Multi-scale MAP Denoising of SAR Images

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Abstract - The SAR images are perturbed by a multiplicative noise called speckle, due to the coherent nature of the scattering phenomenon. The use of speckle reduction filters is necessary to optimize the images exploitation procedures. This paper presents a new speckle reduction method in the wavelets domain using a novel Bayesian-based algorithm, which tends to reduce the speckle, preserving the structural features (like the discontinuities) and textural information of the scene and a new discrete wavelet transform called Diversity Enhanced Discrete Wavelet Transform, DEDWT. The entire class of diversity improved wavelet transforms is characterized and is proved that the averager optimizes the synthesis step for the minimization of the mean square approximation error. A blind speckle-suppression method that performs a non-linear operation on the data, based on a new bishrink filter variant is obtained. Finally, some simulation examples prove the performance of the proposed denoising method. This performance is compared with the results obtained applying state-of-the-art speckle reduction techniques.

I. INTRODUCTION

Some classical estimators, used to denoise SAR images are, [1-2]:

- the Kuan filter (least mean square error linear estimator),
- the Frost filter (Wiener filter adapted to multiplicative noise).

Between the modern estimators can be found:

- the marginal Maximum a Posteriori, MAP, filter (for the maximization of the a posteriori probability), [3],
- the multiresolution MAP filter (a combination between a marginal MAP filter and a multiscale transform), [4].

A new estimators category uses the wavelets theory, [3-6]. The corresponding denoising methods have three steps:

- 1) the computation of the forward wavelet transform, WT, 2) the filtering of the result obtained,
- 3) the computation of the inverse wavelet transform of the result obtained, IWT.

Some comparisons between the application of the classical speckle reduction filters and the application of the denoising methods based on wavelets, in the case of SAR images, were proved the superiority of this last category of methods, [6-7]. Numerous WTs can be used to operate these treatments. The first wavelet transform used in denoising applications was the Discrete Wavelet Transform, DWT. This transform is most commonly used in its maximally decimated form (Mallat's dyadic filter tree), [8-9]. The DWT has two parameters: the mother wavelets, MW and the primary resolution, PR, (number

of iterations). The importance of their selection is highlighted in [1]. The discrete wavelet transform, DWT, realizes a concentration of the energy of the input signal in a small number of coefficients. This concentration enhancement is useful for the reduction of the number of operations in the application considered. For a given different MWs. signal. using different energy concentrations are obtained. So, for a given input image there is a best MW, that realizes the higher energy concentration. Unfortunatelly this MW do not be priori known, esspecialy when the useful image is covered by noise. A relative new DWT, less sensitive to the selection of the MW is the DEDWT, [1]. Numerous filter types can be used in the WT domain: the Wiener filter, [1], that minimizes the mean square estimation error, the hardthresholding filter, [10], that realizes a very simple treatment, the soft-thresholding filter, [4, 7, 10], that minimizes the Min-Max estimation error, the marginal MAP filter, [3], or the bishrink filter, [13]. Some variants of those filters were used in [4,5,7]. In [3] and [6] two special types of MAP filters were used. Unfortunately, these filters have not closed-form input-output relations. Their application requires the use of numerical methods. This paper proposes a new denoising method for SAR images based on the combination of the DEDWT with a variant of bishrink filter. This variant has a closed-form input-output relation. The second section presents the architecture of the proposed denosing system and the third section establishes a statistical analysis of the proposed denoising method. The aim of the forth section is the presentation of some simulation results.

II. THE DENOISING METHOD

The SAR images are perturbed by a multiplicative noise of speckle type,

$$i_r(\tau_1, \tau_2) = i_o(\tau_1, \tau_2) \cdot n_r(\tau_1, \tau_2).$$
 (2.1)

The hypothesis of the independence of the random processes i_0 and n_r , can be adopted, when the speckle is fully developed, [3,6]. The architecture of the denoising system proposed is presented in fig. 1. The WT is applied after the transformation of the multiplicative noise into an additive one. The coefficients of this transform are filtered using the variants of the bishrink filter, proposed in this paper. At the system output, after the computation of the IWT, the logarithm inversion and the mean compensation, the estimation of the useful image, $\hat{i}_0(\tau_1,\tau_2)$, is measured.

