Denoising base-band communication signals

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Abstract — **This paper presents a new denoising method for base-band communication signals corrupted by additive noise. The novelty of this paper is the use of a special MAP filter, called composed bishrink, for communication purposes. A complete statistical analysis of this filter is reported. Some simulations are presented. The results obtained are compared with the results of matched filtering technique.**

Index Terms — **denoising, bishrink, bit error rate, matched filter.**

I. Introduction

Every communication system is composed of an emission unit and a receiver. These two parts are connected through a communication channel. In the communication channel, the signal is perturbed by noise. Generally, the impulse response of the channel and the characteristics of the noise are not known. Due to the non-ideal impulse response of the communication channel the signal to be transmitted is distorted. The information content of the signal at the output of the emission unit is also affected by the channel noise. The complexity of the coding system is selected in accordance with the characteristics of the noise. For more difficult channels, more complex and expensive coding systems must be used. At the input of the receiver, a part of this noise is removed with the aid of a filter. This is the single component of the communication system able to enhance the signal to noise ratio, (SNR). There is also a detection system too. If the communication channel is more difficult than expected then this detection system produces some errors. This is the reason why communication systems must be classified using the bit error rate (BER). A communication system is better than another one if for the same communication channel it has a smaller BER. Generally the BER is expressed as a function of the SNR of the corresponding communication channel and this is a decreasing function. One very efficient channel coding method is based on turbo-coding [1]. Its BER(SNR) characteristic can be approximated with two linear segments. The first line, corresponding to low SNR values, has a slight slope. The second line, corresponding to high SNR values has an important decreasing velocity. Here the effect turbo is present. At the border of the two approximation regions there is a threshold SNR value, SNR_0. Hence the manifestation of the turbo effect is accomplished only if the SNR is superior to SNR_0. So it is very important to obtain a SNR value high enough. The aim of this paper is to propose the inclusion of a denoising system in the structure of the receivers. The proposed place for such a system is at the output of the analog to digital converter, (ADC). The output of the denoising system is connected to the input of the detector. In recent years, the techniques using multiscale and local transform-based algorithms have become popular in noise filtering applications. In particular the use of non-linear filters in the DCT domain was studied. In this paper, we consider local transform based denoising. We propose such an algorithm, based on the discrete wavelet transform, (DWT). The mathematical theory of wavelets is applied to various disciplines of science and engineering including communications over the past two decades. Recent work has suggested the connection between wavelets and different parts of communication theory like for example the analog pulse shapes that satisfy Nyquist first criterion. However, the unfamiliarity of the majority of communication engineers to wavelets prevents its acceptance and wide-spread applications in this field. We treat in this paper only the case of additive noise. We deal with base-band transmissions, where the classical denoising solution is the matched filter. Section II deals with the local DWT-based denoising. In section III, a statistical analysis of the new denoising method is presented. The use of local filters in the DWT domain is described in the following section. These three sections are very important for a good understanding of the present paper. The results described in those three sections were already published by some of the authors of this current paper, [14], in the context of a different application, the SAR images denoising. Those results derivations were accompanied in [14] by diagrams. In section V, numerical simulation results are presented and discussed and comparisons with state of the art filtering techniques (matched filtering) are reported. The last section is dedicated to some concluding remarks.

II. Local DWT-based Denoising

The following model of the observed signal corrupted by additive noise is considered in this paper:

$$x[k] = s[k] + n[k] \tag{1}$$

where s and n represent the useful part and the noise. Of course this hypothesis is not valid for all transmission system. The problem is to estimate s starting from x. The noise is usually considered to be a stationary random process, with a null mean and a variance σ_n^2, uncorrelated with s. To estimate the signal s, Donoho, [2], proposed the following method:

1.The Discrete Wavelet Transform (DWT) of the signal x is computed. The result is the signal $y_i = y + n_y$. The noise n_y converges asymptotically to a Gaussian white one, with the same variance, [3].

2.A non-linear filtering is applied in the wavelet domain:

$$y_0[k] = \begin{cases} \operatorname{sgn}\{y_i[k]\}\left(\left|y_i[k]\right| - t\right), & \left|y_i[k]\right| > t \\ 0, & \text{if not} \end{cases} \quad (2)$$

where t is a threshold. This system is called soft thresholding filter. Because the noise n_y is Gaussian, if $t>3\sigma_n$, the probability $P(n_y>t)$ is very little (the rule of 3 sigmas). So the noise is quasi entirely suppressed. This is the reason why the signal y_0 is a denoised version of the signal y_i. This is a non-linear adaptive filter whose statistic analysis was presented in [4]. The adaptability is due to the selection of the threshold value in function of the noise power.

