SCCC and hybrid HCCC, at the following rates: 1/3, 1/4 and 1/3, respectively, all unpunctured.

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