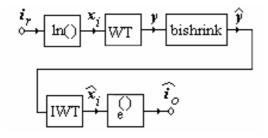


Fig. 1. The architecture of the denoising system.

III. A STATISTICAL ANALYSIS OF THE DENOISING SYSTEM

This analysis will be done block after block, following Fig. 1.

A. The input image

The speckle perturbing a SAR image is a white noise with the energy distributed following a law Gamma:

$$f_{\Gamma}(a) = \frac{L^L}{\Gamma(L)} \cdot a^{L-1} \cdot e^{-La} \quad \text{for} \quad a \ge 0,$$
 (3.1)

where L represents the number of looks. The useful part of the input image has a similar distribution, [1]. The mean of the speckle noise equals 1 and its variance is equal with 1/L.

B. The image at the output of the logarithm computation system

The first block, in fig. 1, transforms the initial probability density function, pdf, into a pdf of $\log -\Gamma$ type:

$$f_{\log-\Gamma}(a) = \frac{L^L}{\Gamma(L)} e^{La} \cdot e^{-Le^a}, \qquad (3.2)$$

with the mean and the variance given by the following relations, [1]:

$$\mu_{\log -\Gamma} = \sum_{k=1}^{L-1} \frac{1}{k} - \gamma - \ln L,$$
(3.3)

where γ is the Euler's number and:

$$\sigma_{\log - \Gamma}^2 = \frac{\pi^2}{6} - \sum_{k=1}^{L-1} \frac{1}{k^2}.$$
 (3.4)

Because the mean in (3.3) is not unitary, the logarithm computation procedure requires a mean correction, [6]. This correction can be realized at the end of the proposed denoising method. The mean of i_r is equal with the mean of i_o . The mean of i_r is estimated before the computation of the logarithm. This value is used after the inversion of the logarithm. The mean of the result obtained is substracted from this result and the mean of i_r is added. So, the mean of the obtained estimation, \hat{i}_o , will be equal with the mean of i_o . Taking into account the strong connection between the DEDWT and the DWT, in the following, a statistical analysis of the DEDWT trough a correspondent analysis of the DWT is

presented.

B. THE DEDWT principle

The DEDWT construction is based on the diversity enhancement principle. Other WTs are also based on this principle. Two examples of such transforms are: the translation invariant discrete wavelet transform, TI DWT, [10] and the direction invariant discrete wavelet transform, DI DWT, [11]. The diversity enhancement is realized computing some slightly different DWTs. Each one is after filtered and the corresponding IDWT is applied, obtaining a different estimation of the useful component of the input image. Finally all these partial results are synthesized and the final result is obtained. Generaly the synthesis is realized by averaging. In section E of this paragraph, the reason to use a runing averager for the synthesis of the partial reslults will be explained. The parameters of the DWT are: the MW, $\psi(t)$, and the number of iterations, M. So the diversity can be enhanced computing for the same image, $x_i[k,l]$, some different discrete wavelet transforms. For each of them a different MW is used in the case of the DEDWT. In the case of the other two WTs, already mentioned (TI DWT and DI DWT) many input images (obtained by the circular translation or rotation of the original input image) are used to obtain the enhancement of the diversity. The DEDWT is a redundant discrete wavelet transform realizing the correspondence between the vector $\mathbf{x}^{T}_{i}[k,l]$ and a three-dimensional matrix \mathbf{y} =**DEDWT**[(k, l), m]. Every column of this matrix represents one of the DWT of the image $x_i^T[k,l]$. Filtering each column with the aid of the bishrink filter variant proposed in this paper, a new three-dimensional matrix, \hat{y} is obtained. Now, the DEDWT can be inverted. Its inverse is called IDEDWT. For every column of the matrix y the corresponding IDWT is computed. A new threedimensional matrix, $\mathbf{E}[(k,l),m]$, is obtained. Every column of this matrix contains an estimation of the image $x_i | k, l |$. Computing the mean of the columns of the matrix $\mathbf{E}[(k,l),m]$ the vector \mathbf{x}_i , is obtained.