3.Taking the inverse DWT (IDWT) of the signal y_0, the denoised version of the signal s, x_0, is obtained.

The principal disadvantage of the already described denoising method is due to the fact that it is based only on the estimation of the noise variance (the useful part of the input signal is ignored) and on hypotheses confirmed only asymptotically. This is the reason why in the following another denoising strategy, based on the use of a Maximum a Posteriori (MAP) filter, will be described.

III. A Statistical Analysis Of The Dwt

The probability density function, (pdf), of the wavelet coefficients at the mth scale, (after m iterations), ${}_xD_m^k$ (k being equal with 1 for detail coefficients and with 2 for approximation coefficients) is given by the following relation:

$$f_{{}_xD_m^k}(a) = \underset{r_1=1}{\overset{N(k)}{*}} \; \underset{q_2=1}{\overset{M_0}{*}} \cdots \underset{q_m=1}{\overset{M_0}{*}} f_d(k, r_1, q_2 \ldots, q_m, a) \quad (3)$$

where:

$$f_d(k, r_1, q_2 \ldots, q_m, a) = G(k, r_1, q_2 \ldots, q_m) \cdot f_x(G(k, r_1, q_2 \ldots, q_m) a) \quad (4)$$

and:

$$G(k, r_1, q_2 \ldots, q_m) = \frac{1}{F(k, r_1) \prod_{l=2}^{m} m_o[q_l]} \quad (5)$$

where:

$$F(k, r_1) = \begin{cases} m_0[r_1] & k = 2 \\ m_1[r_1] & k = 1 \end{cases} \quad (6)$$

M_0 represents the length of the impulse response m_0 (of the low-pass filter used in the computation of the DWT) and M_1 represents the length of m_1 (the high-pass filter used in the computation of the DWT) and the number of the convolutions in the first group from the relation (3) is given by:

$$N(k) = \begin{cases} M_0, & k = 2 \\ M_1, & k = 1 \end{cases} \quad (7)$$

In conformity with (3), the pdf of the wavelet coefficients is a sequence of convolutions. Hence, the random variable representing the wavelet coefficients can be written like a sum of independent random variables. So, the central limit theorem can be applied. This is the reason why the pdf of the wavelet coefficients tends asymptotically to a Gaussian, when the number of convolutions in (3) tends to infinity. This number depends on the mother wavelets used and on the number of iterations of the DWT. For mother wavelets with a long support, this number increases very fast. The slower convergence is obtained for the Haar mother wavelets, which has the shorter support. This is the case analyzed in this paper. The filter used in the DWT domain must be constructed having in mind that after a number of iterations the distribution of the wavelet coefficients can be considered Gaussian. The problem is to establish this number, Nu_1. Another problem is to find the wavelet coefficients distribution law for the first iterations (before to reach the Gaussian law). Generally, this is a heavy-tailed distribution. The correlation of the wavelet coefficients ${}_xD_m^k$ is given by:

$$\Gamma_{{}_xD_m^k}[n_1, p_1] = E\left\{ {}_xD_m^k[n_1] \left({}_xD_m^k[p_1] \right)^* \right\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{p_2=-\infty}^{\infty} \gamma_x\left(2^{-m}\left(w + 2p_2\pi\right)\right) \cdot e^{-jw(n_1-p_1)} \cdot \left|\Im\left\{\psi^k\right\}\left(w + 2p_2\pi\right)\right|^2 dw \quad (8)$$

Because the signals s and n are not correlated it can be written:

$$\Gamma_{{}_xD_m^k} = \Gamma_{{}_sD_m^k} + \Gamma_{{}_nD_m^k} \quad (9)$$

If the input noise is a zero mean white Gaussian, the correlation of its wavelet coefficients becomes:

$$\Gamma_{{}_nD_m^k}[n_1] = \sigma_n^2 \cdot \delta[n_1] \quad (10)$$

At any scale the noise in the wavelets domain is also white, having the same variance. So, a single estimation of its variance, for example using the detail coefficients obtained after the first iteration, is sufficient. For any type of input noise, when m tends to infinity, the relation (8) becomes:

$$\Gamma_{{}_nD_\infty^k}[n_1, p_1] = \gamma_x(0) \cdot \delta[n_1 - p_1] \quad (11)$$

