IMAGE DENOISING USING WAVELET TRANSFORMS WITH ENHANCED DIVERSITY

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ABSTRACT

The performance of image-denoising algorithms using wavelet transforms can be improved significantly by matching the parameters of those transforms with the input image. For every noisy image, there is a best pair of parameters formed by a mother wavelets and a best primary resolution, which maximizes the output Peak Signal to Noise Ratio, PSNR. Unfortunately this best pair is not known in advance. The denoising algorithms are sensitive to the parameters of the wavelet transform used. This sensitivity can be reduced using a diversity enhanced wavelet transform, obtained by computing few wavelet transforms with different parameters. After the filtering of each detail coefficient, the corresponding wavelet transforms are inverted and the estimated image, having a higher PSNR, is extracted. This paper presents a denoising algorithm based on diversity enhanced wavelet transforms. Some comparisons with the best available results will be given in order to illustrate its effectiveness.

1. INTRODUCTION

In 1992 David Donoho has introduced a method of signal denoising based on adaptive nonlinear filtering in the wavelets transform domain, [1]. Let us suppose that the useful signal x is additively perturbed by a noise n_i. To estimate the signal x, from an additive mixture of useful signal and noise, $x_i = x + n_i$, Donoho, [1], proposed the following method:

1. The Wavelet Transform, WT, of the signal x_i is computed.

2. A filtering procedure is applied in the wavelet transform domain, obtaining the signal y_0.

3. Taking the inverse wavelet transform, IWT, of the signal y_0, the denoised version of the signal x_i, named x_0, is obtained.

There are a lot of papers dealing with denoising methods, due to the important number of parameters involved. In this paper the construction of two Diversity Enhanced Wavelet Transforms, DE WTs, will be described. Some simulation results will be given in order to illustrate the effectiveness of the new transforms. Other potentially applications of those new WTs also exist. For example these WTs can be used for watermarking. They can substitute in this respect the Discrete Cosine Transform, DCT or the Discrete Wavelet Transform, DWT, taking into account their advantages: the translations invariance, the enhanced directional selectivity and the low sensitivity with the transforms parameters.

2. SOME WTS USED IN DENOISING

There are two kinds of WTs: redundant and non redundant. The first wavelet transform used in denoising applications was the Discrete Wavelet Transform, DWT. This transform is most commonly used in its maximally decimated form (Mallat's dyadic filter tree [2], [3]). The DWT is an orthogonal (non redundant) WT. It has two main disadvantages, [4-7]: the lack of shift invariance and the poor directional selectivity. It is also dependent on the selection of its parameters.

Some WTs, used in denoising algorithms are presented in table 1. The following denominations were used: Orthogonal Real DWT - OR DWT, Biorthogonal Real DWT - BR DWT, Complex DWT - C DWT, Dual Tree Complex WT - DT CWT, Cycle Spinning OR DWT - CS OR DWT, Undecimated DWT - U DWT and Shiftable Multiscale Transform - SMT. In [8] is proposed a very redundant shift invariant OR DWT, associated to a denoising algorithm named Cycle Spinning, CS. This represents a first example of enhanced diversity in the wavelet domain. Other redundant WTs are proposed in [9] and [10]. All the WTs presented in table 1 have two parameters: the mother wavelets, MW and the primary resolution, PR, (number of iterations). The importance of their selection is highlighted in [11]. For a given noisy image, there is only one pair of such parameters that maximizes the denoising algorithm output PSNR. The selection of the parameters pair that maximizes the output PSNR for a given noisy image is very difficult due to the noise that perturbs the useful image.

Table 1. Different WT used in denoising algorithms.