C. The DWT statistical analysis

This paragraph reproduces the principal results of the DWT statistical analysis obtained in [12]. In any iteration of the DWT, the lines of the input image (obtained at the end of the previous iteration) are low-pass filtered with a filter having the impulse response m_0 and high-pass filtered with the filter with the impulse response m_1 . Then the lines of the two images obtained at the output of the two filters are decimated with a factor of 2. Next the columns of the two images obtained are low-pass filtered with m_0 and high-pass filtered with m_1 . The columns of those four images are also decimated with a factor of 2. Four new images (representing the result of the current iteration) are obtained. The first one, obtained after two low-pass filtering is named approximation image (or LL image). The others three are named detail images: LH, HL and HH. The LL image represents the input for the next iteration. In the following the coefficients of the

DWT will be noted with $_xD_m^k$, where x represents the image who's DWT is computed, m represents the iteration index and k=1, for the HH image, k=2, for the HL image, k=3, for the LH image and k=4, for the LL image. These coefficients are computed using the following relation:

$$_{x}D_{m}^{k}[n,p] = \langle x(\tau_{1},\tau_{2}), \psi_{m,n,p}^{k}(\tau_{1},\tau_{2}) \rangle,$$
 (3.5)

where the wavelets can be factorized:

$$\psi_{m,n,p}^{k}\left(\tau_{1},\tau_{2}\right) = \alpha_{m,n,p}^{k}\left(\tau_{1}\right) \cdot \beta_{m,n,p}^{k}\left(\tau_{2}\right),\tag{3.6}$$

and the two factors can be computed using the scale function $\phi(\tau)$ and the mother wavelets $\psi(\tau)$ with the aid of the following relations:

$$\alpha_{m,n,p}^{k}(\tau) = \begin{cases} \varphi_{m,n}(\tau), & k = 1,4 \\ \psi_{m,n}(\tau), & k = 2,3 \end{cases}$$

$$\beta_{m,n,p}^{k}(\tau) = \begin{cases} \varphi_{m,n}(\tau), & k = 2,4 \\ \psi_{m,n}(\tau), & k = 1,3 \end{cases}$$
(3.7)

where:

$$\varphi_{m,n}(\tau) = 2^{-\frac{m}{2}} \varphi(2^{-m} \tau - n);$$

$$\psi_{m,n}(\tau) = 2^{-\frac{m}{2}} \psi(2^{-m} \tau - n).$$
(3.8)

1) The pdf of the wavelet coefficients

The pdf of the wavelet coefficients, ${}_{x}D_{m}^{k}$, can be expressed with the aid of the pdf of the input image, x, using the relation, [3]:

$$f_{x}D_{m}^{k}(a) = \frac{M(k)N(k)M_{0}M_{0}}{q_{1}=1} \prod_{r_{1}=1}^{r_{1}=1} q_{2}=1r_{2}=1 \dots$$

$$M_{0}M_{0}$$

$$* * f_{d}(k,q_{1},r_{1},...,q_{m},r_{m},a),$$

$$q_{m}=1r_{m}=1$$

$$(3.9)$$

where:

$$f_{d}(k, q_{1}, r_{1}, ..., q_{m}, r_{m}, a) = G(k, q_{1}, r_{1}, ..., q_{m}, r_{m}) \cdot f_{x}(G(k, q_{1}, r_{1}, ..., q_{m}, r_{m}) a),$$
(3.10)

and:

$$G(k, q_1, r_1, ..., q_m, r_m) = \frac{1}{F(k, q_1, r_1) \prod_{l=0}^{m} m_0[q_l] m_0[r_l]},$$
(3.11)

where:

$$F(k,q_{1},r_{1}) = \begin{cases} m_{0}[q_{1}]m_{0}[r_{1}] & \text{for } k = 4\\ m_{0}[q_{1}]m_{1}[r_{1}] & \text{for } k = 3\\ m_{1}[q_{1}]m_{0}[r_{1}] & \text{for } k = 2\\ m_{1}[q_{1}]m_{1}[r_{1}] & \text{for } k = 1 \end{cases}$$
(3.12)

 M_0 represents the length of the impulse response m_0 , M_1 the length of m_1 and the numbers of the first two groups of convolutions in relation (3.9) are given by the relation (3.13).