So, asymptotically, the noise in the wavelets domain becomes white. Unfortunately this is also only an asymptotic result. Combining this result with the result obtained after the pdf analysis, it can be observed that after a given number of iterations, Nu_2, the noise in the wavelet domain is white and Gaussian. The mean of the wavelet coefficients is:

$$E\left\{ {}_x D_m^k [n_1, p_1] \right\} = \begin{cases} 0, & k = 1 \\ 2^{\frac{m}{2}} \cdot \mu_x, & k = 2 \end{cases} \tag{12}$$

In practice the number of iterations of the DWT is high. The length of the approximation coefficients sequence obtained after the last iteration is small. This is the reason why this sequence is not filtered in practice. The variance of the wavelet coefficients is given by:

$$\begin{aligned} \sigma^2_{{}_x D_m^k} &= E\left\{ \left| {}_x D_m^k \right|^2 \right\} = \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \gamma_x(2^m u) \cdot \left| \Im\left\{ \psi^k \right\}(u) \right|^2 du \end{aligned} \tag{13}$$

The correlation of the DWT of the useful part of the input signal, s, is given by:

$$\Gamma_{{}_s D_m^k} [n_1] = 2^m \cdot \Gamma_s \left[2^m n_1 \right] \tag{14}$$

its mean by:

$$E\left\{ {}_s D_m^k [n_1] \right\} = \begin{cases} 0, & k = 1 \\ 2^{\frac{m}{2}} \cdot \mu_s, & k = 2 \end{cases} \tag{15}$$

and its variance by:

$$\sigma^2_{{}_s D_m^k} = 2^m \cdot \sigma_s^2 \tag{16}$$

So, the variances of the detail wavelet coefficients sequences of the useful component of the input signal increases when the iteration index increases.

IV. MAP Filters Exploiting The Interscale Dependency Of The Detail Coefficients

In conformity with (8), there is an important correlation between a wavelet coefficient at a given scale and the same coefficient situated in the same position at the next scale (named the parent of the considered coefficient). This correlation can be exploited to construct adaptive filters acting at a given scale and using for the estimation of their parameters information obtained at the next scale, [5]. Using the parent and child wavelet coefficient of the input signal it is possible to estimate the child coefficients of the DWT of the useful part of the input signal, with the aid of a bishrink filter, [5]. Let ${}^1 y_i$ be the considered detail coefficient and ${}^2 y_i$ its parent. The statistical parameters of the child coefficients can be determined using their parent coefficients and the neighbor child coefficients, located in a window with a length of 3, centered on the current child coefficient. It can be written:

$$\mathbf{y}_i = \mathbf{y} + \mathbf{n}_y \tag{17}$$

where:

$$\mathbf{y}_i = \left({}^1 y_i, {}^2 y_i \right); \mathbf{y} = \left({}^1 y, {}^2 y \right); \mathbf{n}_y = \left({}^1 n_y, {}^2 n_y \right) \tag{18}$$

The MAP estimation of **y**, realized using the observation $\mathbf{y}_i$, is given by:

$$\hat{\mathbf{y}}(\mathbf{y}_i) = \arg\max_{\mathbf{y}} \left\{ \ln\left(f_{\mathbf{n}_y}(\mathbf{y}_i - \mathbf{y}) \cdot f_{\mathbf{y}}(\mathbf{y}) \right) \right\} \tag{19}$$

In the following, we will consider that the DWT of the noise is distributed following a zero mean Gaussian:

$$f_{\mathrm{n}_y}(\mathrm{n}_y) = \frac{1}{\sqrt{2\pi}\sigma_n} \cdot e^{-\frac{\left({}^1 n_y\right)^2 + \left({}^2 n_y\right)^2}{2\sigma_n^2}} \tag{20}$$

Concerning the model of the DWT of the useful component, in the case of the composed bishrink filter, for the first Nu_2 iterations, a Laplace distribution (this a heavy-tailed one) will be considered (like in the case of the bishrink filter, [5]):

$$f_{\mathrm{y}}(\mathrm{y}) = \frac{\sqrt{3}}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{\sqrt{3}}{\sigma} \cdot \sqrt{\left({}^1 y\right)^2 + \left({}^2 y\right)^2}} \tag{21}$$

and for the other iterations, a Gaussian distribution will be used:

$$f_{\mathrm{y}}(\mathrm{y}) = \frac{1}{\sqrt{2\pi {}^1\sigma {}^2\sigma}} \cdot e^{-\frac{\left({}^1 y\right)^2 + \left({}^2 y\right)^2}{2 {}^1\sigma {}^2\sigma}} \tag{22}$$