Type	Ref.	Ortho.	Algo.
OR DWT	[2,3]	yes	Mallat
BR DWT	[2]	no	Mallat
C DWT	[5,6]	yes	Mallat
DT CWT	[4,7]	yes	Mallat
CS OR DWT	[8]	yes	Mallat
U DWT	[9]	yes	à trous
SMT	[10]	no	Simoncelli

The sensitivity of the output PSNR with the parameters selection can be reduced by using diversity enhanced WT, obtained by computing few WTs with different parameters.
After the filtering of each detail coefficient, the corresponding WTs are inverted and the estimated image, having a higher PSNR, is extracted. Being redundant, these WTs assure more room for the mark insertion, increasing the capacity of the corresponding watermarking system. Another interesting feature for watermarking applications of some of those diversity enhanced WTs is their translation invariance.

3. FILTERS FOR DETAIL WAVELET COEFFICIENTS

Some recent research addressed the development of statistical models for the wavelet coefficients of natural images and application of these models to image denoising [12-17], on the basis of Maximum A Posteriori, MAP, filters. In this paper, the denoising of an image corrupted by additive independent white Gaussian noise with variance σ_n^2 will be considered. In this case the noise probability density function, pdf, p_{n_y} is a Gaussian with zero mean and variance σ_n^2. The selection of the model must be matched to the WT used. For example for the SMT described in [10] the appropriate model is the Gaussian mixture described in [13] and [14]. In this paper the case of the OR DWT and of the DT CWT are considered. For these transforms there are strong dependencies between neighbor coefficients from adjacent scales such as between a coefficient and its parent (adjacent coarser scale locations). In [16] statistical models for the useful image are proposed, which take into account the dependency between the detail coefficients and their parents. Let y_2 represent the parent of y_1 (y_2 is the wavelet coefficient at the same position as y_1, but at the next coarser scale.) Then:

$$\boldsymbol{y}_i = \boldsymbol{y} + \boldsymbol{n}_y. \qquad (1)$$

where: $\boldsymbol{y}_i = (y_{i1}, y_{i2})$; $\boldsymbol{y} = (y_1, y_2)$ and $\boldsymbol{n}_y = (n_{y1}, n_{y2})$. The MAP filter input-output relation becomes, [16]:

$$\hat{\boldsymbol{y}}(\boldsymbol{y}_i) = \underset{\boldsymbol{y}}{\operatorname{argmax}}\left[p_{\boldsymbol{n}_y}(\boldsymbol{y}_i - \boldsymbol{y}) \cdot p_{\boldsymbol{y}}(\boldsymbol{y})\right], \qquad (2)$$

Assuming the noise samples (n_y) are independent and identical distributed, i.i.d., following a Gaussian pdf and using the next statistical model for the useful component (y):

$$p_{\boldsymbol{y}}(\boldsymbol{y}) = \frac{3}{2\pi\sigma^2} \cdot \exp\left(-\frac{\sqrt{3}}{\sigma}\sqrt{y_1^2 + y_2^2}\right), \qquad (3)$$

the bishrink filter is constructed in [16]. It has the following input-output relation:

$$\hat{y}_1 = \frac{\left(\sqrt{y_{i1}^2 + y_{i2}^2} - \frac{\sqrt{3}\hat{\sigma}_n}{\hat{\sigma}}\right)_+}{\sqrt{y_{i1}^2 + y_{i2}^2}} \cdot y_{i1}, \qquad (4)$$

where:

$$(g)_+ = \begin{cases} 0, & \text{if } g < 0 \\ g, & \text{otherwise} \end{cases}. \qquad (5)$$

If the useful component (y) is supposed to be Gaussian distributed at all scales, then the solution of the problem in (2) is the zero order Wiener filter. To use (4), the variances of the useful component and of the noise component must be estimated. The global estimation of the noise variance can be performed using the diagonal detail image obtained at the finest scale (after the first iteration), [17]:

$$\hat{\sigma}_n^2 = \frac{\operatorname{median}(|y_i|)}{0.6745} \quad y_i \in \text{ first subband HH.} \qquad (6)$$

To estimate the variance of the useful component, first the variance of $y_i\,[p, q]$ is empirically estimated into a square moving window, $N\,[p, q]$, centered in (p, q):

$$\hat{\sigma}_{y_i}^2 = \frac{1}{M} \cdot \sum_{y_i \in N[p,q]} y_i^2, \qquad (7)$$

where M represents the size of the neighborhood $N\,[p, q]$. Then, σ can be estimated as:

$$\hat{\sigma} = \sqrt{\left(\hat{\sigma}_{y_i}^2 - \hat{\sigma}_n^2\right)_+}. \qquad (8)$$

The precision of this estimation could be improved, using the method proposed in [18].