In conformity with (3.9), the pdf of the wavelet coefficients is a sequence of convolutions. Hence, the

random variable representing the wavelet coefficients can be written like a sum of independent random variables. So, the central limit theorem can be applied. This is the reason why the pdf of the wavelet coefficients tends asymptotically to a Gaussian, when the number of all convolutions in (3.9) tends to infinity. This number depends on the mother wavelets used and on the number of iterations of the DWT.

$$M(k) = \begin{cases} M_0 & \text{for } k = 4\\ M_0 & \text{for } k = 3\\ M_1 & \text{for } k = 2\\ M_1 & \text{for } k = 1 \end{cases}$$
and
$$N(k) = \begin{cases} M_0 & \text{for } k = 4\\ M_1 & \text{for } k = 3\\ M_0 & \text{for } k = 2\\ M_1 & \text{for } k = 1 \end{cases}$$
(3.13)

For mother wavelets with a long support, this number becomes high very fast (for a small number of iterations). For the first two iterations, heavy-tailed models must be considered. Finer analysis, measuring the distance between the real pdfs and Gaussians, are performed in [3], and [6].

2) The correlation of the wavelet coefficients

The input image, x, represents the sum of the logarithm of the useful image, s, and of the logarithm of the speckle image, n. Because these two random signals are not correlated, the correlation of the wavelet coefficients of the image x, can be written in the following form:

$$\Gamma_{v,D_{m}^{k}} = \Gamma_{v,D_{m}^{k}} + \Gamma_{v,D_{m}^{k}}. \tag{3.14}$$

Taking into account the fact that the input noise is white, with a variance $\sigma_{\log - \Gamma}^2$, the expression of the wavelet coefficients of the input noise image correlation function is, [13]:

$$\Gamma_{nDm}^{k}[n_{1}, p_{1}] = \left(\frac{\pi^{2}}{6} - \sum_{l=1}^{L-1} \frac{1}{l^{2}}\right) \cdot \delta[n_{1}] \cdot \delta[p_{1}]. \tag{3.15}$$

So, the wavelet coefficients sequences of the noise component of the input image are white noise images having the same variance. The first and second order moments of the wavelet coefficients of the input noise image can be computed using the following relations, [12]:

$$E\left\{{}_{n}D_{m}^{k}\left[n_{1}, p_{1}\right]\right\} = \begin{cases} 0, & k = 1, 2, 3\\ 2^{m}\left(\sum_{l=1}^{L-1} \frac{1}{L} - \gamma - \ln L\right), & k = 4 \end{cases}$$
(3.16)

and:

$$\sigma_{n}^{2} D_{m}^{k} = \begin{cases} \frac{\pi^{2}}{6} - \sum_{l=1}^{L-1} \frac{1}{l^{2}}, & k = 1, 2, 3\\ \frac{\pi^{2}}{6} - \sum_{l=1}^{L-1} \frac{1}{l^{2}} - 2^{2m} \left(\sum_{l=1}^{L-1} \frac{1}{l} - \gamma - \ln L\right)^{2} & k = 4. \end{cases}$$
(3.17)

The correlation of the DWT of *s* is given by:

$$\Gamma_{sD_{m}^{k}}[n_{1}, p_{1}] = 2^{2m} \cdot \Gamma_{s}[2^{m}n_{1}, 2^{m}p_{1}],$$
(3.18)

its mean by:

$$E\left\{_{s}D_{m}^{k}[n_{1}, p_{1}]\right\} = \begin{cases} 0, & k = 1, 2, 3\\ 2^{m} \cdot \mu_{s}, & k = 4, \end{cases}$$
(3.19)

and its variance, by:

$$\sigma_{sD_{m}^{k}}^{2} = 2^{2m} \cdot \sigma_{s}^{2}. \tag{3.20}$$

So, the variance of the detail wavelet coefficients sequences, obtained starting from the useful component of the input image, increases when the iteration index increases.