(like in the case of the Wiener filter, [6]). For the models in (20) and (21) the solution of the maximization problem in (19), named bishrink filter, is:

$$\widehat{{}^1 y} = \frac{\left(\sqrt{\left({}^1 y_i\right)^2 + \left({}^2 y_i\right)^2} - \frac{\sqrt{3}\hat{\sigma}_n^2}{\hat{\sigma}} \right)_+}{\sqrt{\left({}^1 y_i\right)^2 + \left({}^2 y_i\right)^2}} \cdot {}^1 y_i \tag{23}$$

where:

$$\hat{\sigma}^2 = \widehat{{}^1\sigma} \cdot \widehat{{}^2\sigma} \tag{24}$$

and:

$$(g)_+ = \begin{cases} g, & g > 0 \\ 0, & if \quad not \end{cases} \tag{25}$$

and for the models in (20) and (22) the solution of the maximization problem in (19) is:

$$\widehat{{}^1 y} = \frac{\widehat{{}^1\sigma} \cdot \widehat{{}^2\sigma}}{\widehat{{}^1\sigma} \cdot \widehat{{}^2\sigma} + \hat{\sigma}_n^2} \cdot {}^1 y_i \tag{26}$$

So, the input-output relations of the composed bishrink filter are (23) and (26). The noise variance is estimated using the details obtained after the first iteration and the variances $\widehat{{}^1\sigma}$ and $\widehat{{}^2\sigma}$ are estimated in moving windows centered on the current child and parent coefficients. First the means are estimated in each window and second the variances. But, applying the relation (16), a different estimation of the local variance of the child coefficients can be obtained:

$$\widehat{{}^{1}\sigma_{d}} = \frac{\widehat{{}^{2}\sigma}}{\sqrt{2}} \qquad (27)$$

To profit of these two estimations of the local variances, obtained at two successive scales, it can be written:

$$\widehat{\widehat{{}^{1}\sigma}} = \frac{\widehat{{}^{1}\sigma} + \frac{\widehat{{}^{2}\sigma}}{\sqrt{2}}}{2} \qquad (28)$$

This estimation will be used in (24) and (26), substituting $\widehat{{}^{1}\sigma}$, for the input-output relations of the composed bishrink filter.

V Simulation Results

A useful input signal constant within intervals was considered. This is a data sequence, specific for the communication in the base-band. This sequence has a number of 16384 symbols, each having *b*=128 samples. A portion of this signal is represented in fig. 1. Taking into account the waveform of this signal, the Haar mother wavelets must be used. The DWT was computed on blocks, each having a length of 4096 samples. The maximal number of iterations (equal with 12) was used for the computation of each DWT. For the implementation of the composed bishrink filter, a value of Nu_2=8, was used. In the following fig. is represented the dependency between the output and input SNRs for different denoising methods. The filter used in the wavelets domain gives the difference. It can be observed that these dependencies are linear. The curves describing the bishrink filter and the composed bishrink filter are superposed. These filters give the better results. These are superior to the results obtained using the soft-thresholding filter. The poor results are obtained using the Wiener filter. Finally, a comparison between the use of a matched filter (adapted to the input impulse train) and the proposed denoising system, into a communication application is discussed. Each of the two systems are connected at the output of a communication channel, that adds a zero mean white noise to the data sequence, which beginning is represented in fig. 1. The first system is a matched filter (adapted to a rectangle), having a duration equal with *b*. Six experiments are made, with different noise variances. At the output of the investigated system (adapted filter or denoising system), an ideal sampling system is connected. Three hypotheses,

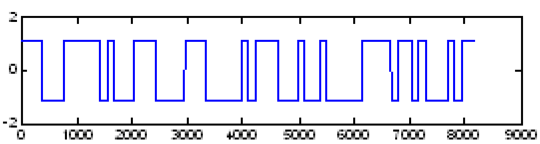

Figure 1. The waveform of the useful component of the input signal.