4. THE DE DWT

The architecture of the DE DWT is presented in the next figure. The DE DWT of the input image, x_i, is a generalized matrix. Each column of this matrix is an image, representing a different OR DWT or DT CWT of the image x_i. The first

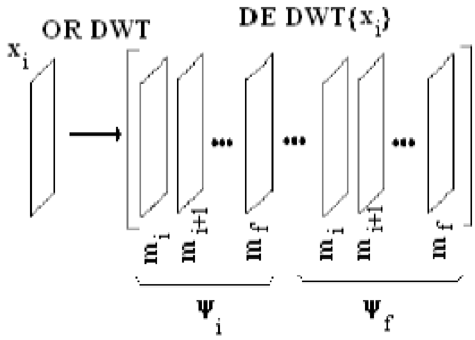

Figure 1. The construction of the DE DWT.

column contains the image obtained using the parameters pair (ψ_i,m_i), the second column contains the image obtained using the parameters pair (ψ_i,m_{i+1}) and so on. The function ψ_i represents the first MW and the function ψ_f the last one. m_i denoted the initial number of iterations and m_f the final one. The redundancy of the DE DWT, presented in figure 1, is of $(f - i + 1) \cdot (m_f - m_i + 1) \cdot RWT$, where RWT represents the redundancy of the WT used (OR DWT or DT CWT). This transform can be included in a denoising algorithm by filtering each column of the matrix **DE DWT {*xi*}** with a bishrink filter. A new matrix, $\boldsymbol{y_0}$, is obtained. The corresponding inverse OR DWT or DT CWT are computed for each column of $\boldsymbol{y_0}$. A new matrix, $\boldsymbol{x_0}$, is obtained. Replacing this matrix with the arithmetic mean of its columns, the image representing the estimation of the input image, x_0, is finally obtained.
The DE DWT obtained starting from the DT CWT is translation invariant and has an enhanced directional selectivity. These are good reasons to use it in watermarking applications also.

5. RESULTS

Two extreme cases of the DE DWT are investigated. In the first experiment the nine compactly supported Daubechies' extreme phase MWs with a number of vanishing moments between 2 and 10 were used to enhance the diversity in the OR DWT domain. In this case i = 2 and f = 10. All the nine corresponding DWTs of the same input image have the same primary resolution, being computed with a number of 5 iterations. In this case $m_i = m_f = 5$. The redundancy of this DE DWT is of 9. In the second experiment the Kingsbury's MW, [4 page 246], was used. In this case i = f. Three DT CWTs of three realizations of the same input image (the sums of the same useful image with three realizations of a white Gaussian noise with the same variance), with different primary resolutions, corresponding to 4, 5 and 6 iterations were computed. In this case $m_i = 4$ and $m_f = 6$. The redundancy of this transform is of 12, but the diversity was also enhanced in the domain of the input image. The dimensions of *N [p, q]* were of 7 × 7 in each case. The PSNR values are listed in table 2 (the values are taken from the corresponding papers). Two images, Lenna and Barbara, each of 512 × 512 pixels, contaminated with additive white Gaussian noise with different variances, σ_n^2, were used. We compared the results obtained using the two DE DWTs, already described, and named DE OR DWT and DE DT CWT, to other effective systems. Some of them use redundant WT, i.e., the U DWT in [19], the SMT in [13] and the DT CWT in [16]. In [20] is proposed an image denoising method that improves the diversity in the wavelets domain with the aid of several MWs. The filtering in the wavelets domain is realized using the projection on convex sets. Analyzing table 2, it can be observed the superiority of the bishrink filter (1'st exp) versus the projection on convex sets ([20]) or versus the Wiener filter ([18], despite of the variable moving window used for the variance estimation). In addition the output PSNR obtained using the DE OR DWT was compared to the output PSNR issued by the application of the bishrink filter to each particular OR DWT that composes the DE OR DWT. Each time the output PSNR of the method corresponding to our first experiment was superior to the various PSNRs obtained using the component OR DWTs. The superiority of the association of DT CWT with the bishrink filter, ([16], second experiment) versus the associations based on other WTs (SMT in [13], U DWT in [19]) is also obvious. The best results, in table 2, correspond to our second experiment, proving the importance of the diversity enhancement.