D. The bishrink filter

Let 1y be the considered detail coefficient and 2y its parent (the detail coefficient at the same position but at the following iteration). In fact for a given parent corresponds a zone composed by four child coefficients. This is the reason why every image containing parent coefficients will be over sampled to have the same number of pixels like the corresponding image formed with child coefficients. The statistical parameters of the child coefficients (mean, variance), will be estimated using the parent coefficients having the same position and the neighbor child coefficients, located in a rectangular window, centered on the current child coefficient. It can be written:

$${}^{1}v = {}^{1}s + {}^{1}n \tag{3.21}$$

and:

$$^{2}y = ^{2}s + ^{2}n, (3.22)$$

or, with vectorial notations:

$$y = s + n. ag{3.23}$$

The MAP estimation of s, realized using the observation y, is given by, [13]:

$$\hat{s}(y) = \underset{s}{\arg\max} \left\{ \ln \left(p_n \left(y - s \right) \cdot p_s \left(s \right) \right) \right\}. \tag{3.24}$$

Tacking into account the considerations already made, in the following we will consider that the DEDWT of the noise component is distributed following a Gaussian with a null mean, [3,5,6], the model of the first two iterations of the DEDWT of the useful image will be a Laplace distribution, [13], and for the other iterations this model will be Gaussian. The noise variance estimation can be done using the relation, [14]:

$$\hat{\sigma}_n^2 = \frac{median(|y[n_1, p_1]|)}{0.6475} \quad (n_1, p_1) \in HH.$$
 (3.25)

The useful component DEDWT variance must be estimated locally. In this estimation process the correlation between the values of the same wavelet coefficient computed at two successive scales can be exploited. For this purpose the following relations can be used. First the local mean of the DEDWT of the useful component must be estimated:

$$(3.18) \quad \mu_{\binom{b}{s}}[n_1, p_1] = \frac{1}{(2P+1)^2} \cdot \sum_{(n_1, p_2) \in {}^b W_{m, p_1}} {}^b y[n_2, p_2] \quad (3.26)$$

where b=1 or 2.

Then, the variance of the DEDWT of the input image, contained in the moving window ${}^bW_{n_1,p_1}$, can be computed:

$${}^{b}\hat{\sigma}_{y}^{2}[n_{1}, p_{1}] = \frac{1}{(2P+1)^{2}} \sum_{(n_{2}, p_{2}) \in {}^{b}W_{n_{1}, p_{1}}} {}^{b}y[n_{2}, p_{2}] \cdot -\mu_{(b_{s})}[n_{2}, p_{2}]^{2}$$
(3.27)

Using these values, the useful component DEDWT variance is given by:

$${}^{b}\hat{\sigma}^{2}[n_{1}, p_{1}] = \max\left(0, {}^{b}\hat{\sigma}_{x}^{2}[n_{1}, p_{1}] - \hat{\sigma}_{n}^{2}\right)$$
(3.28)

But, applying the relation (3.20), a theoretical result of this paper, a different estimation of the local variance of the child coefficients can be obtained:

$${}^{1}\hat{\sigma}_{d} = \frac{{}^{2}\hat{\sigma}}{2} \tag{3.29}$$

To profit of these two estimations of the useful component DEDWT local variances, obtained at two successive scales, it can be written:

$$\hat{\hat{\sigma}} = \frac{\hat{\hat{\sigma}} + \frac{\hat{\hat{\sigma}}}{\hat{\sigma}}}{2} \tag{3.30}$$

and the input-output relations of the two variants of the bishrink filter that will be used in the following becomes:

$${}^{1}\hat{s} = \frac{\left(\sqrt{(1x)^{2} + (2x)^{2}} - \frac{\sqrt{3}\hat{\sigma}_{n}^{2}}{1\hat{s}}\right)_{+}}{\sqrt{(1x)^{2} + (2x)^{2}}} \cdot {}^{1}x, \qquad (3.31)$$