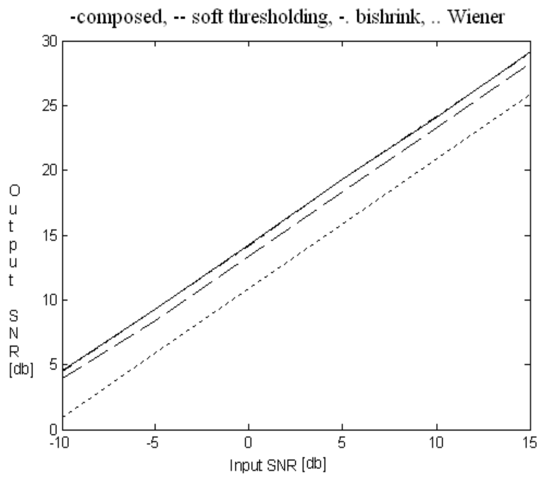

Figure 2. The dependence of the output SNR of the input SNR.

concerning the synchronization, are used. The first hypothesis supposes a perfect synchronization. The second hypothesis accepts a little loss of synchronization (the sampling moments are delayed with $3 \cdot b/8$) and the third hypothesis accepts a more important loss of synchronization (the sampling moments are delayed with $b/2$). This third hypothesis is the more realistic when the channel SNR is very small. The output of the sampling system is connected to the input of a comparator. The output of this comparator represents the output of the simulated receiving unit. The denoising system uses a soft thresholding filter when the input SNR is inferior to –16.65 dB and a composed bishrink filter when the input SNR is superior to –16.65 dB. The better result is obtained with the matched filter with perfect synchronization (ad.filt.perf.sync, the continuous line in fig. 3). For the other hypotheses, an analysis, taking into account the value of the input SNR, must be made. The synchronization losses do not affect the performances of the denoising system (denois.perf.syn, the dashed line in fig. 3, denois.part.sync.1 and denois.part.sync. 2) but affect the hypothesis, the denoising system is better than the matched filter (ad.filt.part.sync 1, the dash dot line in

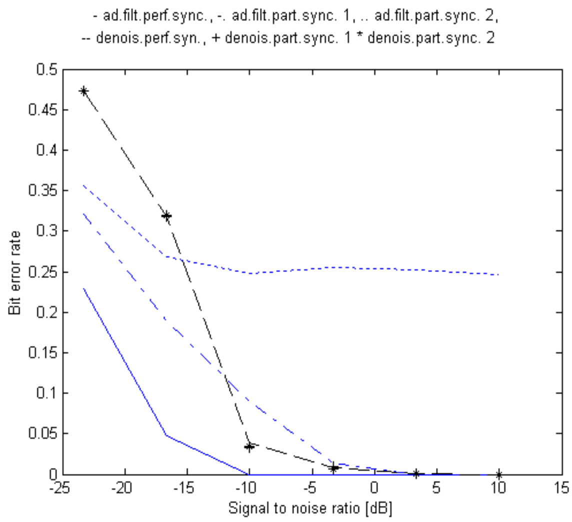

Figure 3. A comparison between the use of an adapted filter and a denoising system, in a base-band communication application.

fig. 3) for input SNRs superior to –10 dB. Also, in the third hypothesis, the denoising system is superior to the adapted filter (ad.filt.part.sync 2, the dotted line in fig. 3), for input SNRs superior to –10.47 dB. Practically in this case the matched filter cannot be used. We have repeated the experiment already described for other types of noise: uniform, Rayleigh, Cauchy. The results obtained are similar with those presented in fig. 2 and 3, concerning the efficiency of the compared treatment methods, but obviously the values of the output SNR and BER are different. In fact a continuous degradation was observed. The better results were obtained for white Gaussian noise followed by the results obtained for uniform noise (a loss in performance of 7 dB versus the results presented in fig. 2) and the results obtained for Rayleigh noise (a loss in performance of 11 dB). The poorer results are obtained for the Cauchy noise (a loss in performance of 12,5 dB). This conclusion was expected because the matched filter is constructed in the hypothesis that the noise is AWGN and for the construction of our MAP filter we have made the same hypothesis (see (20)). The performance evaluation procedure already described is presented in fig. 4.

VI CONCLUSION

In this paper is proposed the application of a new denoising method based on the use of the composed bishrink filter in the wavelets domain, in base-band communications. This method takes into account the statistics of the useful part of the input signal. In fact the construction of the MAP filter associated to the proposed denoising method is based on these statistics. This is the reason why the proposed denoising method performs better than the denoising method using the soft thresholding filter for low input SNRs (superior to –10 dB). This assumption is illustrated in fig.2. The denoising method proposed can be used in base-band communications, replacing the matched filter when the input SNR is low (when the synchronization is difficult). The signal at the output of the matched filter represents the correlation of its input signal. Because the samples of a white noise are not correlated, this kind of noise is completely rejected by the matched filter. The proposed denoising system cannot reject all the noise. This is the reason why its performance is inferior to the performance of the matched filter when the perfect synchronization is supposed.