Table 2. PSNR values (in dB) of denoised images for different test images and noise levels (σ_n) of noisy, the system in [20], the system in [18], our first experiment, the system in [19], the system in [13], the system in [16] and our second experiment.

σ_n	Noisy	[20]	[18]	1'st exp	[19]
Lena					
10	28.18	-	34.49	35.00	-
15	24.65	-	32.48	33.14	33.41
20	22.14	-	31.06	31.84	32.12
25	20.17	29.41	29.94	30.72	31.11
Barbara					
10	28.16	-	32.73	33.17	-
15	24.63	-	30.34	30.86	31.14
20	22.14	-	28.73	29.16	29.52
25	20.18	26.72	27.50	27.88	28.33

σ_n	Noisy	[13]	[16]	2'nd exp
Lena				
10	28.18	35.31	35.34	37.20
15	24.65	33.55	33.67	35.51
20	22.14	32.31	32.40	34.34
25	20.17	31.33	31.40	33.30
Barbara				
10	28.16	33.45	33.35	35.86
15	24.63	31.22	31.31	33.62
20	22.14	29.71	29.80	32.50
25	20.18	28.57	28.61	30.60

6. CONCLUSION

This paper presents an effective and low complexity image-denoising algorithm using DE DWTs and the bishrink filter. The proposed technique consists in applying several variants of the same WT to the noisy image, filtering each result, inverting each WT and taking the average of the results. The different variants of the same WT are obtained by selecting different parameters (MW and/or PR) of the WT. This is the source of the diversity enhancement. For example, the diversity is enhanced, in the case of CS, by translating the input image (for each translated image the same DWT is used). This approach is not equivalent with several separate filtering of the same image. The output PSNR obtained in every experiment reported is higher than any partial output PSNR obtained making one of the separate filtering of the input image that compose the denoising method corresponding to the considered experiment.

Our results, obtained using the DE DT CWT, outperform the results obtained processing the same images with CS (which is also sensitive to the DWT parameter selection). So the selection of WT parameters (problem considered for the first time in this paper) is very important. We presented our results for both DE OR DWT and DE DT CWT and compared it with other published results. The comparison suggests that the new denoising results are competitive or better than the best wavelet-based results already reported in the literature, from the output PSNR point of view. Some comparisons with respect to the results of some simple and classical filters used for image denoising are already reported in a previous paper of the first two authors, [21]. The image Lena, perturbed with speckle noise, was denoised using: a running averager, a median filter, the Lee's filter, the Kuan's filter, the Gamma filter and the Frost's filter but the results obtained are inferior to the results obtained using wavelet based denoising techniques. The reduced redundancy of the DE DWTs proposed in this paper (the CS ORDWT and the U DWT are more redundant) assures a reduced computational complexity for the corresponding denoising methods.

Taking into account the properties of the DE DWT obtained starting from the DT CWT: its reduced redundancy, its low sensitivity with the parameters selection, its translation invariance and its improved directional selectivity, we believe that this transform can be also successfully used in watermarking applications. This is a future research direction for our team.

ACKNOWLEDGMENT

The research with the results reported in this paper was realized in the framework of the Programme de recherche franco-roumain Brancusi with the title: Débruitage des images SONAR en utilisant la théorie des ondelettes: applications aux systèmes d'aide à la décision pour la classification.

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