for the first two iterations of the DEDWT and:

$$\hat{s} = \frac{\hat{\sigma} \cdot \hat{\sigma}}{\hat{\sigma} \cdot \hat{\sigma}} \cdot 1x$$

$$\hat{s} = \frac{\hat{\sigma} \cdot \hat{\sigma}}{\hat{\sigma} \cdot \hat{\sigma} + \hat{\sigma}_{n}} \cdot 1x$$
(3.32)

for the following iterations.

E. Synthesis by averaging

In figure 2 is presented a denoising system based on a diversity enhanced WT. This system corresponds to the sub-system in Figure 1 composed by the second, the third and the fourth blocks. It is composed by N different denoising systems, each one corresponding to a different DWT. The synthesis of the result can be done with the relation:

$$\widehat{X}_{i} = \sum_{l=1}^{N} \beta_{l} X_{l}$$
 (3.33)

where the coefficients β_i are constrained to satisfy the

perfect reconstruction condition:

$$\sum_{I=1}^{N} \beta_I = 1 \tag{3.34}$$

These coefficients are selected to minimize the mean square error between X and \widehat{X}_i . This error has the expression:

$$MSE = E\left\{ \left(\widehat{X}_i - X\right)^2 \right\} = \sigma_X^2 - 2E\left\{ X\widehat{X}_i \right\} + \sigma_{\widehat{X}_i}^2$$
 (3.35)

This minimization can be done using the Lagrange multipliers method. The associated functional has the expression:

$$F(\beta_1, \beta_2, ..., \beta_N) = \sigma_X^2 - 2E\{X\widehat{X}_i\} + \sigma_{\widehat{X}_i}^2 - \lambda \left(\sum_{l=1}^N \beta_l - 1\right)$$

$$(3.36)$$

The minimization condition is:

$$\frac{\partial F}{\partial \beta_k} = 0 \tag{3.37}$$

equivalent with:

$$-2E\{XX_{k}\} + 2E\left\{\left(\sum_{l=1}^{N} \beta_{l} X_{l}\right) X_{k}\right\} - \lambda = 0.$$
 (3.38)

Using the notations:

$$E\{XX_k\} = i_k , E\{X_lX_k\} = c_{l,k} ,$$

the last relation becomes:

$$\sum_{l=1}^{N} \beta_{l} c_{l,k} = \frac{\lambda}{2} + i_{k} \quad k = \overline{1, N} \quad . \tag{3.39}$$

The partial results, X_l , are estimations of the same useful image, X_l , obtained using a denoising method based on the DWT and a variant of bishrink filter. Because the noise in the DWT domain is white and Gaussian, it can be supposed that the partial results are of the form:

$$X_l = X + \varepsilon_l, \quad l = \overline{1, N}$$
 (3.40)

where ε_l represent N realizations of a white Gaussian noise. This is the reason why it can be written:

$$i_k = \sigma_X^2, \quad c_{l,k} = \begin{cases} \sigma_X^2 + \sigma_{\varepsilon}^2, & \text{for } l = k \\ \sigma_X^2, & \text{in rest} \end{cases}$$
 (3.41)

and the relation (3.43) becomes the system of equations described in relation (3.42). Using the constraint from (3.34) the last system of equations takes the simplified form in relation (3.43), with the solutions in relation (3.44). Or taking into account once more the constraint (3.34) the solutions become those described in relation (3.45). So, the best synthesis solution for the minimization of the approximation mean square error is the averager.