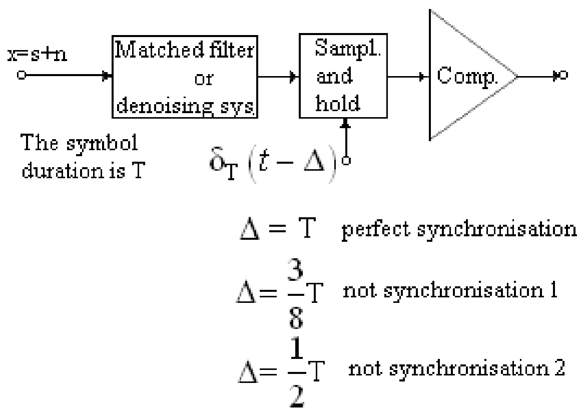

Figure 4. The performance evaluation procedure.

But the synchronization is very difficult when the useful signal is covered by noise. In these cases an important jitter can appear. The waveform at the output of the matched filter is linear for any input symbol. Due to the sign changes in the data, triangular atoms compose the global waveform at the output of the matched filter (the correlation of a train of pulses is a train of triangles). The picks appear at moments that are entire multiple of T. The detector (the block Comp. In fig. 4) computes the sign of its input signal. If the waveform at the output of the matched filter is not sampled at moments that are entire multiple of T then the sign can change between these two moments producing detection errors. Contrary to the matched filter the denoising system try to conserve at its output the rectangular waveform of the useful signal. This is the reason why the proposed denoising system manages better the synchronization errors, especially when the jitter is important.

There are some papers dealing with the application of the denoising methods in communications, [7-9]. In [7] is presented a transmission technique based on a multi-resolution analysis. This approach can be included in the framework of Donoho's denoising methods, if a system that rejects all the detail coefficients is considered in the place of the filter used in the second step of the algorithm in paragraph II. The techniques used in [8] and [9] are closer to the Donoho's framework. The single difference versus the denoising technique proposed in this paper is the filter used in the wavelet domain, an adaptive hard-thresholding one. This filter works iteratively. In each iteration is filtered the result obtained at the end of the previous iteration, using the same threshold, an estimation of the output SNR and a comparison with the value estimated for the SNR in the previous iteration are done. In the first iteration a hard thresholding filter with a small threshold is used. The iterative procedure is stopped when for the first time the actual SNR estimated value is smaller then the value estimated at the previous iteration. So, this denoising method does not take into account the statistics of the useful component of the input signal.

Due to the use of the proposed MAP filter, the results obtained here are superior versus the results presented in [7-9]. The simulation results presented in this paper are very promising. This research will be continued on the following directions:

-the comparison of the proposed denoising method with the matched filter for colored noise;

-the study of the sensitivity of the proposed denoising method versus the length of the input symbols, *b*;

-the optimization of the proposed denoising by selecting the better wavelet transform (it is possible that another wavelet transform, a redundant one maybe, to minimize the BER);

-the simulation of a synchronization system followed by the comparison of the proposed denoising method with

the matched filter made with the aid of this synchronization;
-the diversification of the channel models; we intent to continue the comparison of a new version of the proposed denoising method and the matched filter for fading channels (Rayleigh, Ricci, Nakagami…) flat or selective;
-the comparison of the matched filter with denoising methods based on wavelets for transmission strategies requiring different modulation types;
-the construction of a new denoising method based on the exact pdf of the useful component in the input signal. This pdf can be modeled or estimated. Applying the relation (3) the exact pdf, $f_y(y)$, can be obtained. Starting from this relation and from (20), a new MAP filter can be constructed.
-the study of the wavelet modulation technique, [10], and its connections with the denoising framework in the context of OFDM communications. In fact a wavelet modulator consists in a system for the computation of the IDWT and a wavelet demodulator is a system computing the DWT. So, if the architecture of the receiver is completed with a filter, then the signal to be detected is denoised. The proposed denoising method or variants of this method (based on other wavelet transforms) were already used with promising results in other fields of signal processing theory like: speech processing [11], biomedical signal processing [12] and image processing [13-15].

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