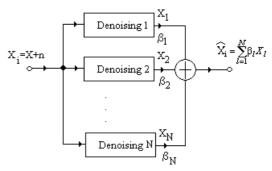


Fig. 2. An equivalent representation of the denoising system

$$\begin{cases} \beta_{1}\left(\sigma_{X}^{2}+\sigma_{\varepsilon}^{2}\right)+\beta_{2}\sigma_{X}^{2}+...+\beta_{N}\sigma_{X}^{2}=\frac{\lambda}{2}+\sigma_{X}^{2}\\ \beta_{1}\sigma_{X}^{2}+\beta_{2}\left(\sigma_{X}^{2}+\sigma_{\varepsilon}^{2}\right)+...+\beta_{N}\sigma_{X}^{2}=\frac{\lambda}{2}+\sigma_{X}^{2}\\ & \cdot\\ & \cdot\\ \beta_{1}\sigma_{X}^{2}+\beta_{2}\sigma_{X}^{2}+...+\beta_{N}\left(\sigma_{X}^{2}+\sigma_{\varepsilon}^{2}\right)=\frac{\lambda}{2}+\sigma_{X}^{2} \end{cases}$$

$$(3.42)$$

$$\sigma_X^2 + \beta_k \sigma_\varepsilon^2 = \frac{\lambda}{2} + \sigma_X^2, \quad k = \overline{1, N}$$
 (3.43)

$$\beta_1 = \beta_2 = \dots = \beta_N = \frac{\lambda}{2\sigma_s^2}$$
, (3.44)

$$\beta_1 = \beta_2 = \dots = \beta_N = \frac{1}{N}$$
 , (3.45)

IV. SIMULATION RESULTS

A comparison between the mean square errors obtained applying different speckle reduction methods is presented in the following table. The image Lena was perturbed with a multiplicative Rayleigh noise, obtaining the input image. This image was treated using: a running averager, a median filter, the Lee's filter, the Kuan's filter, the Gamma filter, the Frost's filter and the proposed denoising method. The first parameter of each filter represents its window size. The second one is specific for each filter. These quantities were selected to minimize the mean square error of the result for the considered image. All the parameters of the proposed denoising method are selected automatically. Finally, a real image, where the speckle is fully developed, was treated. Analyzing the result it can be observed that the noise was practically entirely removed and the fact that the details of the useful part of the input image (textures or edges) were not affected by the treatment proposed. An objective measure of the performance of a denoising method for the homogeneous regions of a SAR image is the enhancement of the equivalent number of looks, ENL. The ENL is defined with the following relation:

TABLE I A Comparison Of Seven Speckle Reduction Methods

WIETHODS	
Noisy Image	3635
Averager 5	571.7
Median 7	569.8
Lee 7-5	807.5
Kuan 9-5.5	732.8
Gamma 5-1.5	595.5
Frost 5-1	566
Proposed	287.4

$$ENL = \left(\frac{\text{mean}}{\text{standard deviation}}\right)^2. \tag{4.1}$$

Considering the same homogenous region in the original and result images, we have obtained for the input ENL a value of 15 and for the output ENL a value of 62.

V. CONCLUSION

A new denosing method for the processing of SAR images was proposed. It is based on the use of the DEDWT and of two original variants of bishrink filter. This method permits to retain coefficients produced by significant structures present in the useful part of the input image and suppress those produced by the speckle noise. A complete statistical analysis of this method was reported. Its hypotheses and results were confirmed by simulations. From $\log - \Gamma$ assumptions for the pdf of the reflectivity and the speckle we have expressed the pdf of the wavelet coefficients. For the first two iterations of the DEDWT, those pdfs corresponds to heavy-tailed distributions. We have approximated those distributions with a Laplace pdf. For the following iterations these pdfs can be considered Gaussians. Using these hypotheses a MAP filter with closed-form input output relation, that takes into account the interscale dependency of the wavelet coefficients was derived. Its parameters are locally estimated. Because two different estimations of the local variance of the child wavelet details of the useful component of the input image are at our disposal, they are combined to increase the precision of this estimation. An important theoretical result reported in this paper is the proof of the averager optimality for the synthesis of the result that minimizes the mean square approximation error. An adaptive mean correction method was also proposed. We evaluated the results on both synthetic data and real SAR images, validating the theoretical hypotheses used. Further improvements could be obtained if a better WT and a 3D bishrink filter would be used. The latter avenue is currently under investigation and results will be reported soon.

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